Stochastic Resonance Phenomenon of Two-coupled Duffing Oscillator and its Application on Weak Signal Detection

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Abstract: By taking two-coupled duffing oscillator system as the research object, the study found under the synergistic effect of weak periodic signal and noise, the two-coupled duffing oscillator system has typical stochastic resonance and even could achieve better output of stochastic resonance than single duffing oscillator. Based on this phenomenon, this paper proposes a new method to detect the weak periodic signal through the stochastic resonance of two-coupled duffing oscillator. The experimental study indicates that we could achieve excellent detection effect by using the method we mentioned in this paper at low SNR (signal-to-noise ratios), especially, it has excellent filter effect for filtering and shaping of weak square signals and has potential values in the application of digital communications. Copyright © 2014 IFSA Publishing, S. L.

Keywords: Two-coupled duffing oscillator, Stochastic resonance, Weak signal detection.

1. Introduction

The detection technique for weak signal has always been the research hotspot for scholars [1-5]. It has been widely used in communication, vibration measurement, biomedical signal, fault diagnosis of power system etc. In recent years, the rapid development of nonlinear science provides a new idea for the detection of weak signal and can achieve better effect than traditional detection methods. Currently, the method to detect weak signal by using nonlinear system can mainly be divided into two types: One is represented by scholars as Wong Guanyu, Li Yue [6-9]. They proposed to use the perturbation of weak to cause the chaotic oscillator’s conversion between chaotic state and large scale periodic state. That is phase transition, to detect the signal through phase transition. This method used the features of nonlinear system as ultra sensitive to small signal and has strong immunity to noise. The system it mainly used is duffing oscillator system including single duffing oscillator and coupled duffing oscillator. Another method [10-12] has significantly enhanced the output SNR of the system by making use of the synergistic effect between nonlinear system with weak periodic signals and noise to produce the same frequency as weak signal but larger range’s periodic oscillation that is stochastic resonance. This method doesn’t restrain the noise but used the theory to transform noise energy to signal energy, at this moment, the noise has a positive effect on the detection of signals. Those two methods have simulated the rapid development of method of nonlinear detection significantly.

The previous studies about stochastic resonance mainly concentrated on the one-dimensional
Langevin system, but relatively fewer researched on duffing oscillator system, for which research have mostly involved the method of moment and bifurcation theory to explain the phenomenon as stochastic resonance of single duffing oscillator system but mainly used the single duffing oscillator system. In this paper, during the research on the detection used the features of nonlinear system as ultra sensitive to small signal and has strong immunity to noise, the author have found that annular coupling duffing oscillator’s periodic phase are more stable and has stronger noise immunity at the definite system parameters [13, 14].

Therefore, on the basis of the previous researches, the paper studied the stochastic resonance features of coupled duffing oscillator by making use of the amplification coefficient of spectral power and SNR and discussed the correlation between system parameters (damping ratio, coupling coefficient, the frequency of measured signal) and stochastic resonance, obtained the change rule of stochastic resonance under different parameters. And then compared the anti-noise performance and stochastic resonance of two-coupled duffing oscillator and single duffing oscillator, analyzed the reasons why two-coupled duffing oscillator can achieve better output effect of stochastic resonance and has a comparison of sinusoidal signal and square signal detection. The results shows, when we detected weak sinusoidal signal by two-coupled duffing oscillator, the output signal are magnified more significant and achieved higher SNR, and two-coupled duffing oscillator has more qualified features of filter shaping for weak square signal, which has a higher quality output signal and has potential values in the application of digital communications.

2. The Characteristics of Stochastic Resonance of Two-coupled Duffing Oscillator

The equation Holmes’ duffing oscillator system driven by weak periodic signal and noises as follows:

\[ x + k x - x + x^3 = s(t) + n(t) \]  

where \( K \) is the damping ratio, \(-x + x^3\) is the nonlinear restoring force, \( s(t) = A \cos(2\pi f_0 t) \) is the measured periodic signals with which amplitude is \( A \), frequency is \( f_0 \), \( n(t) = \sqrt{2D\xi(t)} \) is the Gaussian White Noise whose intensity is \( D \) and in which \( \xi(t) \) is the Gaussian White Noise whose mean value is 0 and variance is 1.

