ISSN 1726-5479

SENSORS & TRANSDUCERS

vol. 119
8 /10

Sensors and Transducers Applications

International Frequency Sensor Association Publishing

Sensors & Transducers

Volume 119, Issue 8, August 2010

www.sensorsportal.com

ISSN 1726-5479

Sensors & Transducers Journal (ISSN 1726-5479) is a peer review international journal published monthly online by International Frequency Sensor Association (IFSA).
Available in electronic and on CD.

Sensors & Transducers Journal

Contents

Volume 119
Issue 8
August 2010

www.sensorsportal.com

ISSN 1726-5479

Research Articles

Authors are encouraged to submit article in MS Word (doc) and Acrobat (pdf) formats by e-mail: editor@sensorsportal.com
Please visit journal's webpage with preparation instructions: http://www.sensorsportal.com/HTML/DIGEST/Submition.htm

International Frequency Sensor Association (IFSA).

Sensors & Transducers Journal, Vol. 119, Issue 8, August 2010, pp. 151-161

Sensors & Transducers

ISSN 1726-5479

http://www.sensorsportal.com

Identification and Speed Control of PMDC Motor Using Time Moments

Prasanta SARKAR, Sagarika PAL, Swadhin Sambit DAS
Department of Electrical Engineering, National Institute of Technical Teachers' Training and Research, Kolkata (under MHRD, Govt. of India),
Block-FC, Sector-III, Salt Lake City, Kolkata-700106, India
E-mail: sarkarprasant@yahoo.com, spal922@yahoo.co.in, swadhinsambit@rediffmail.com

Received: 23 April 2010 /Accepted: 17 August 2010 /Published: 31 August 2010

Abstract: In this paper identification and speed control of Permanent Magnet DC Motor is presented. A combination of output error identification technique and method of time moments is used for identification and speed control. The time constraint is expressed using equality between the time moments of the closed loop system and that of a reference model. The reference model is developed from the classical time, frequency and complex domain specifications which guarantee both stability and performance in a model matching framework. Both the simulation and experimental validation show the usefulness of the proposed work. *Copyright © 2010 IFSA.*

Keywords: PMDC motor, Time moments, Output error identification technique.

1. Introduction

This paper proposes a scheme for identification and speed control of permanent magnet DC motor, in which output error estimation technique [1] is used to identify the parameters of the PMDC motor. Subsequently, the method of time moment [2] is used for speed control. Many techniques have been proposed for the parameters estimation and speed control of DC motors. Time moment, which is a traditional tool extensively used in reduced order modeling literature, has been successfully used by many authors in parameter identification and control system design. Partial time moments is used in [3] for parameter identification, in which identification calculations were achieved on sets of sampled measurements that provide values of the physical parameters based on signal representative of the process behavior. Parameter identification utilizing the concept of sensitivity points was used in [4], in which steepest descent estimation method was used. The efficacy of any identification experiment relies on how accurately the identified model is close to the actual one, an accurate model of an actual geared PMDC motor and the difference in the respective results between the ideal theoretical equation and model with an enhanced equation to an actual measurement was presented in [5]. In [6], a method of poles position identification of axial flux permanent magnet machine for starting the permanent magnet motor in the demanded direction was presented. A high performance autotuned speed control loop for a permanent magnet motor is studied [7], in which optimal degree and tuning of the RST digital controller and the identification conditions were analyzed. A cost effective speed control method based on RLS algorithm in a model reference adaptive framework was proposed in [8], in which the speed regulator was designed on feedback of armature voltage and current of PMDC motor.

