Analysis to Effect of Non-linear Axial Force on the MEMS Clamped-clamped Switch Beam

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Abstract: The effect of non-linear axial force, which is caused by both temperature change of environment and the length change of the beam, on the behavior of the switch is quite heavy because of the larger slenderness ratio of the MEMS clamped-clamped switch beams. By using the 1D beam model, the behaviors of electrostatic MEMS clamped-clamped switch beams with both the temperature change and the length change for three different stages were studied. In the paper, the governing equation of the elastic curve of the beam with the effect of non-linear axial force in the beam was given. A three-fold method of bisection was used to solve the unknown axial force in the governing equation, applied voltage and one boundary condition of the beam. On obtaining the axial force, the applied voltage and the boundary condition, the deflections at any cross-section could be obtained easily by using the numerical method to the ordinary differential equation. Incorporating the numerical method to ordinary differential equations with some optimization method, pull-in voltage and the corresponding deflection could be distinguished and obtained automatically on a computer with the specifically designed program. In examples, the entire mechanical behavior of the MEMS clamped-clamped switch beam over the operational voltage ranges was investigated and the effects of the axial force on pull-in voltage at several temperature changes with different geometry dimensions of the beam were studied. Copyright © 2014 IFSA Publishing, S. L.

Keywords: Temperature change, Axial force, Pull-in voltage, Clamped-clamped switch beam.

1. Introduction

Since the first micro-electromechanical systems (MEMS) switch was reported by Petersen in 1979 [1], the MEMS switch has been one of the most important devices in MEMS system because of some advantages, such as low consumption, low driving power, relative ease of fabrication, light in weight, high reliability, and so on. From the mechanics point of view, models of MEMS switches are divided into three classes, simple lumped element models with a single degree of freedom [2-4], 1D beam models [5-9], and 3D models [10-12]. Simple lumped element models of MEMS switches result in easy calculations but fail to capture details of the deformation behavior of MEMS switches. 3D models lead to a detailed and accurate prediction about the deformation and stress behavior of MEMS switches, but simulations are expensive in time and computation. 1D beam models are an intermediate level of complexity, and can give useful results, which are validated with 3D simulations [6]. Incorporating the numerical method to ordinary differential equations with a two-fold method of bisection, analyses to three possible static configurations, which are called floating, pinned and
flat configuration respectively, of a MEMS cantilever switch are conducted in [5]. Using the finite difference approximations of derivatives and the boundary conditions, both the static behavior and dynamic stability of cantilever beam electrostatic actuators beyond pull-in are studied in [6]. Incorporating the numerical method in [5] with some optimization method, the pull-in voltages of MEMS clamped-clamped switches can be distinguished and obtained automatically on a computer with the specifically designed program [7]. The effect of temperature (without the length change of the beam) on the behavior for the three stages and on the pull-in voltages of MEMS clamped-clamped switches are analyzed in [8] because of the larger slenderness ratio of micro beams. The pull-in behavior of the multilayered micro beam type MEMS switches are studied with the nonlinear finite difference method in [9]. In this paper, the governing equation of the elastic curve of the beam with the effect of both the temperature change and the length change in the beam is first given. A three-fold method of bisection is then used to solve the unknown axial force in the governing equation, applied voltage and one boundary condition of the beam for the three different stages. Lastly, incorporating the numerical method to the ordinary differential equation with some optimization method, pull-in voltage and the corresponding deflection could be distinguished and obtained automatically on a computer with the specifically designed program. In examples, the entire mechanical behavior of the MEMS clamped-clamped switch beam over the operational voltage ranges is investigated and the effects of the axial force on pull-in voltage at several temperature changes with different geometry dimensions of the beam, such as the length of the beam, the depth of the beam and the initial gap, are studied. The analysis in the paper will be helpful to understand the effect of the temperature change and the axial force on the behavior related MEMS switches.

2. Differential Equation of the Elastic Curve of the Beam with Temperature Change and Axial Force

A typical capacitive RF MEMS switch is mainly composed by a clamped-clamped beam [13], the transmission line, and an insulator or dielectric layer fabricated on the transmission line, the side view of which is shown in Fig. 1. The mechanics model of the switch is shown in Fig. 2. The beam and the insulator length are both \( l \). The depth and width of rectangle cross-section of the beam are \( t \) and \( B \) respectively. The initial gap between the beam and the insulator is \( G_0 \). The depth and width of the insulator layer are \( t_1 \) and \( B \) respectively.