When there is no extrinsic motivation, that is \( s(t) = 0 \), \( n(t) = 0 \) and the potential function of the system is \( U(x) = -\frac{1}{2} x^2 + \frac{1}{4} x^4 \). It represents a nonlinear dual potential field that contains two equilibrium points \((x = \pm 1)\) and one unstable equilibrium point \((x = 0)\) and constructed two potential well that separated by two potential barrier, as shown in Fig. 1.

![Fig. 1. Potential function of duffing oscillator system.](image)

When the measured periodic signal and noise are added into the duffing oscillator system, due to the synergistic effect of signal, noise and nonlinear systems, in some intensity area, the noise will transfer partial energy to signal, thus the signal will enhanced. The system output will form a large-scale jump motion within the two potential wells that is synchronous with signals. Once the output SNR maximized, the stochastic resonance state will emerge.

2.1. Two-coupled Duffing Oscillator Model

This study shows that coupled duffing oscillator system has more sufficient dynamics behaviors than the same kind of single duffing oscillator. The synchronization and control process have already been widely used in many fields. This paper, a two-coupled duffing oscillator system has constructed by two single duffing oscillators through linear coupling.

\[
\begin{align*}
\dot{x}_1 + k x_1 - x_1 + x_1^3 + \xi_1(x_1 - x_2) &= s(t) + n(t) \\
\dot{x}_2 + k x_2 - x_2 + x_2^3 + \xi_2(x_2 - x_1) &= s(t) + n(t)
\end{align*}
\]

where, \( \xi_1(x_1 - x_2) \) and \( \xi_2(x_2 - x_1) \) are the linear coupling terms among oscillators, \( \xi \) is the coupling coefficient represents the coupling strengths. In the system, the oscillators controlled and interacted with each other through coupling terms and always kept consistent in coupling strengths. The structure is as shown in Fig. 2.

Meanwhile, to study the phenomenon of stochastic resonance of two-coupled duffing oscillator system, the paper has introduced the amplification coefficients of spectral power and SNR to have quantitative description and analysis.

SNR has used the generally definition in signal processing field
Fig. 2. Structure chart of two-coupled duffing oscillator.

\[
\text{SNR} = \frac{S(f_0)}{P - S(f_0)},
\]

where \(S(f_0)\) is the signal power; \(P\) is the total output power of the system, including the noise and signal power. \(P - S(f_0)\) is the noise power. Supposed the output of the coupled system is \(x(t)\), then carries on the FFT transformation on \(x(t)\), we will obtained a spectrum distribution.

\[
X(k) = \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}
\]

Therefore, the signal’s unilateral spectral amplitude value of the system’s output is \(X(k_o)\), where \(k_o = \frac{f_o}{f_s}\). \(f_s\) is the sample frequency, \(N\) is the signal length. According to the formula (3), we can get the output SNR:

\[
\text{SNR} = \frac{2|X(k_o)|^2}{\sum |X(k)|^2 - 2|X(k_o)|^2}
\]

The definition of amplification coefficient of spectral power is:

\[
\eta = \frac{|X(k_o)|^2}{|Z(k_o)|^2},
\]

where \(Z(k_o)\) and \(X(k_o)\) are the signal’s unilateral spectral amplitude value at \(f_0\) in the input and output frequency spectrum.

In essence, the amplification coefficient of spectral power and SNR are stochastic variable, thus, the curve chart of this indexes in this paper are the arithmetic mean from multiple calculation and then have a curve fitting.

2.2. The Phenomenon of Stochastic Resonance

Firstly, analyzed the process of noise transformed to signal in two-coupled duffing oscillator system from the perspective of time domain and frequency domain. Taking the two-coupled duffing oscillator system in formula (2) as an example, supposed the damping ratio \(k=0.7\), \(\xi=0.3\), \(A=0.35\), \(f_0=0.01\) Hz, noise strengths \(D=0.2\), sample frequency \(f_s=5\) Hz, solved numerically with four stages of Runge-Kutta methods. The results are as shown is Fig. 3.

Fig. 3. The two-coupled duffing stochastic resonance. (a) The waveform of the measured signal and noise; (b) The spectrum of the measured signal and noise; (c) The system output waveforms; (d) the system output spectrum; (e) The synchronization errors between two oscillators.