The fundamental requirement for a closed loop system is to ensure guaranteed stability and performance and to verify time performances which can be characterized by the settling time and the damping ratio of the step response and to reject some particular disturbances[9, 10]. This can be achieved after framing a reference model which embodies time, frequency and complex domain specifications of the overall control system with the augmented controller. The reference model is thus chosen to be in the form $M(s) = \frac{1}{(1+sT)^2}$ [11, 12]. The closed loop control system is shown below in Fig. 1:

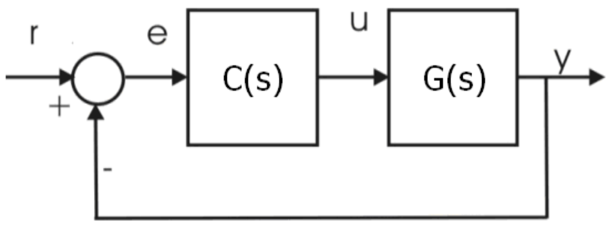

Fig. 1. Closed loop system.

Where C(s) represents the controller and G(s) represents the motor. The controller has to be determined in order that the closed loop transfer function $\mathrm{T(s)}$ approximates its reference model that is ideally expressed by the equality:

$$\mathrm{T(s) = (1 + G(s)C(s))^{-1}\, G(s)C(s) = M_{ref}(s)} \tag{1}$$

This corresponds to

$$\mathrm{G(s)C(s)(1 - T_{ref}(s)) = M_{ref}(s)} \tag{2}$$

Using state model of $C(s)$

$$\mathrm{G(S)(C_c(sI - A_c)^{-1}B_c + D_c)(1 - M_{ref}(s)) = M_{ref}(s)} \tag{3}$$

The time characteristics of the reference model are described by time moments [13-15]. This technique can be applied to a large variety of systems such as electrical motors. Thus, this application is dedicated to the design of PI controller in order to control a PMDC motor. The synthesis of PI controllers is not a specific problem. It can be considered as a particular case of a wider problem, *i.e.* the synthesis of fixed structure controllers. The fixed structure controller is directly designed. The stability and performances of a closed loop are necessary requirements for the choice of the design technique. The PMDC motor parameters are estimated using Output Error Identification technique. The parameters of the output error model structure are estimated using prediction error [16] where the data object is a set of input-output data.

The paper is organized as follows: in section 2, a brief introduction of PMDC motor is presented. In section 3, a brief account of time moment is discussed with particular reference in identification and controller design. In section 4, estimation and control of the proposed PMDC motor using time moments is presented. In section 5, controller implementation is proposed. Finally, in section 6, validity of the proposed scheme is discussed.

2. PMDC Motor Model

Fig. 2 represents a PMDC Motor. As the field excitation is constant, the armature controller depends on armature voltage only. This scheme shows an electrical part which represents an armature and a mechanical part represented by T_L and J.

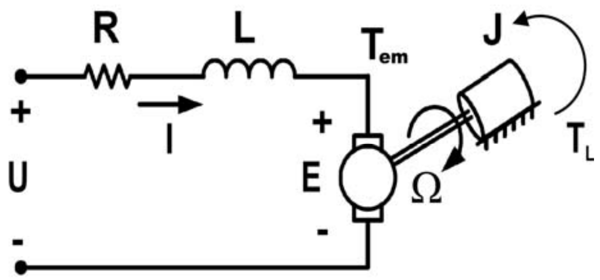

Fig. 2. PMDC motor.

Where: I is the armature current (A), U is the armature voltage (V), R is the armature resistance (Ω), L is the armature inductance (H), K is Torque and back electromagnetic constant ($Nm.A^{-1}$), Ω is the rotor angular speed ($rad.\sec^{-1}$), T_{em} is the electromagnetic torque ($N.m$), T_L is the total load torque ($N.m$) and J is the rotor inertia ($Kg.m^2$).

