Numerical Simulation and Verification of Weight Function of Electromagnetic Flow Meter with Non-insulation Pipe Wall

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Received: 29 April 2014   /Accepted: 30 June 2014   /Published: 31 July 2014

Abstract: Weight function distributions for an electromagnetic flow meter with a non-insulating pipe are numerically simulated and verified. The weight function distribution is very important for solution of flow velocity of distribution. In this paper, a finite element method (FEM) is used for solving the weight function distribution for this novel electromagnetic flow meter. First of all, a numerical simulation model of an electromagnetic flow meter with non-insulating pipe wall is built by the Comsol Multiphysics software. The Boundary condition of weight function is researched. Secondly, weight values for different kinds of ‘non-insulating pipe electromagnetic flow meters’ are analyzed. These non-insulating pipe electromagnetic flow meters have pipes with different conductivities and thicknesses. At lastly, inducted electromotive force between the pair of electrodes of the novel EMFM is researched in this paper. The result of distribution of virtual current is the same as weight value distribution has been proved under situation of uniform magnetic field. This method can be used in analysis different weight functions in other fields like propulsion of surface pipes and electrode poles of electromagnetic flow meter. It is very meaningful in medical and industrial field.

1. Introduction

Based on the Faraday’s electromagnetic induction theory, the Electromagnetic Flow Meter (EMFM) has been widely used in the field of flow measurement [1]. The weight function is a very important concept and absolutely necessary for analysis of flow meters. It shows that amount by which the flow velocity at each point in the cross-section contributes to the voltage signal at the electrodes and it follows the certain rules [2, 3]. Analytical solution of weight function shows the value at the pipe centre is 1, value of circle (except electrodes) is 0.5, the maximum value at electrodes [4, 5]. Ideal electromagnetic flow meters have an insulating pipe or a layer of insulating material at the inner wall of the pipe [3]. In many conditions, the surface of pipe is not insulating or becomes non-insulating because of pollution. Research on the weight functions of EMFMs with non-insulating pipe wall is rare. It introduces some problems. Such as, measuring blood velocity using EMFM in medical, researching the effect of measure results because of the pipe or the electrodes are polluted.
This paper points out the difficulty of solving weight function of EMFs with non-insulating pipe wall and shows that the electrical conductivity of pipe must be considered. Traditional method of analyzing weight function is solving Laplace equation by means of Green’s function.

For solving weight value distribution of EMFM with non-insulating pipe wall the virtual current density distribution has been researched. Changing of induced electromotive force between the pair of electrodes is pointed out. The analysis method put forward in this paper would solve the difficulties in medical and industrial field.

The most important thing is that the electrodes of the novel EMFM need not be embedded in the pipe, in other words is that the electrodes is not fixed on a certain location. However electrodes are on the outside of the pipe.

The design in the paper makes collecting the signals easy.

2. Model of EMFM with Non-insulating Pipe Wall

Some reasonable assumptions for theoretical analysis:
1) Laminar flows in EMFM;
2) The electrodes are considered as point electrodes;
3) The imposed magnetic field is uniform.

The geometry of the EMFM in this paper is a cylindrical cavity body with non-insulating pipe wall. Inner diameter is \( R_2 \), outer diameter is \( R_1 \) \(( R_2 < R_1 \) ), pipe wall thickness is \( h \), and \( h = R_1 - R_2 \).

Smaller cylindrical cavity (diameter is \( R_2 \) ) filled with liquid of conductivity \( \sigma_2 \), pipe conductivity is \( \sigma_1 \).

Establishing coordinate system, the origin is located in the circle center, the electrodes are located on the Y axis as shown in Fig. 1.

The two electrodes are located at the outer diameter of the pipe wall which is the advantage of the novel EMFM design.