With the original point of the horizontal \( x \)-axial being at the center of the beam, the electrostatic force per unit length is

\[
q(x) = \frac{1}{2} \varepsilon_0 \varepsilon_r \frac{BU^2}{2(G - v(x))^2}, \quad |x| \leq \frac{l}{2},
\]

where \( \varepsilon_0 \) is the vacuum permittivity; \( \varepsilon_r \) is the relative permittivity of the dielectric between the beam and the insulator, if in air it is approximately equal to 1.0; \( U \) is applied voltage; \( v(x) \) is the deflection at \( x \); \( G = G_0 + t_1/\varepsilon_1 \) is the initial equivalent gap between the two electrodes; \( \varepsilon_1 \) is the relative permittivity of the insulator.

The deformation of the clamped-clamped beam subjected to the electrostatic force with the temperature change and axial force as shown in Fig. 2 can also be divided into three stages [8] in the entire operational range as follows.

1) First stage: At the stage the clamped-clamped beam is only subjected to electrostatic force without any constrain at the center, and at the end of this stage the deflection at the central point reaches \( G_0 \). Suppose the clamped-clamped beam is a symmetrical one. As a result the half of the beam can be seen a cantilever beam, in which the free end of the cantilever beam with zero angle of rotation is
the central point of the clamped-clamped beam. Taking the right half of the clamped-clamped beam with the temperature change and axial force for example, the deformation and the loads at the first stage are shown in Fig. 3;

2) Second stage: At the stage the beam is subjected to a concentrated force at the central point besides the electrostatic force with a constraint at the center, and at the end of the second stage the moment at the central point is zero. The deformation and the loads of the cantilever beam at the second stage are shown in Fig. 4;

3) Third stage: At the stage the beam is subjected to both the electrostatic force and a concentrated force with a given displacement and a zero angle of rotation over the central part of the beam. The deformation and the loads of the cantilever beam at the third stage are shown in Fig. 5.

The first two stages represent boundary value problems while the third stage represents a free boundary problem [14].

The elastic curve of the beam in Fig. 3 and Fig. 4 is governed by the fourth-order non-linear differential equation,

\[ D \frac{d^4v}{dx^4} - F_N \frac{d^2v}{dx^2} + F_T \frac{d^2v}{dx^2} = q(x) \]  

(2)

where \( D = \frac{Bt^3 E}{12(1-\mu^2)} \), \( E \) is the Young’s modulus or modulus of elasticity of the beam, \( \mu \) is the Poisson’s ratio, \( F_T = \frac{\alpha T E B t}{1-\mu^2} \) is the axial force resulted by the temperature change \( T \) [8], \( \alpha \) is the coefficient of linear thermal expansion, and \( F_N \) is the axial force caused by the length change of the beam, which can be calculated with [9, 15]

\[ F_N = \frac{EBt}{(1-\mu^2)} \int_0^\frac{l}{2} \left( \frac{dv}{dx} \right)^2 dx \]  

(4)

3. Solution to the Non-linear Differential Equation

In generally, if the applied voltage, the temperature change \( T \), the axial force \( F_N \) and four boundary conditions (the deflection and its first three derivatives) at the point \( x = 0 \) or at the point \( x = \Delta \) of the beam shown in Figs. 3, 4 or 5 are known, the deflection and the first three derivatives at any point then can easily be obtained numerically from the differential governing equation (2) by using some numerical technique such as the adaptive Runge-Kutta-Fehlberg algorithm [16] which can automatically adjust the step size and stop the calculations when a certain prescribed error criterion, such as \( 10^{-6} \), is met. Due to the different boundary conditions at the three different stages, the unknowns are also different. The procedure of the numerical solution to equation (2) or
equation (3) for the three stages is stated respectively as follows.

### 3.1. Solution at the First Stage

At the stage the boundary conditions are

\[
\begin{align*}
\nu(0) &= \nu_0 \\
\frac{dv(0)}{dx} &= \nu'(0) = 0 \\
\frac{d^2\nu(0)}{dx^2} &= \nu''(0) = -\frac{M_0}{D}, \\
\frac{d^3\nu(0)}{dx^3} &= \nu'''(0) = 0
\end{align*}
\]

where \(M_0\) is the bending moment at \(x = 0\).

When \(\nu_0\) is given, the unknown applied voltage \(U\), bending moment \(M_0\) and axial force \(F_N\) can be determined with a three-fold method of bisection based on the shooting method under the conditions \(v(l/2) = v'(l/2) = 0\) and equation (4). The flow diagram for the numerical procedure is shown in Fig. 6.

### 3.2. Solution at the Second Stage

At the stage the boundary conditions are

\[
\begin{align*}
\nu(0) &= G_0 \\
\frac{dv(0)}{dx} &= \nu'(0) = 0 \\
\frac{d^2\nu(0)}{dx^2} &= \nu''(0) = -\frac{M_0}{D}, \\
\frac{d^3\nu(0)}{dx^3} &= \nu'''(0) = -\frac{P_0}{D}
\end{align*}
\]

where \(P_0\) is the shearing force at \(x = 0\).