The electrical and mechanical equations describing this system can be written as follows [17, 18] with the following assumptions that include losses torque in load torque and by neglecting viscous friction constant:

$$U = RI + L\dot{I} + E \tag{4}$$

$$J\dot{\Omega} = T_{em} - T_L \tag{5}$$

with

$$E = K\Omega \tag{6}$$

$$T_{em} = KI \tag{7}$$

The control input is armature voltage U; the total load T_L is the disturbing input. The two state variables are armature current I and angular speed Ω. Then the previous equations lead to the state space model of DC motor:

$$\begin{bmatrix} \dot{I} \\ \dot{\Omega} \end{bmatrix} = \begin{pmatrix} -\frac{R}{L} & -\frac{K}{L} \\ \frac{K}{J} & 0 \end{pmatrix} \begin{bmatrix} I \\ \Omega \end{bmatrix} + \begin{pmatrix} \frac{1}{L} & 0 \\ 0 & -\frac{1}{J} \end{pmatrix} \begin{bmatrix} U \\ T_L \end{bmatrix} \quad (8)$$

We are interested by the angular speed in order to perform a speed regulator. So Ω is considered as the output of the system and U is the input. Considering only these two system variables, the transfer function of the DC motor is:

$$H(s) = \frac{\Omega(s)}{U(s)} = \frac{1}{K} \frac{1}{(1 + \frac{RJ}{K^2} s + \frac{LJ}{K^2} s^2)} \quad (9)$$

The two time constants are defined as:

$$\tau_e = \frac{L}{R} \text{ - electrical time constant} \quad (10)$$

$$\tau_{em} = \frac{RJ}{K^2} \text{ - electromechanical time constant} \quad (11)$$

So,

$$H(s) = \frac{\Omega(s)}{U(s)} = \frac{1}{K} \frac{1}{(1 + \tau_{em} s + \tau_{em} \tau_e s^2)} \quad (12)$$

This transfer function corresponds to the following state model:

$$x_{sys} = \begin{bmatrix} \dot{\Omega} \\ \Omega \end{bmatrix}; A_{sys} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{\tau_{em}\tau_e} & -\frac{1}{\tau_e} \end{bmatrix} \quad (13)$$

$$B_{sys} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; C_{sys} = \begin{bmatrix} \frac{1}{K\tau_{em}\tau_e} & 0 \end{bmatrix}; D_{sys} = 0 \quad (14)$$

3. Time Moments

3.1. Definition of Moments

Consider a Linear Time Invariant system whose transfer function L(s) is analytic in the right half complex plane, and characterized by its impulse response $l(t)$, such as:

$$L(s) = \int_0^\infty l(t) e^{-st} dt \quad (15)$$

Expanding e^{-st} in Taylor series about $s=0$ yields:

$$L(s)=\int_0^\infty \sum_{n=0}^{\infty}(-1)(s)^n \frac{t^n}{n!} l(t)dt \tag{16}$$

$$L(s)=\sum_{n=0}^{\infty}(-1)^n (s)^n M_{l,n} \tag{17}$$

where

$$M_{l,n}=\int_o^n \frac{t^n}{n!} l(t)dt \tag{18}$$

$M_{l,n}$ is the n^{th} order time moment of $l(t)$

- The 0^{th} order time moment $M_{l,0}$ represents the area of the impulse response. It is also equal to the static gain of the transfer function.
- The 1^{st} order time moment $M_{l,1}$ represents mean time of the impulse response. It also characterizes the response time of the system.
- The 2^{nd} order time moment $M_{l,2}$ characterizes the dispersion of the impulse response around its mean time.

The first three time moments $M_{l,o}, M_{l,1}$ *and* $M_{l,2}$ are sufficient to describe the time characteristics of a system.

3.2. Moments and State Space Control

Consider a system $L(s)$ whose state model is:

$$L(s)=C_l(sI-A_l)^{-1}B_l+D_l, \tag{19}$$

where the matrices A_l, B_l, C_l and D_l are known and are of appropriate dimension. Expanding $L(s)$ in Taylor series about $s=0$, we get

$$L(s)=-C_l(\sum_{n=1}^{\infty}(s)^n (A_l)^{-(n+1)})B_l+(-C_l(A_l)^{-1})B_l+D_l) \tag{20}$$