The amplitude of the measured signal \(s\) is small and almost being buried in the Gaussian white noise \(n\), as shown in Fig. 3(a). From the input spectrum, we found that, the peak values of the measured signal are small, as shown in Fig. 3(b). After the measured signal and noise are added into the two-coupled duffing oscillator system, the two-coupled duffing oscillator system will produce similar square output signals contains small noise, and its period is basically the same as the measured signals, as shown in Fig. 3(c).

From the frequency spectrum graph of the system output 3(d), we can found it more obvious that the peak value of the spectra in the output frequency spectrum at the signal frequency has improved significantly while the high frequency component in
noise energy declined dramatically, this means parts of the high-frequency component has transferred to low-frequency signal, achieved the transformation between noise energy and signal energy and improved the quality of the output signal. Therefore, the two-coupled duffing oscillator system has the features of Lorentz distribution as the noise energy centralized to the low frequency area.

Besides, through the analysis on the synchronous errors of the trajectory of two oscillators, they are always in synchronization state. That is the outputs of two oscillators are consistent, as shown in Fig. 3(e). Therefore, we can analyze it by any one of oscillators.

In order to study the change rule of amplification coefficient of spectral power and output SNR of the coupled system as the changes of noise intensity under the condition that other parameters the same, but changes the input noise intensity D, we can obtained the change curves as shown in Fig. 4.

According to the change graph of amplification coefficient of spectral power, we could know that, the output signal’s spectral amplitude value increases as the noise intensity enhance. When D=0.5, it reaches its maximum peak that is formant, and then the output signal’s spectral amplitude value decreases gradually as the noise intensity increase but decreases slowly, as shown in Fig. 4(a).

From the change graph of the system output SNR, we can know that the formant shift right, the SNR reaches its peak when D=0.7, as shown in Fig. 4(b). It is because when D=0.5, though the output signal’s spectral amplitude value is the maximum value, the output noise also relatively large, so there is no maximum value of the SNR. While it reaches the peak as D=0.7. Meanwhile, after the formant emerged, the disordered interference of noise become dominant which caused the component of noise in system output increased rapidly and SNR declined dramatically, thus the formant is relatively narrow.

![Fig. 4. The change rule of system output. (a) Amplification coefficient of spectral power η; (b) Output SNR.](image)

The above two indexes reflect the change rule of system output from different perspective, though the result is slightly different, partial maximization phenomenon appears that is to say, the coupled duffing oscillator system has the same typical features of stochastic resonance.

### 2.3. The Influence of Damping Ratio, Signal Frequency and Coupling Coefficient

Through the study we found, the stochastic resonance phenomenon of two-coupled duffing oscillator system not only related to noise intensity but also the parameter of coupled system. Therefore, this section mainly analyzes the influence of damping ratio, signal frequency and coupling coefficient to the features of stochastic resonance.

Taking the two-coupled duffing oscillator system in format (2) for example, supposed the parameters of the system $\zeta=0.3, A=0.35, f_0=0.01$ Hz, with different damping ratio, the curve chart of output SNR and amplification coefficient of spectral power as the noise intensity changes, as shown in Fig. 5.

![Fig. 5. The change rule of system output with different damping ratio. (a) Amplification coefficient of spectral power η; (b) Output SNR.](image)

From the Fig. 5(a), with the damping ratio $k$ increases, the formant get higher gradually. When $k>0.7$, the formant become wider but peak value keeps the same and the position of the peak shifts to the right, this demonstrated that the positive function of ranges of noise intensity became wider. From the Fig. 5(b), with the damping ratio $k$ increases, the formant get higher rapidly, the position of the peak shifts to the right, more importantly, SNR>1, this demonstrated that through the influence of coupled system, the signal energy is larger than noise energy, which is the symbol of the positive function of stochastic resonance system in signal processing.

These results suggest, with large damping ratio, the high-frequency component of noise tends to transfer to low frequency area to strengthen the energy of output signal. Thus, when the input noise intensity D increases, one can improve the noise energy transfer to signal by increasing the damping ratio $k$ and then the coupled system can will produce stochastic resonance in a large noise environment. Thus, the weak signal in the context with strong noise intensity can be detected.