When \(\nu''(0)\) is given, the unknown applied voltage \(U\), shearing force \(P_0\) and axial force \(F_N\) can also be determined with the three-fold method of bisection under the conditions \(v(l/2) = v'(l/2) = 0\) and equation (4). The flow diagram for the numerical procedure is similar to that shown in Fig. 6 (except \(M_0\) for \(P_0\) and equation (5) for equation (6)).

### 3.3. Solution at the Third Stage

Because of \(\nu(x) = G_0\) in the range \([0, \Delta]\) at this stage, we should only calculate the deformations in the range \([\Delta, l/2]\), and boundary conditions at \(x = \Delta\) are

\[
\begin{align*}
\nu(\Delta) &= G_0 \\
\frac{dv(\Delta)}{dx} &= \frac{d^2\nu(\Delta)}{dx^2} = 0 \\
\frac{d^3\nu(\Delta)}{dx^3} &= -\frac{P_\Delta}{D}
\end{align*}
\]

where \(P_\Delta\) is the shearing force at \(x = \Delta\).

When \(\Delta\) is given, the unknown applied voltage \(U\), shearing force \(P_\Delta\) and axial force \(F_N\) can be determined with the three-fold method of bisection under the conditions \(v(l/2) = v'(l/2) = 0\) and equation (4). The flow diagram for the numerical procedure is also similar to that shown in Fig. 6 (except \(M_0\) for \(P_\Delta\), equation (2) for equation (3) and equation (5) for equation (7)).

### 4. Examples

#### 4.1. Entire Deformation Analysis of a MEMS Clamped-clamped Switch Beam

In this example, an entire analysis for the three different stages will be performed. The material and geometrical parameters [8] in the example are listed

![Fig. 6. Flow diagram for the solution to the three unknowns.](image-url)
in Table 1. Fig. 7 shows the applied voltage versus the center deflection of the beam at the first stage for three different temperature changes. The bending moment $M_0$ versus the center deflection $v_0$ of the beam is shown in Fig. 8 (the bending moment $M_0$ versus the center deflection $v_0$ of the beam without the temperature change or the axial force is omitted because of a small difference). The axial force $F_N$ (the difference is quite small at different temperature changes) versus the center deflection $v_0$ of the beam is shown in Fig. 9. The pull-in voltages $U_{PI}$ are about 31.90 V at temperature change -20 °C, 25.33 V at temperature change 0 °C, 16.14 V at temperature change 20 °C, and 24.49 V without temperature change or axial force, respectively.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Quantity</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>Modulus of elasticity</td>
<td>80.0 GPa</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Poisson’s ratio</td>
<td>0.45</td>
</tr>
<tr>
<td>$\varepsilon_0$</td>
<td>Vacuum permittivity</td>
<td>8.854 pF·m^{-1}</td>
</tr>
<tr>
<td>$\varepsilon_r$</td>
<td>Relative permittivity of the dielectric layer</td>
<td>9.72</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>the coefficient of linear thermal expansion</td>
<td>3.8×10^{-6} /°C</td>
</tr>
<tr>
<td>$\varepsilon_r$</td>
<td>Relative permittivity of air</td>
<td>1.0</td>
</tr>
<tr>
<td>$B$</td>
<td>Width of the beam</td>
<td>90 μm</td>
</tr>
<tr>
<td>$t$</td>
<td>Depth of the beam</td>
<td>2.0 μm</td>
</tr>
<tr>
<td>$l$</td>
<td>Length of the beam</td>
<td>280 μm</td>
</tr>
<tr>
<td>$t_1$</td>
<td>Thickness of the insulator</td>
<td>0.15 μm</td>
</tr>
<tr>
<td>$G_0$</td>
<td>The initial gap</td>
<td>1.5 μm</td>
</tr>
</tbody>
</table>

Fig. 7. Applied voltage $U$ versus center deflection $v_0$.

Fig. 8. The bending moment $M_0$ versus center deflection $v_0$.

Fig. 9. The axial force $F_N$ versus center deflection $v_0$.

Fig. 10 shows the applied voltage versus the center bending moment of the beam at the second stage for three different temperature changes. The axial force $F_N$ versus the center bending moment of the beam for three different temperature changes is shown in Fig. 11.

Fig. 10. Applied voltage $U$ versus bending moment $M_0$. 
Fig. 11. The axial force $F_N$ versus bending moment $M_0$.

Fig. 12 shows the applied voltage versus the contact length at the third stage for three different temperature changes (the applied voltage versus the contact length $\Delta$ without the temperature change or the axial force is omitted because of a small difference). The pull-in voltage 31.90 V at temperature change -20 °C is corresponding to a contact length 80.85 μm, 25.33 V at temperature change 0 °C to a contact length 74.06 μm, 16.14 V at temperature change 20 °C to a contact length 58.46 μm, and 24.49 V without temperature change or axial force to 73.90 μm, respectively.