3. Analysis on the Weight Function of EMFM with Non-insulation Surface Pipe

3.1. Solution Weight Function Based on the Laplace’s Equation

The Green’s function is used to solve the Laplace’s equation. It is the normal method for obtaining weight function distributions for EMFM with insulating pipe walls [6].

Ohm’s law is [7-9]:

\[
\mathbf{j} = \sigma (\mathbf{E} + \nabla \phi) + \mathbf{B},
\]

(1)

If the displacement current is ignored:

\[
\nabla \cdot \mathbf{j} = 0,
\]

(2)

Substituting the equation (1) into the equation (2), we can get:

\[
\sigma \left( \nabla \times (\nabla \times \mathbf{B}) + \nabla (\nabla \cdot \mathbf{B}) \right) = 0,
\]

(3)

If the conductivity of the liquid is uniform:

\[
\nabla \sigma = 0,
\]

(4)

Using the Green’s function gets the weight function analytical solution of EMFM (long pipe) with insulation pipe wall [2]:

\[
W = \frac{R_2^2 \left( R_1^2 + x^2 - y^2 \right)}{R_1^4 + 2R_1^2 \left( x^2 - y^2 \right) + \left( x^2 + y^2 \right)^2},
\]

(5)

where \( R_1 \) is the radius of the pipe.

For an EMFM with non-insulating pipe wall, equation (5) cannot apply. When the conductivity of the pipe wall is not equal with the conductivity of the liquid it is difficult to solve equation (3) to get the weight function distribution for an EMFM with a non-insulating pipe wall.

3.2. Solving Weight Function Distribution Based on Virtual Current Density Distribution

The virtual current density defined by Bevir is determined by the shape of the electrodes and the conductivity of measurement pipe wall.

So the weight function is given by [10, 11]:

\[
W = \mathbf{B} \times \mathbf{j}_v,
\]

(6)

where \( \mathbf{j}_v \) corresponds to the electrical current distribution in the fluid at rest applying the unit current between the electrodes.
The models in this paper, electrodes are point-electrodes.

So the virtual current of electrode is:

$$ \frac{\partial G_j}{\partial \tau} = \pm \frac{\delta}{R} $$

Pipe wall is non-insulating, so the virtual current density in normal direction is not zero.

If the virtual current density in electrode is $1 \text{ (A/m}^2\text{)}$, virtual current on inner wall point B (see in Fig. 1) is $j (0 < j < 1)$.

It projects on non-insulation pipe wall is

$$ j \sin \alpha (\text{A/m}^2), $$

As shown in the Fig. 2, $\overline{BM}$ is the virtual current, $\overline{BN}$ is the tangent of circle by point B. Angle $\alpha$ between $\overline{BM}$ and $\overline{BN}$, So $\overline{MN}$ is Projection of $\overline{BM}$ in normal direction.

![Fig. 2. Schematic diagram of virtual current projection.](image)

The Ohm’s law:

$$ j = \sigma_1 E $$

where $\sigma_1$ is the conductivity of the pipe. It shows that current density is proportional to size of electric field.

Electric field intensity on point A is:

$$ \overline{E} = k \frac{Q}{h^2}, $$

$$ k = \frac{1}{4\varepsilon \pi} \varepsilon = 8.8542 \times 10^{12} \text{ F/m} $$

where $h$ is the thickness of the pipe, $Q$ is the electric quantity of the electrode.

Substituting the equation (9) and the equation (10) into the equation (8):

$$ j = \sigma_1 k \frac{Q}{h^2} \sin \alpha, $$

The equation (11) shows that the virtual current density is directly proportional to $\sigma_1$ and inversely proportional to $h^2$.

$$ W = B \times \sigma_1 k \frac{Q}{h^2} \sin \alpha, $$

Induced potential between $A$ and $A'$ is [9]:

$$ E_{AA'} = j \cdot (\overline{B} \times \overline{F}) \cdot \overline{dV}, $$

The analysis of novel EMFM in layer flow field, so:

$$ \nu = \frac{1}{2} \nu_{\text{max}}, $$

$$ \nu_{x} = \nu_{\text{max}} \left[1 - \left(\frac{r_x}{r_c}\right)^2\right], $$

where $\nu$ is the average velocity, $\nu_{\text{max}}$ is the liquid velocity on the center of the pipe, $r_c$ is the radius of the pipe, $r_x$ is the distance of central axis of the pipe on the radius direction. $\nu_x$ is the flow velocity on $r_x$.