Without changing other parameters, supposed the system’s parameters $k=0.7, \zeta=0.3, A=0.35, f_0$ was 0.03, 0.09, 0.15, 0.21, the curve chart of output SNR and amplification coefficient of spectral power as the noise intensity changes, as shown in Fig. 6.
From Fig. 6, we could know, with the increase of input-signal frequency $f_0$, the formant in the curve chart of SNR and amplification coefficient of spectral power declines rapidly. When $f_0$>0.15, the formant disappear, it means at this moment, the coupled system can not produce stochastic resonance. The reason for this phenomenon is the increase of this input-signal frequency caused the response of the coupled system lags behind gradually. Therefore, a signal with larger amplitude is needed to form transition and then stimulate the phenomenon of stochastic resonance.

Therefore, just as the single duffing oscillator system, the two-coupled duffing oscillator system applies to the detection of low-frequency signal. If the frequency of the measured signal is relatively higher, then we need to transfer the frequency of measured signal into low frequency with variable metric method.

Without changing other parameters, supposed the system’s parameters $k=0.7$, $A=0.35$, $f_0=0.01$ Hz, coupling coefficient $\xi$ is 0.1, 0.5, 1, the curve chart of output SNR and amplification coefficient of spectral power as the noise intensity changes, as shown in Fig. 7.

From Fig. 7, we could know, if $\xi=0.1$, the output signal peak value is relatively smaller, but the formant still appears in the curve chart of SNR and amplification coefficient of spectral power. The study shows, the coupling coefficient has some effect on the features of stochastic resonance. With the coupling coefficient increases, the resonance peak increases gradually. It means the quality of output signal in stochastic resonance is improved.

The above study showed that the two-coupled duffing oscillator system also has the typical stochastic resonance and the change rule of stochastic resonance is related to damping ratio, coupling coefficient and the frequency of measured signal.

3. The Algorithm of Difference Distribution

This section used the fourth-order Runge-Kutta method to solve the duffing oscillator and discovered the change rule between the system solution $x$ and $y$ and proposed a method to distinguish the system’s state with the difference distribution of system solution thus to reduce the calculation amount of the state discrimination.

3.1. Experimental Research

The last section we have studied the features of stochastic resonance in two-coupled duffing oscillator system, this section has compared it with single duffing oscillator system, analyzed their anti-noise performance, and features of stochastic resonance and the effect for the detection of sinusoidal signal and square signal.

3.1. Comparison of Anti-noise Performance

In order to compare the anti-noise performance of those two oscillators, using the formula (7) to get the trajectory of oscillator’s displacement from the original position under the effect of the Gaussian noise and take this as the standard to measure the anti-noise performance.

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N-1}[x_{o}(t)-x_{n}(t)]^2}$$

where $N=8000$, $x_{o}(t)$ represents the movement locus without noise, $x_{n}(t)$ represents the movement locus with noise. The result is as shown in Fig. 8.

As can be seen from Fig. 8, if the SNR in single duffing oscillator system is relatively high and the system has strong inhibition, with the noise intensity increase, the movement locus fluctuated greatly, in some noise intensity; the track displacement from the original position is relatively serious. While the movement locus of two-coupled duffing oscillator is relatively stable. With the noise intensity increases, there is no big fluctuation, when the SNR is -20 dB, the track displacement is relatively small.
Fig. 8. The degree of deviation of two different oscillators in different noise intensity.

These results suggest, the movement locus of two-coupled duffing oscillator is relatively stable and its inhibition to noise is stronger than single duffing oscillator system.

3.2. Comparison of Stochastic Resonance

In order to compare the features of stochastic resonance of those two oscillators, by taking formula (1) and (2), supposed the system parameters $k=0.7$, $\xi=0.3$, $f_0=0.01$ Hz, $A=0.35$, we could get the curve chart of those two oscillators’ output SNR and amplification coefficient of spectral power with the change of the noise intensity, as shown in Fig. 9.

Fig. 9. The change rule of two oscillators in different noise intensity. (a) Amplification coefficient of spectral power $\eta$; (b) output SNR.

As can be seen from Fig. 9(a), both two oscillators can produce stochastic resonance and the amplification of the output signal in stochastic resonance is roughly equivalent. It means the noise energy which transfers to signal energy is basically the same. But as we can seen from the curve chart Fig. 9(b) of SNR, though the amplification of two oscillators’ output signal are basically the same, the coupled system has stronger ability for noise inhibition, more high-frequency noise are weakened. Thus, the coupled system can obtain higher SNR in stochastic resonance. Combined with the research of 2.3 (The influence of damping ratio), for the stochastic resonance of two-coupled duffing oscillator, part of the high frequency noise that controlled by damping ratio was transferred to output signal while another part of high frequency noise was restrained by coupled system. Therefore, we could obtain higher output SNR.