Fig. 13 shows the applied voltage $U$ versus contact length $\Delta$.

Fig. 14 show the axial force (the difference is quite small for different temperature changes) versus the contact length at the third stage.

4.2. Effect of Some Parameters on the Pull-in Voltage

The effects of some different geometry dimensions of the beam with the axial force on pull-in voltage at several temperature changes will be studied in the example. Note please that in the example, all parameters are the same as the Table 1 except for the parameter under discussion.

The relation between the length of the beam and the pull-in voltage is shown in Fig. 14 (the critical length, below which the beam is stable, is about 411.0 μm at temperature change 20 °C with the axial force $F_N$, and the critical length is only 345.6 μm at temperature change 20 °C without the axial force $F_N$). Fig. 14 shows that the effect of temperature change on pull-in voltage increases with the increase in length.

The relation between the depth of the beam and the pull-in voltage is shown in Fig. 15 (the critical depth, below which the beam is unstable, is about 0.97 μm at temperature change 20 °C with the axial force $F_N$, and the critical depth is 1.62 μm at temperature change 20 °C without the axial force $F_N$). Fig. 15 shows that the effect of temperature change on pull-in voltage decreases with the increase in depth.
The relation between the initial gap and the pull-in voltage is shown in Fig. 16. Fig. 16 shows that the effect of temperature change on pull-in voltage increases with the increase in initial gap.

![Figure 16](image16.png)

**Fig. 16.** Pull-in voltage versus initial gap.

The relation between the temperature change of environment and the pull-in voltage is shown in Fig. 17, and the difference between the two pull-in voltages with the axial force $F_N$ and without the axial force is shown in Fig. 18, which shows the effect of axial force on pull-in voltage increases with the increase in temperature change.

![Figure 17](image17.png)

**Fig. 17.** Pull-in voltage versus temperature change.

![Figure 18](image18.png)

**Fig. 18.** Difference between the two pull-in voltages versus temperature change.

5. Comparison with Some Other Results

For clamped-clamped beams without the temperature change or insulator, some pull-in voltages predicted by employing the 2D distributed model, 3D MEMCAD simulation [17], and the finite difference method [9] are compared with those obtained using the presented method to demonstrate the feasibility. The geometric and material properties [17] are: modulus of elasticity $E$ is 169 GPa, Poisson's ratio is 0.06, the width of the beam $B$ is 50 μm, the depth of the beam $t$ is 3 μm, the initial gap $G_0$ is 1 μm, and the vacuum permittivity is $8.854 \text{ pF} \cdot \text{m}^{-1}$. Four clamped-clamped beam models with two different lengths of the beam were simulated and compared in Table 2. The four models agree to within 3.0%.

<table>
<thead>
<tr>
<th>Model</th>
<th>$L=250$ μm</th>
<th>$L=350$ μm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pull-in voltage (2D)</td>
<td>39.50 V</td>
<td>20.20 V</td>
</tr>
<tr>
<td>Pull-in voltage (MEMCAD)</td>
<td>40.10 V</td>
<td>20.30 V</td>
</tr>
<tr>
<td>Pull-in voltage (difference method)</td>
<td>39.13 V</td>
<td>20.36 V</td>
</tr>
<tr>
<td>Pull-in voltage (Proposed method)</td>
<td>39.58 V</td>
<td>20.20 V</td>
</tr>
</tbody>
</table>

6. Conclusions

By using the 1D beam model, the behaviors of electrostatic MEMS clamped-clamped switch beams with both the temperature change and the length change for three different stages were studied in the paper. A three-fold method of bisection was suggested to solve the unknown axial force in the governing equation, applied voltage and one boundary condition of the beam. On obtaining the axial force, the applied voltage and the boundary condition, the deflections at any cross-section could be obtained easily by using the numerical method to the ordinary differential equation. A specifically designed program incorporating the numerical method to the ordinary differential equation with some optimization method is used to obtain the pull-in voltage and the corresponding deflection automatically on a computer. In examples, the entire mechanical behavior of the MEMS clamped-clamped switch beam over the operational voltage ranges was investigated and the effects of some different geometry dimensions of the beam with the axial force on pull-in voltage at several temperature changes were studied. It can be seen from the results that the effect of temperature change on pull-in voltage increases with the increase in length or in initial gap, decreases with the increase in depth,
and the effect of axial force on pull-in voltage increases with the increase in temperature change. The analysis in the paper will be helpful to understand the effect of the temperature change and the axial force on the behavior related MEMS switches.

References


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