Identification of each term yields:

$$M_{l,0}=-C_l(A_l)^{-1}B_l+D_l \tag{21}$$

$$M_{l.n}=(-1)^{n+1}C_l(A_l)^{-(n+1)}B_l;\ \text{n=1} \ldots \infty \tag{22}$$

4. Estimation and Control using Time Moments

4.1. Algorithm for Estimation and Control

The algorithmic flow chart shown below in Fig. 3. shows the steps used for identification and control of the PMDC motor:

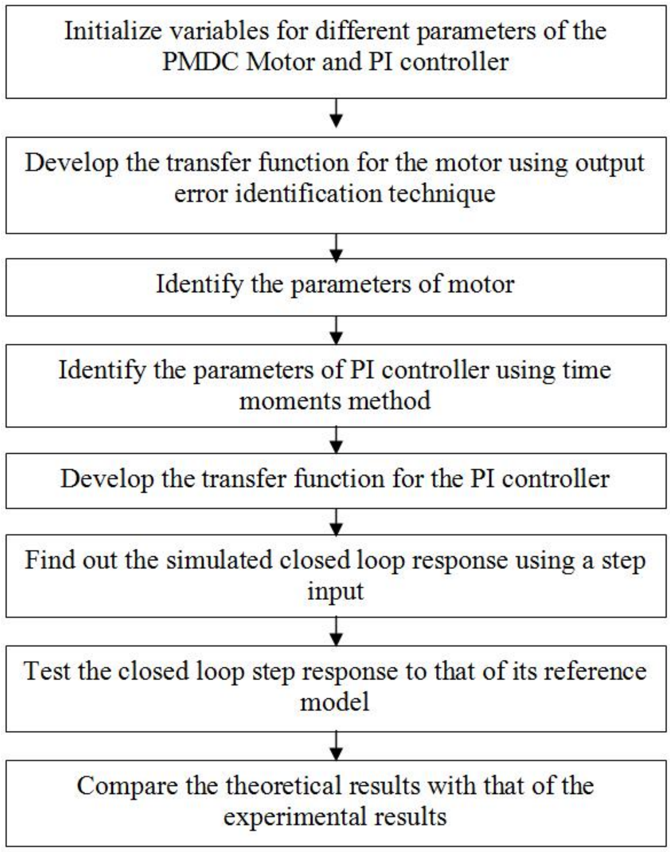

Fig. 3. Algorithmic flowchart.

A 12 V PMDC motor is used. Firstly, a set readings showing input voltage of the motor versus output speed were taken. Fig. 4 below shows the same.

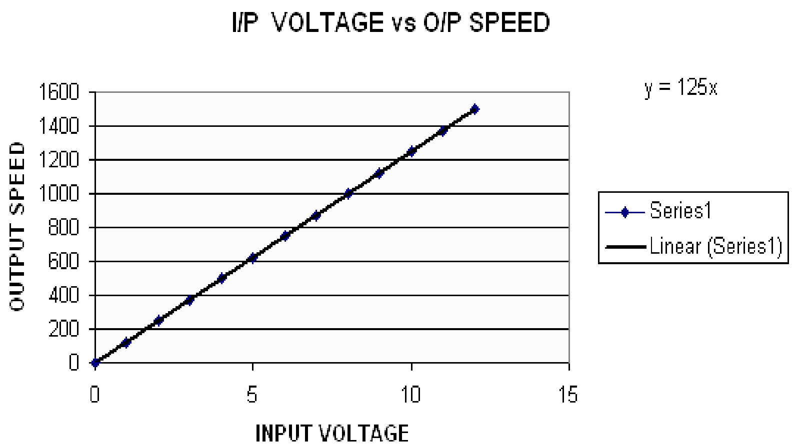

Fig. 4. Input voltage vs. output speed relationship.

