Denoising for Different Noisy Chaotic Signal Based on Wavelet Transform

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Abstract: In a complete Chaotic radar ranging system, its effective range is often limited by the randomness of the chaotic signal itself and other transmission channel noises or interferences. In order to improve the precision and accuracy of radar ranging system, wavelet transform is proposed to remove different kinds of noise embedded in chaotic signals. White Gaussian noise, colored Gaussian noise as well as sine-wave signal are respectively applied for simulation analysis. Applied for simulation analysis, the experimental results show that wavelet transform can not only remove the chaotic signal mixed in some of the different types of noise, and can also improve the noise ratio.

Keywords: Chaotic signal, Wavelet transform, White Gaussian noise, Different types of noise, Colored Gaussian noise.

1. Introduction

In the past few years, chaotic signal which are produced by a deterministic system have received a great deal of attention in many fields such as communication signal detection [1]. In the process of the generation and transmission of chaotic signal, the existence of noise which masks the inherent dynamic characteristics of the system affects calculating the chaotic parameters [2, 3]. It follows that preprocessing the chaotic signals to reduce the noise without distorting the dynamics of the underlying signal is therefore of highly significance.

Due to the chaotic signal and the noise often having overlapping bandwidth, conventional methods like linear low-pass filtering do not work well for chaotic data [5-10]. Recently, wavelet theory which is a new time-frequency analysis method has been extensively applied in noise reduction [13, 14]. It is very suitable for analyzing and processing Colpitts chaotic which is non-stationary signal.

The wavelet theory and its method provide a new tool that reduces the noise from a chaotic sequence [15] and improve the signal-to-noise ratio in chaotic laser radar [4]. This method is considered nonparametric and is applicable to nonlinear noisy data even without prior information of their dynamics. Dual-lifting Wavelet Transform and new wavelet modulus maximum are used for studying denoising of Lorenz chaotic signal [6-9], but it only discuss the chaotic signals in which the white Gaussian noise is embedded. Genetic algorithms and lifting wavelet transforms is used to remove two kinds of noise which are embedded in Lorenz chaotic signals [5]. However, the frequency of Colpitts chaotic signal is larger than the Lorenz chaotic signal. The closer to the noise signal, the more difficult to denoise.
The rest of the paper goes as follows: Section I presents the introduction. The idea of wavelet denoising is presented in Section II. Section III states the denoising parameters that should be optimally selected to give the best denoising results. Section IV presents Simulations of Colpitts dynamic systems. Section V represents the conclusions of this paper.

2. Wavelet Transform Theory

Consider a drive Colpitts chaotic system which produces transmitted chaotic signal $X(t)$. This signal is sent through a noisy channel to the receiver, so the signal at the receiver is

\[ X_e(t) = X(t) + e(t), \]  

(1)

where $e(t)$ is the noise signal and it is taken in this study as Gaussian white noise and Gaussian colored noise. Thus, the function of equalizer is reducing the noise effect on $X(t)$ as much as possible after the signal output from the receiving antenna. This paper is intended to design an efficient denoising technique that produces output $X_e(t)$ very close to the original signal $X(t)$.

As shown in Fig. 1, wavelet denoising is performed by taking the wavelet transform $X_{e1}(t)$ of the noisy signal $X_e(t)$, and then zeroing out the detail (typically high-pass) coefficients that fall below a certain threshold. An inverse wavelet transform is applied to the thresholded signal $X_{e2}(t)$ to yield the final estimate $X_{e3}(t)$.

The general wavelet denoising procedure involves three steps as follows [10]:

A. Decompose.

Here, we have to choose a wavelet form and determine the decomposition level $M$. Then the wavelet decomposition of the signal $X_e(t)$ at level $M$ should be computed.

B. Threshold detail coefficients.

In this step, for each level from 1 to $M$, we select a threshold and apply thresholding to the detail coefficients.

C. Reconstruct.

Finally, the original signal is reconstructed by using the original approximation coefficients of level $M$ and the modified detail coefficients of levels from 1 to $M$.

3. Optimal Parameters Selection for Wavelet Denoising

This paper gives a detailed study about chaotic wavelet denoising using Colpitts system taking under consideration the selection of wavelet parameters such as wavelet form, level of decomposition ($M$), and threshold method.

A. Selecting the wavelet form.

In general, the more a wavelet resembles the signal, the better it denoises the signal. That means when we want to denoise the Colpitts chaotic signal, the better wavelet for use is Colpitts signal itself. However, because it is very difficult, and simulation time consuming to use wavelet form based on Colpitts signal, therefore for simplicity it is preferred that wavelets which will be considered for use will be firstly defined.