3.3. The Comparison of the Detection Performance of Weak Signal

From above study, it would be concluded that anti-noise performance and stochastic resonance effect is two-coupled duffing oscillator was superior to single duffing oscillator. We detected the different weak signals by those two oscillators and compared their effect.

1) Detection of sinusoidal signal. The frequency of the measured sinusoidal signal $f_0=0.01$ Hz, amplitude value $A=0.35$. At the same time, adds Gaussian White Noise with the intensity $D=0.6$ to test the influence of noise to system output, as shown in Fig. 10(a). The frequency spectrum as shown in Fig. 10(b), we can see that the peak value of the measured signal $f_0=0.01$ Hz is small.

Fig. 10. The detection result of sinusoidal signal. (a) The waveform of the measured sinusoidal signal and noise; (b) The frequency spectrum of the measured sinusoidal signal and noise; (c) The output frequency spectrum of single duffing oscillator; (d) the output frequency spectrum of two-coupled duffing oscillator.

Detected the sinusoidal signal with formula (1) and (2), supposed the system parameter $k=0.7$, $\xi=0.3$, then the output frequency spectrum of two oscillators’ system will be as what the Fig. 10(c), (d) shown. We can see that, the output signal of single duffing oscillator system was magnified while the component of high frequency noise declined. This demonstrated that some part of the noise energy was transferred to signal, while the output signal of two-
coupled duffing oscillator system was magnified more significantly. The component of high frequency noise becomes smaller. It means the quality of the output signal is improved.

2) Detection of square signal. The period of the measured square signal $T=100$ s, amplitude value $A=0.35$, as shown in Fig. 11(a). Add Gaussian White Noise with the intensity $D=0.5$, as shown in Fig. 11(b). The square signal almost overwhelmed by noise. And it becomes difficult to identify the signal accurately and the square signal has a broad frequency band. The effect of filtering is not significant.

The detection formula is the same as the above. The system output as shown in Fig. 11(c), (d). We can see that, the system output of the single duffing oscillator has kept the basic profile of the waveform, but there are too many small burrs which will caused a misjudgment while the system output waveform of two-coupled duffing oscillator are relatively more smooth, which has shows the waveform’s profile further. It shows that it has the features of stronger ability in filtering and shaping the waveform.

The study shows, the output of both oscillators has the feature of transient transition that is the same as the temporal characteristics of square signals. Thus, we can filter and shape the square signal by duffing oscillator. This has potential applications in digital communication.

Fig. 11. The detection result of square signal. (a) The waveform of the measured square signal; (b) The frequency spectrum of the measured square signal; (c) The output frequency spectrum of single duffing oscillator; (d) The output frequency spectrum of two-coupled duffing oscillator.

In conclusion, compared with the single duffing oscillator, two-coupled duffing oscillator system has stronger anti-noise performance in the detection of weak sinusoidal signal and square signal and has better detection effect. The quality of two-coupled duffing oscillator’s output SNR is higher than the single duffing oscillator.

4. Conclusions

1) The study found that, under some certain parameters, the two-coupled duffing oscillator system can also produce a periodic oscillation with the same frequency as input measured signal but larger amplitude that is the stochastic resonance. The change rule of the stochastic resonance is related to system parameter (damping ratio, the coupling coefficient, the frequency of the measured signal); 2) Through the comparison of the anti-noise performance and stochastic resonance characteristics of two-coupled duffing oscillator and single duffing oscillator. We found that the two-coupled duffing oscillator has better output characteristics in stochastic resonance and can obtain higher output SNR.

3) By making full use of the excellent characteristics of the two-coupled duffing oscillator in filtering and shaping of square signal, we can detect the weak square signals from the background with strong noise intensity.

This paper has studied the two-coupled duffing oscillator’s stochastic resonance phenomenon and detection performance to weak signal and accumulated some experience for the further study of stochastic resonance phenomenon in high-dimensional systems or complex network system.

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