Thus, its transfer function is identified using Output Error Identification technique through MATLAB program. The transfer function of the motor was found out to be

$$H(s) = \frac{0.02057}{s^2 + 0.6206\, s + 0.6092} \tag{23}$$

The three parameters of the motor were thus identified as

$$K = 29.61;\ \tau_{em} = 1.02;\ \tau_e = 1.61 \tag{24}$$

And the corresponding state model of the motor was:

$$A_{sys} = \begin{bmatrix} 0 & 1 \\ -0.61 & -0.62 \end{bmatrix};\ B_{sys} = \begin{bmatrix} 0 \\ 1 \end{bmatrix};\ C_{sys} = \begin{bmatrix} 0.02 & 0 \end{bmatrix} \tag{25}$$

The reference model is chosen to be a second order system with unity gain given by the transfer function $T_{ref}(s) = \frac{1}{4s^2 + 4s + 1}$.

The chosen controller is of PI type. As the total number of moments (N+1) used in the quadratic criterion has to be at least equal to the number of controller parameters, the number of time moments used in time criterion is given below (Table 1):

Table.1. Choice of number of moments for usual controllers.

Controller	**(N+1)**
PI	2
PID	3

The simulated output in MATLAB is presented as below in Fig. 5:

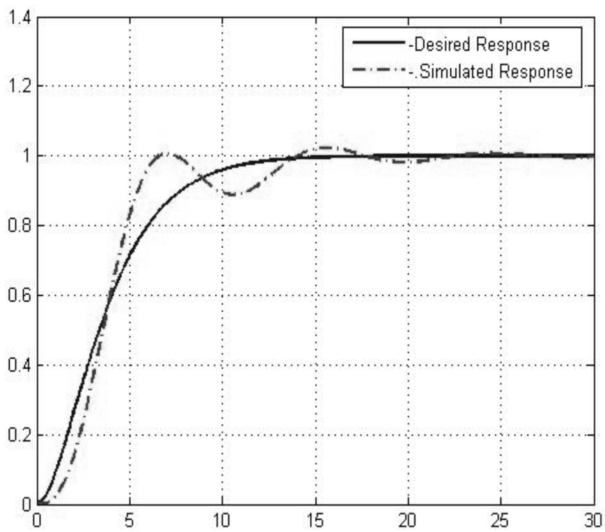

Fig. 5. Closed loop response for PI controller (theoretical).

Subsequently, the transfer function in case of a PI controller

$$K(s) = 0.13855 + \frac{7.404}{s} \tag{26}$$

5. Controller Implementation

The theoretical analysis was confirmed with the help of a practical circuit [19], [20] where a PI controller is implemented for the DC motor. The block diagram for the circuit implementation is shown below in Fig. 6.

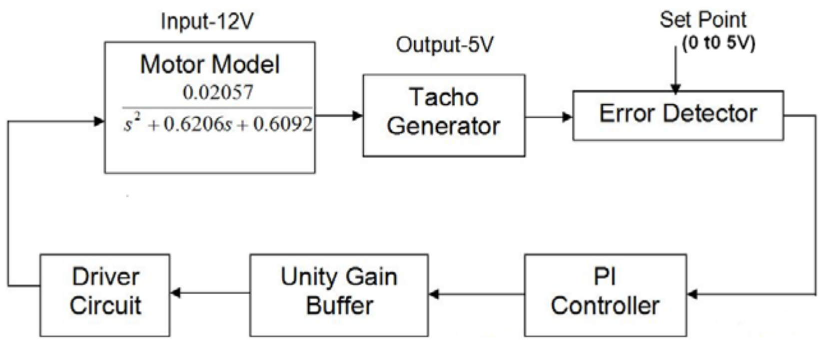

Fig. 6. Block diagram of the closed loop circuit.