In order to obtain perfect reconstruction results, only orthogonal wavelet will be considered. The orthogonal wavelet has certain benefits; it is very concise, allows for perfect reconstruction of original signal and is not very difficult to process therefore the simulation time is significantly reduced. So in this work only Daubechies and Symlets families will be considered (in Matlab, db=daubechies, sym=symlet) [7, 8].

![Fig. 1. Denoising by thresholding wavelet coefficients.](image)

B. Thresholding.

The wavelet transform is applied to the signal and all coefficients below a certain size are discarded. This technique makes use of the fact that some of the decomposed wavelet coefficients correspond to signal averages and others are associated with details on the original signal. If the smaller details are eliminated from the signal decomposition, the original signal can be extracted from the remaining coefficients and the main signal characteristics will remain intact because an orthogonal wavelet transform compresses the energy of the signal into few large components and the white noise is very disordered so it is scattered throughout the transform in few coefficients.
Mainly there are two types of threshold; hard threshold and soft threshold. Let \( \lambda \) denote the threshold:

The hard threshold signal is

\[
\omega_{h} = \begin{cases} 
\omega, & \omega \geq \lambda \\
0, & \omega < \lambda 
\end{cases}
\]  

The soft threshold signal is

\[
\omega_{s} = \begin{cases} 
\text{sign}(\omega)(|\omega| - \lambda), & |\omega| \geq \lambda \\
0, & |\omega| < \lambda 
\end{cases}
\]  

There are four threshold selection rules that are available to use in Matlab wavelet toolbox and are listed in Table 1. These threshold selection rules use statistical regression of noisy coefficients over time to obtain a non-parametric estimation of the reconstructed signal without noise [10].

**Table 1. Computing method of threshold.**

<table>
<thead>
<tr>
<th>Threshold name</th>
<th>Computing method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sqtwolog</td>
<td>Fixed from threshold equal to the square root of two times the logarithm of the length of the signal</td>
</tr>
<tr>
<td>Rigrsure</td>
<td>Selection using the principle of Stein’s Unbiased Risk Estimate (SURE)</td>
</tr>
<tr>
<td>Heursure</td>
<td>Selection using a mixture of the first two options mentioned</td>
</tr>
<tr>
<td>Minimaxi</td>
<td>Threshold Selection using the minimax principle</td>
</tr>
</tbody>
</table>

4. Simulations of Colpitts Dynamic Systems

To investigate Wavelet Transform for different kinds of noise, noisy chaotic signals are simulated by corrupting the chaotic signals with Gaussian white noise and Gaussian colored noise.

4.1. Colpitts Model

A well-known and studied chaotic system, now called the Colpitts system, has been used to produce identical chaotic signals in both transmitter and receiver.

Consider the Colpitts dynamic equation: in which

\[
x_n = x_{n-1} - x_n - (\frac{g^*}{Qk} - 1)x_n - \frac{1}{Qk}(x_n - x_n^2) + \frac{1}{Qk}y_n x_n
\]  

4.2. Gaussian White Noise Modeling

The Gaussian white noise is generated by using a Box–Mueller algorithm [5]. The pseudo-steps are summarized as follows. This paper adds Gaussian white noise generated as Fig. 2 to chaotic signals.

4.3. Gaussian Colored Noise Modeling

The Gaussian colored noise is generated according to the following four pseudo-steps [5]. This paper adds Gaussian colored noise generated as Fig. 3 to chaotic signals.

**Fig. 2. Gaussian white noise.**
4.4. The Standard of Denoising

The signal–noise ratio (SNR) and the root mean square error (RMSE) are adopted to evaluate the performance of noise reduction methods, the formula of them are defined respectively as follows:

\[
\text{SNR} = 10 \log \left( \frac{\text{var} (X(t))}{\text{var}(Xe3(t) - X(t))} \right),
\]

\[
\text{RMSE} = \sqrt{\frac{1}{2N} \sum_{t=1}^{N} (Xe3(t) - X(t))^2},
\]

where var(\(X(t)\)) is the Variance of signal, var(Xe3(t) - X(t)) is the Variance of noise, \(N\) is the length of the signal, \(X(t)\) and Xe3(t) represent the clean data and the denoised data, respectively.

4.5. Results of the Simulation

First of all, this paper chooses different wavelet forms with the same length and similar properties; then we give the discrete wavelet transform of the noisy Colpitts signal with three scales. To find the best wavelet form, we study two aspects of the signal–noise ratio (SNR) and the root mean square error (RMSE). Table 2 lists the result of selection of Wavelet form.

From Table 2 we can see: when the Wavelet form is sym5, the SNR change from 12.4190 dB to 17.9353 dB, and RMSE=0.8516 is smaller than others. So the optimal Wavelet form is sym5 for chaotic signal with Gaussian white noise.

This paper selects different thresholds of Wavelet with the same length and similar properties; then we give the discrete wavelet transform of the noisy Colpitts signal with three scales. To find the best wavelet threshold, we discuss two aspects of the signal–noise ratio (SNR) and the root mean square error (RMSE). Table 3 lists the result of selection of Wavelet threshold.

