Complete Shaking Moment Balancing of N-Bar Planar Mechanisms

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Abstract: A new method of shaking moment balancing of force balanced linkages is presented in the paper. The shaking moment balancing is gained based on the angular momentum principle. This technique can be used for balancing of n-bar planar linkages which have a revolute input kinetic pair. The method transforms the balance problem into a profile curve synthesis of a planar cam. Therefore, it is simple and convenient to use. As a numerical example, the method is used to entirely balance shaking moment of a six-bar mechanism.

Keywords: Complete balancing, Method of shaking moment balancing, Planar linkages, Shaking moment, Angular momentum, Profile curve synthesis.

1. Introduction

During machines run, all components produce inertia forces and moment. The forces and moments may cause noise, wear, vibration and make the input crank wind up. In 1971, Berkof and Lowen dealt with the shaking moment optimization [1]. In 1973, Berkof represented a complete force and moment balance of an inline four-bar linkage [2]. In 1982, Begci gave the method to completely balancing shaking force and moment of linkages using balance idler loop [3]. In 1985, Huang presented the method of synthesis of a dyad to balance inertia input torque [4]. In 1991, Kochev dealt with the theory of torque, shaking force and shaking moment balancing of planar linkages [5]. In 2001, Arakelian and Dahan dealt with the partial shaking moment balancing of fully balanced linkages [6]. In 2012, Static balancing of a four-bar linkage and its cognates was presented [7].

As mention before, balance of shaking moment of planar linkages is usually accomplished by means of appropriately chosen counterweights or attached motive components. The method of choosing appropriate counterweights is usually used for four-bar linkages. The method of attaching motive components has been taken to balance shaking moment of four-bar and six-bar linkages. But it usually makes the mechanism become very complex.

In this paper, a new method of synthesizing a planar groove cam is taken to completely balance shaking moment of n-bar planar linkages.

2. The Principle of Balancing

It is well known that when external loading is not included, the shaking moment of a linkage with one-DOF can be described as the time rate change of the total angular momentum with respect to the reference point o. It is

\[ M_o = -\dot{H}_o, \] (1)
where $\mathbf{M}_a$ is the shaking momentum with respect to point $o$; $\dot{H}_a$ represents the time rate of change of angular momentum of all the moving links, also taken with respect to point $o$. The angular momentum of \(n\)-bar linkage is

$$H_o = \sum_{i=1}^{n-1} \left[ m_i (X_i \dot{Y}_i - Y_i \dot{X}_i) + I_{ii} \dot{\phi}_i \right], \quad (2)$$

where $X_i$ and $Y_i$ are the coordinates of the mass center of the $i^{th}$ link with respect to the point $o$; $m_i$ is the mass of the $i^{th}$ link; $I_{ii}$ is the moment of inertia of the $i^{th}$ link with respect to its mass center; $\dot{\phi}_i$ is the angular velocity of the $i^{th}$ link.

Shaking force balancing of \(n\)-bar linkage can be accomplished by using linearly independent vectors to force the center of mass of the linkage to be stationary. Then its angular momentum $H_o$ has nothing to do with reference point $o$. From equation (1) we know if the angular momentum equals a constant, the shaking mordent equals zero. Therefore, we may add a rotary link to the original linkage, making the fluctuation of the angular momentum offset the fluctuation of that of the original linkage, which is

$$H_o + H_r = C_o, \quad (3)$$

where $H_r$ is the angular momentum of the added link; $C_o$ is the appropriately selected constant. When the mass center of the rotary link (or roll) coincides with its rotating pivot, $H_r$ can be expressed as follows:

$$H_r = I_r \dot{\phi}_r, \quad (4)$$

where $I_r$ is the moment of inertia of the roll; $\dot{\phi}_r$ is the required angular velocity of the roll. Substituting equation (4) into (3), we have

$$I_r \dot{\phi}_r = C_o - I_o \dot{\phi}_1, \quad (5)$$

where $I_o = H_o / \dot{\phi}_1$.

Rearranging equation (5), we have

$$\frac{\dot{\phi}_r}{\dot{\phi}_1} = \frac{C - I_o}{I_r} \quad (6)$$

Then, integrating equation (6), we have

$$\phi_r - \phi_{r_0} = \int_{\phi_1}^{\phi_r} \frac{C - I_o}{I_r} d\phi_1 \quad (7)$$

$I_o$ can not be chosen freely after construct $C$ is selected. If we want the input crank to rotate $360^\circ$, while the roll also rot $360^\circ$, then

$$I_r = C - \frac{1}{2\pi} \int_0^{2\pi} I_o (\phi) d\phi_1 \quad (8)$$

Suppose

$$G = \frac{\dot{\phi}_r}{\dot{\phi}_1} = \frac{C - I_o}{I_r} \quad (9)$$

Therefore, we have

$$G_{\text{max}} = \frac{C - I_o_{\text{min}}}{I_r} \quad (10)$$

$$G_{\text{min}} = \frac{C - I_o_{\text{max}}}{I_r} \quad (11)$$

Solving the system of simultaneous equations, yields

$$C = \frac{1}{G_{\text{max}} - G_{\text{min}}} (G_{\text{max}} I_o_{\text{max}} - G_{\text{min}} I_o_{\text{min}}) \quad (12)$$

Constant $C$ can be calculated by equation (12) after selecting the appropriate $G_{\text{max}}$ and $G_{\text{min}}$.