The detailed circuit diagram as shown in Fig. 7 is as follows:

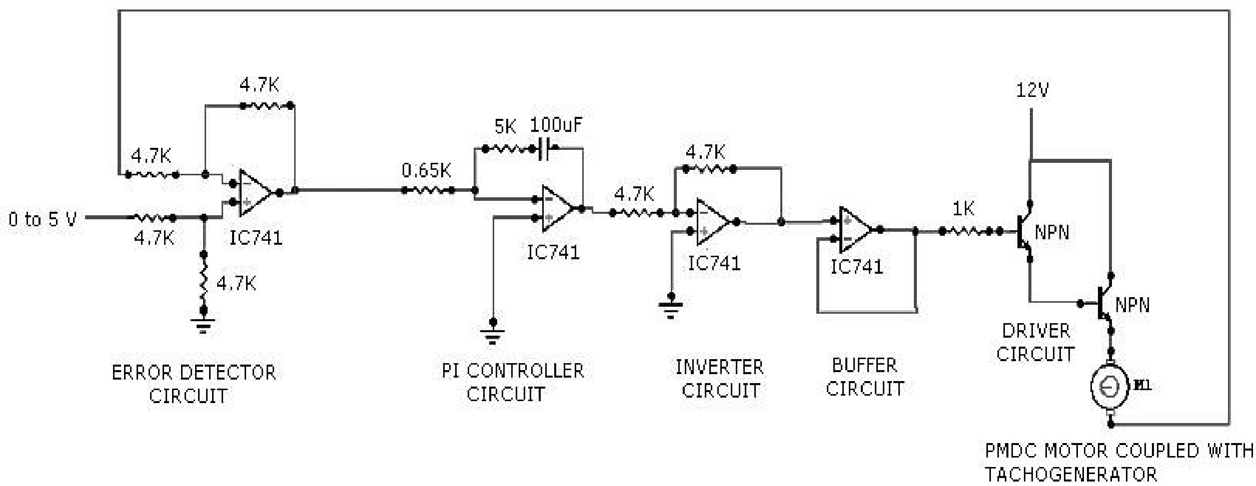

Fig. 7. Detailed circuit.

R_1, R_2 and C were chosen as per the PI controller transfer function obtained theoretically.

Thus,

$$R_1 = 650\Omega \qquad R_2 = 5K\Omega \qquad C = 100\mu F$$

The PI controller output was taken from CRO. The controller output was thus found out to be as in Fig. 8:

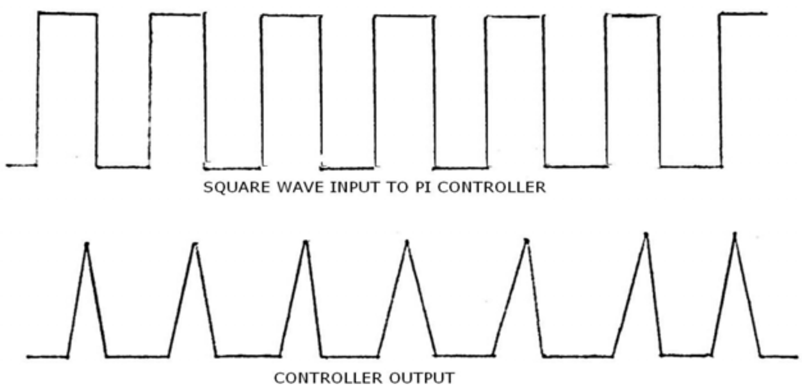

Fig. 8. Response of PI controller using CRO.

It is very well found from the figure that if a square wave input is given to the controller the output generated is a triangular wave. Thus, we can conclude that the controller is of PI type.

Varying the set point value of the Error Detector from 0V to 5V, a set of input voltage versus output speed readings for the practical circuit were taken. Simultaneously the transfer function of the motor was calculated to be

$$H(s) = \frac{0.0215}{s^2 + 0.7308\, s + 0.5675} \tag{27}$$

The simulated output is presented as below in Fig. 9:

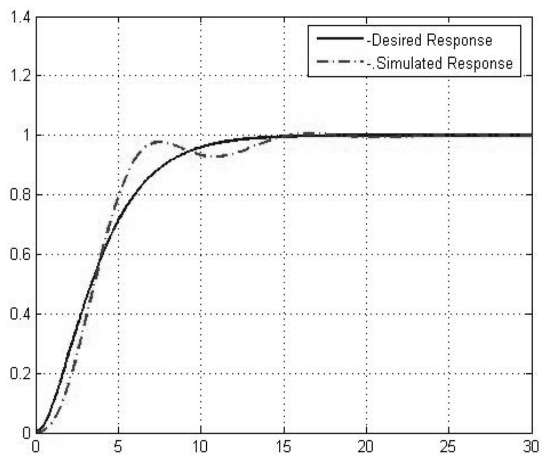

Fig. 9. Closed loop response for PI controller (practical setup).

Subsequently, the transfer function of the controller was found out to be

$$K(s) = 0.18988 + \frac{6.5988}{s} \tag{28}$$

6. Validation of Results

The experimental results were compared with that of the theoretical results in order to validate the performances of the PMDC motor. The table below shows a comparative study of the present work.

Table 2. Validation of results.

	Theoretical Results	Experimental Results
Transfer Function of the Motor	$\frac{0.02057}{s^2 + 0.6206\,s + 0.6092}$	$\frac{0.0215}{s^2 + 0.7308\,s + 0.5675}$
Transfer Function of the Controller	$0.13855 + \frac{7.404}{s}$	$0.18988 + \frac{6.5988}{s}$
K_P	0.13855	0.18988
K_I	53.44	34.75
DC gain	0.03376	0.03788

Thus, we can confirm that the theoretical as well as experimental results were approximately same for both the cases.

7. Conclusion

This paper describes a new design procedure to synthesize a PI controller satisfying time requirements, specified by a reference model. First of all, the motor model is identified using Output Error Identification technique and method of time moments is used for design of PI controller. The parameters of the controller were found out by matching the time moments of the reference model and that of the closed loop plant model with augmented PI Controller. The simulation results were subsequently verified with developing an experimental setup in the laboratory. Both the simulated and experimental results were very close to each other demonstrating the efficacy of the identification and control scheme presented for parameter estimation and speed control of PMDC motor. The estimation scheme for parameters identification was implemented from a batch of input-output measurement data and the controller design method was an offline procedure, therefore an on-line counterpart can be reformulated in future work.

References

[1]. N. Maamri, J. P. Gaubert, J. C. Trigeassou and S. Moreau, Speed Controller Using Time Constrained Output Feedback for PMDC Motor, in *Proceedings of the IEEE International Symposium on Industrial Electronics* (ISIE), 2007, pp. 1153 – 1158.

[2]. J. Pal, Control System Design using Approximate Model Matching, *System Science*, Vol. 19, 1993, pp. 3-23.

[3]. J. D. Gabano, J. P. Pincon and G. Champenois, Parameter Estimation and Sequential Test of Wald for DC Motor Monitoring, in *Proceedings of the IEEE Conference on Control Applications*, 1995.

[4]. J. Y. Hung, Parameter Estimation using Sensitivity Points: Tutorial and Experiment, *IEEE Transactions on Industrial Electronics*, 2001.

[5]. P. Wolm, X. Q. Chen, J. G. Chase, W. Pettigrew and C. E. Hann, Analysis of a PMDC Motor Model for Application in Feedback Design for Electric Powered Mobility Vehicles, in *Proceedings of the International Conference on Mechatronics and Machine Vision in Practice*, 2008.

[6]. P. Jakubowski, W. Koczara and N. Al-Khayat, Method of Poles Position Identification for Brushless Axial Flux Permanent Magnet Motor Drive System, *European Conference on Power Electronics and Applications,* 2005.

[7]. F. Gagneux, D. Jouve and J.P. Rognon, Permanent Magnet Motor Speed Control Loop Autotuning, *Applied Power Electronics Conference and Exposition,* 1991.

[8]. H. Zhou, Model Based Regulator in PMDC Motor, in *Proceedings of IEEE International Conference on Systems, Man and Cybernetics,* 2004.