From Table 3 we can see: when the Wavelet threshold is Sqtwolog, the SNR change form 12.4190 dB to 17.9353 dB, and RMSE=0.8516 is smaller than others. So the optimal Wavelet threshold is sqtwolog for Chaotic signal with Gaussian white noise.

To investigate wavelet transform for various kinds of noise, the noisy chaotic signal is simulated by adding Gaussian colored noise or sine-wave signal to the chaotic signal.

As the Fig. 4 and Table 4 are shown, the wavelet denoising technique significantly removes the different noises and recovers the original signal with insignificant distortion. At the same time, it also improves the SNR.
Table 2. Analysis selection of Wavelet form.

<table>
<thead>
<tr>
<th>Wavelet form</th>
<th>Chaotic signal with Gaussian white noise(after de-noising)</th>
<th>SNR (dB)</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>db 1</td>
<td></td>
<td>16.0124</td>
<td>1.0326</td>
</tr>
<tr>
<td>db 2</td>
<td></td>
<td>17.4936</td>
<td>0.8699</td>
</tr>
<tr>
<td>db 3</td>
<td></td>
<td>17.6477</td>
<td>0.8724</td>
</tr>
<tr>
<td>db 4</td>
<td></td>
<td>17.6659</td>
<td>0.8710</td>
</tr>
<tr>
<td>db 5</td>
<td></td>
<td>17.3300</td>
<td>0.8976</td>
</tr>
<tr>
<td>db 6</td>
<td></td>
<td>17.4898</td>
<td>0.8655</td>
</tr>
<tr>
<td>db 7</td>
<td></td>
<td>17.3032</td>
<td>0.9174</td>
</tr>
<tr>
<td>db 8</td>
<td></td>
<td>17.4940</td>
<td>0.8689</td>
</tr>
<tr>
<td>sym2</td>
<td></td>
<td>17.4936</td>
<td>0.8699</td>
</tr>
<tr>
<td>sym3</td>
<td></td>
<td>17.6477</td>
<td>0.8724</td>
</tr>
<tr>
<td>sym4</td>
<td></td>
<td>17.6997</td>
<td>0.8652</td>
</tr>
<tr>
<td>sym5</td>
<td></td>
<td>17.9353</td>
<td>0.8516</td>
</tr>
<tr>
<td>sym6</td>
<td></td>
<td>17.5037</td>
<td>0.8623</td>
</tr>
<tr>
<td>sym7</td>
<td></td>
<td>17.5788</td>
<td>0.8731</td>
</tr>
<tr>
<td>sym8</td>
<td></td>
<td>17.5566</td>
<td>0.8598</td>
</tr>
</tbody>
</table>

Table 3. Analysis from threshold value of Wavelet.

<table>
<thead>
<tr>
<th>Threshold name</th>
<th>Chaotic signal with Gaussian white noise(after de-noising)</th>
<th>SNR (dB)</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sqtwolog</td>
<td></td>
<td>17.9353</td>
<td>0.8516</td>
</tr>
<tr>
<td>Rigrsure</td>
<td></td>
<td>12.4952</td>
<td>1.3572</td>
</tr>
<tr>
<td>Heursure</td>
<td></td>
<td>12.4952</td>
<td>1.3572</td>
</tr>
<tr>
<td>Minimaxi</td>
<td></td>
<td>16.4123</td>
<td>0.9187</td>
</tr>
</tbody>
</table>

Fig. 4. The results of simulation ((a) Chaotic signal with Gaussian white noise, (b) Chaotic signal with Gaussian colored noise, (c) Chaotic signal with sine-wave signal, (d) The denoised data of (a), (e) The denoised data of (b), (f) The denoised data of (c)).

Table 4. SNR and RMSE.

<table>
<thead>
<tr>
<th>Noise type</th>
<th>SNR (Before de-noising)</th>
<th>SNR (After de-noising)</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian white noise</td>
<td>12.3428</td>
<td>15.6145</td>
<td>1.6608</td>
</tr>
<tr>
<td>Gaussian colored noise</td>
<td>6.9839</td>
<td>11.3448</td>
<td>1.9487</td>
</tr>
<tr>
<td>Sine-wave signal</td>
<td>11.8235</td>
<td>17.4213</td>
<td>1.4987</td>
</tr>
</tbody>
</table>
5. Conclusions

This research proposes using Wavelet technique to reduce the superimposed different noises on transmitted chaotic signal and reconstruct the original chaotic signal. Applied for simulation analysis, the experimental results show that wavelet transform can not only remove the chaotic signal mixed in some of the different types of noise, and can also improve the noise ratio and reduce the root mean square error.

Acknowledgements

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