3. Adding Motive Components for Balancing Shaking Moment

For a mechanism to be balanced, the balancing components added are shown in Fig. 1. The gear 1 is fixed; the gear 2, to which a planar groove cam called as balancing cam is fixed, is connected to the crank AB of the original mechanism with a joint at point B. The gear 2, engaged with the gear 1, is a planet gear and the number of its teeth is equal to that of the gear 1. The mass centre of the component 2 is at the point B. The added component AE is a roller, driven by the cam 2, for balancing the shaking moment. The mass centre of AE is at the rotating centre A while the component 2 is moving; it makes the roller AE rotate at variable angular velocity. If the profile of the cam is properly designed, the expected angular velocity of the roller can be obtained. So that, the angular momentum of the roller may be equal to $H_r$ in equation (3).
From equation (7) we know that $\phi_1$ is known for given $\phi_0$. Because the balancing cam rotates at constant angular velocity with respect to the crank AB, the relative rotating angle between the roller and the cam may be determined. We choose coordinate system X*BY* which is fixed to the crank AB as shown in Fig. 1. So

$$\delta = \phi_1 - \phi_{10}$$  

(13)

$$\phi = \phi_1 - \phi_0 - \phi_o$$,  

(14)

where $\delta$ is the relative rotating angle of the cam with respect to the crank AB; $\phi_0$ and $\phi$ are the original angular position and angular displacement, at some time, of the crank AB with respect to the roller AE respectively.

Substituting equation (7) into equation (14), yields

$$\phi = \phi_1 - \left[ \phi_{10} + \int_{\delta_o}^{\delta} \left( \frac{C - L_o}{L_r} \right) d\phi_1 \right] - \phi_o$$ .  

(15)

After choosing the original angular position of the roller AE, $\phi_{10}$, and that of the crank AB, $\phi_{10}$, from equation (14):

$$\phi_o = \phi_{10} - \phi_{10}$$ .  

(16)

From equation (13) and (15), a series of values of $\phi$ and $\delta$ can be obtained for given a series of values of $\phi_1$. Coordinate values of the theoretical outline curve of the cam in system X*BY* are

$$X^* = L'_1 \sin \delta - R \sin(\delta + \phi + \phi_o)$$

$$Y^* = L'_1 \cos \delta - R \cos(\delta + \phi + \phi_o)$$ .  

(17)

where $L'_1$ is the length of the crank AB; $R$ is the length of the roller AE.

4. A Numerical Example

To take a six-bar linkage as an example. The linkage is shown in Fig. 2. It is force-balanced by applying the method of linearly independent vectors. The dimensions and parameters of it are listed in Table 1. The linkage is assumed to run at constant input speed of 120 r.p.m. The curve of the angular momentum is shown in Fig. 3. The chosen and calculated parameters for the synthesis of the balancing cam are listed in Table 2. It is necessary to point out that the value of $\theta'_1$ in Table 2 is arbitrary. The outline curve of the cam is shown in Fig. 4 (only 36 points are shown). The mechanism added the balancing components are shown in Fig. 5.
The balancing cam not only can completely balance shaking moment, but also has many advantages such as being easy to design and realize in structure.

References


Table 1. Dimensions and parameters of the six-bar linkage.

<table>
<thead>
<tr>
<th>L1</th>
<th>L2</th>
<th>L3</th>
<th>L30</th>
<th>L4</th>
<th>L5</th>
<th>L6</th>
<th>L60</th>
<th>R1</th>
<th>R2</th>
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<td>0.02</td>
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<td>0.09</td>
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<td>0.06</td>
<td>0.08</td>
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<td>0.1</td>
<td>0.06</td>
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<td>R4</td>
<td>R5</td>
<td>θ1</td>
<td>θ2</td>
<td>θ3</td>
<td>θ4</td>
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<td>-π/6</td>
<td>-π/6</td>
<td>π/2</td>
<td>π/12</td>
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<tr>
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<td>m3</td>
<td>m4</td>
<td>m5</td>
<td>I1</td>
<td>I2</td>
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<td>I4</td>
<td>I5</td>
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<td>30</td>
<td>80</td>
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<td>0.14</td>
<td>0.16</td>
<td>0.07</td>
<td>0.07</td>
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Table 2. Parameters used for the synthesis of the cam.

<table>
<thead>
<tr>
<th>C (kg.m²)</th>
<th>L (kg.m²)</th>
<th>R (m)</th>
<th>L' (m)</th>
<th>Θ1̇</th>
<th>Ψp</th>
<th>Ψrp</th>
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<td>5.1552</td>
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<td>0.1</td>
<td>0</td>
<td>154°</td>
<td>140°</td>
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</table>

5. Conclusion

From theoretical analyzing and numerical calculating, we know that the technique presented in this paper for balancing shaking moment is practical.

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