If magnetic field is uniform, the velocity of flow is fixed. The change of weight function reflected as change of induction electromotive force between electrodes.

4. Simulation of EMFM with Non-insulation Pipe Wall

The AC/DC module of COMSOL Multiphysics is used. Helmholtz coils are introduced to producing uniform magnetic fields [12-14]. The flow field is laminar and the average velocity is 2 m/s. The radius of the pipe is 1 m, the material is PTFE, and the electrodes are copper. The flow in the pipe is water at 25°C, and the water conductive is $1.0 \mu\text{s/cm}$. The theory is verified by simulation. Compare simulation result is shown in Fig. 3(a) and the result of solving Laplace’s Equation are shown in Fig. 3(b).

![Fig. 3 (a). Simulation results of weight function: diagram of weight function of EMFM with insulation pipe wall.](image)
The result shows that the theory of this paper is correct; because of the simulation result is the same as the result solved by Laplace’s equation.

4.1. Simulation for different conductive of surface pipe ($\sigma_1$)

The novel model will be widely used in medical science. It could be used to measure blood velocity exactly. Models with different conductive pipe material are discussed.

Fig. 4 shows the simulation results of the weight function of EMFM (three different values for the conductivity of the pipe wall have been chosen namely $1.035\times10^{-12}$ s/m, $1.537\times10^{-7}$ s/m, and $4.5\times10^{-3}$ s/m).

Equation (13) is used to calculate the flow induced potential difference $U$ between the electrodes. The result is shown in Fig. 5.

As can be seen from Fig. 5, the flow induced electromotive force decreases as the conductivity of the pipe wall is increased.

It also shows virtual current density changes with the conductive of the pipe. In another words, virtual current density is direct proportion with conductive of pipe. Equation (11) is verified.

4.2. Simulation of Weight Function of EMFM with Different Thickness ($h$) of Pipe

The model discussed in this paper will be widely used in industrial field to measure the flow velocity when electrodes or pipes are polluted.

Using the same simulation method, different thickness of pipe ($\sigma_1=0.45\times10^{-3}$ s/m) produce different simulation results of weight function distribution (shown in Fig. 6).

The induced electromotive force is calculated. The result is shown in Fig. 7.

Fig. 7 shows that if the thickness of the pipe wall is increased, virtual current density is decreased. In another words, the induced potential is increased. Equation (11) is verified. The theory of this paper that the virtual current density is inverse proportion with the thickness of the pipe is proved.
5. Conclusion

The weight function shows the velocity of different location on across-section pipe contributes to different signals of the electrodes. The uniform weight function means every point has the same contribution.

Low unevenness is very important to the design of EMFM.

The virtual current density on juncture between non-insulation cavity and flow is \( \sigma k \frac{Q}{h^2} \sin \alpha \) can get a uniform weight function.

1) The methods in this paper of calculate virtual current density to get weight function is correct.
2) The equation (11) is verified by simulation and analysis.
3) The weight function of EMFM with non-insulation surface pipe changes with the conductive and the thickness of pipe.

The induce electrode force is decreased as conductive increased and the thickness of pipe decreased.

The finite element method is used to account virtual current density to get weight value distribution in this paper.

The method is very important to the research on weight function of EMFM with non-insulation surface pipe.

The novel model of EMFM is very meaningful to solve problems according by pollution of electrodes and pipe surface in measurement. The novel EMFM with non-insulation pipe wall will play an important role in the future.

References