[9]. A. Bentayeb, N. Maamri and J. C. Trigeassou, The Moments in Control: a tool for Analysis, Reduction and Design, *International Journal of Computers, Communications and Control*, Vol. II, No. 1, 2007, pp. 17-25.

[10].P. M. Da Costa and J. C. Trigeassou, Controller Design by Moments placement: An Application to a Flexible system, *IEEE Transactions on Automatic Control*, 1995.

[11].L. S. Sheikh, Y. P. Wei, H. Chow and R. E. Yates, Determination of Equivalent Dominant Poles and Zeros using Industrial Specifications, *IEEE Transactions on Industrial Electronics and Control Instrumentation*, 1979.

[12].C. K. Sanathanan and S. B. Quinn, Controller Design via the Synthesis Equation, *Journal of Franklin Institute,* 1987.

[13].N. Maamri and J. C. Trigeassou, Design of PID Controllers for Delayed MIMO plants using Moments Based Approach, in *Proceedings of the European Control Conference*, Groningen, Holland, 1993.

[14].N. Maamri and J. C. Trigeassou, Controllers Design by Moments Placement, *European Control Conference,* Roma, Italy, 1993.

[15].M. Hadef, A. Bourouina and M. R. Mekideche, Parameter Identification of a DC Motor using Moments Method, *International Journal of Electrical and Power Engineering*, Medwell Journals, 2007.

[16].L. Ljung, System Identification Toolbox: For use with MATLAB, User's Guide, *The Math Works Inc.*, 1995.

[17].R. Krishnan, Electric Motor Drives: Modelling, Analysis and Control, *Prentice Hall*, 2001.

[18].W. Leonhard, Control of Electrical Drives, *Springer*, 2001.

[19].Curtis D. Johnson, Process Control Technology, *Prentice Hall*, 2008.

[20].K. Krishnaswamy, Process Control, *New Age International*, 2008.

Guide for Contributors

Aims and Scope

Sensors & Transducers Journal (ISSN 1726-5479) provides an advanced forum for the science and technology of physical, chemical sensors and biosensors. It publishes state-of-the-art reviews, regular research and application specific papers, short notes, letters to Editor and sensors related books reviews as well as academic, practical and commercial information of interest to its readership. Because it is an open access, peer review international journal, papers rapidly published in *Sensors & Transducers Journal* will receive a very high publicity. The journal is published monthly as twelve issues per annual by International Frequency Association (IFSA). In additional, some special sponsored and conference issues published annually. *Sensors & Transducers Journal* is indexed and abstracted very quickly by Chemical Abstracts, IndexCopernicus Journals Master List, Open J-Gate, Google Scholar, etc.

Topics Covered

Contributions are invited on all aspects of research, development and application of the science and technology of sensors, transducers and sensor instrumentations. Topics include, but are not restricted to:

- Physical, chemical and biosensors;
- Digital, frequency, period, duty-cycle, time interval, PWM, pulse number output sensors and transducers;
- Theory, principles, effects, design, standardization and modeling;
- Smart sensors and systems;
- Sensor instrumentation;
- Virtual instruments;
- Sensors interfaces, buses and networks;
- Signal processing;
- Frequency (period, duty-cycle)-to-digital converters, ADC;
- Technologies and materials;
- Nanosensors;
- Microsystems;
- Applications.

Submission of papers

Articles should be written in English. Authors are invited to submit by e-mail editor@sensorsportal.com 8-14 pages article (including abstract, illustrations (color or grayscale), photos and references) in both: MS Word (doc) and Acrobat (pdf) formats. Detailed preparation instructions, paper example and template of manuscript are available from the journal's webpage: http://www.sensorsportal.com/HTML/DIGEST/Submition.htm Authors must follow the instructions strictly when submitting their manuscripts.

Advertising Information

Advertising orders and enquires may be sent to sales@sensorsportal.com Please download also our media kit: http://www.sensorsportal.com/DOWNLOADS/Media_Kit_2009.pdf