Adaptive Maneuvering Target Tracking Algorithm

1 Chunling Wu, 1 Yongfeng Ju, 1 Panzhi Liu, 2 Chongzhao Han
1 Chang’an University, Middle Section of Nan Erhuan Road, Xi’an City, Shaanxi, 710064, P. R. China
2 Xi’an Jiaotong University, No. 28 Xianning West Road, Xi’an, Shaanxi, 710049, P. R. China

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Abstract: Based on the current statistical model, a new adaptive maneuvering target tracking algorithm, CS-MSTF, is presented. The new algorithm keep the merits of high tracking precision that the current statistical model and strong tracking filter (STF) have in tracking maneuvering target, and made the modifications as such: First, STF has the defect that it achieves the perfect performance in maneuvering segment at a cost of the precision in non-maneuvering segment, so the new algorithm modified the prediction error covariance matrix and the fading factor to improve the tracking precision both of the maneuvering segment and non-maneuvering segment; The estimation error covariance matrix was calculated using the Joseph form, which is more stable and robust in numerical. The Monte-Carlo simulation shows that the CS-MSTF algorithm has a more excellent performance than CS-STF and can estimate efficiently. Copyright © 2014 IFSA Publishing, S. L.

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1. Introduction

In maneuvering target tracking, to establish practical target motion model has been a difficult and hot problem. In the past thirty years, all kinds of mathematical model [1, 2] that is used to describe the changes of maneuvering target motion. The flaw of Uniform motion model (CV) and uniformly acceleration motion model (CA) is that target motion form in model is single. Zhou Dong-hua proposed strong target tracking filter (STF) [2, 3]. STF can adjust the corresponding gain matrix online and force the residuals orthogonal to each other through introducing a time-varying suboptimal fading factor \( \lambda(k+1) \) into the covariance of KF or EKF. STF is robust for the model mismatch and can be used to track the maneuvering target. But, the tracking accuracy of STF is not ideal. Moreover, the computation complexity of state estimation is too much [4, 5]. In order to resolve this problem, this paper proposed a novel adaptive maneuvering target tracking algorithm, current statistical model-modified strong tracking filter (CS-MSTF), which improves the tracking performance by several kinds of modification. Firstly, this paper gives us the current statistical model and its tracking filter. Then, give us the modified algorithm. Lastly, the simulation results are given.

2. The Modified Strong Tracking Algorithm Based on Current Statistical Model

2.1. The Ordinary Strong Tracking Filter Based on Current Statistical Model

Because of the flaw of current statistical model adaptive filter, the literature [4] proposed the following current statistical model strong tracking filter (CS-STF) to track the maneuvering target. After introducing the suboptimal fading factor, STF can adaptively adjust gain according residuals, which
enhance the tracking ability of the system to the mutation status.

The discrete state equation and observation equation of current statistical model are given bellow [2], respectively.

State equation:
\[ X(k+1) = F(k)X(k) + U(k)\bar{\pi}(k) + w(k) , \]

In this equation, \( X(k) = [x(k), \bar{x}(k), \bar{x}(k)]^T \) is the system state, \( F(k) \) is the state transition matrix, \( U(k) \) is the state input matrix, \( \bar{\pi}(k) \) is the mean of "current" acceleration, \( w(k) \) is Gaussian white noise sequence with zero-mean, covariance \( Q(k) = E[w(k)w^T(k)] \).

Observation equation:
\[ Z(k+1) = H(k+1)X(k+1) + \nu(k+1) , \]

where \( H(k) \) is the measurement matrix, and \( H(k) = [1 0 0] \) when only the target position can be observed. Measurement noise, \( \nu(k) \), is the Gaussian white noise sequence with zero mean and covariance \( R(k) = E[\nu(k)\nu^T(k)] \).

According to state equation (1) and measurement equation (2), the step of CS-STF algorithm is below.

\[
\hat{X}(k+1/k) = F(k)\hat{X}(k/k) + U(k)\bar{\pi}(k) ,
\]

\[
P(k+1/k) = \lambda(k+1)F(k)P(k/k)F^T(k) + Q(k) ,
\]

\[
K(k+1) = P(k+1/k)H^T(k+1)\left[H(k+1)\right. \\
\left. \times P(k+1/k)H^T(k+1) + R(k+1)\right]^{-1} ,
\]

\[
r(k+1) = Z(k+1) - H(k+1)\hat{X}(k+1/k) ,
\]

\[
\hat{X}(k+1/k+1) = \hat{X}(k+1/k) + K(k+1)r(k+1) ,
\]

\[
P(k+1/k+1) = [I - K(k+1)H(k+1)]P(k+1/k) ,
\]

In equation (4), \( \lambda(k+1) \) is the suboptimal fading factor, and can be computed like this:
\[
\lambda(k+1) = \begin{cases} \lambda_0, & \lambda_0 \geq 1 \\ 1, & \lambda_0 < 1 \end{cases} ,
\]

In this equation,
\[
\lambda_0 = \frac{\text{tr}[N(k+1)]}{\text{tr}[M(k+1)]} ,
\]

\[
N(k+1) = V_0(k+1) - \beta R(k+1) \\
- H(k+1)Q(k)H^T(k+1) ,
\]

\[
M(k+1) = H(k+1)F(k)P(k/k)F^T(k)H^T(k+1) ,
\]

\[
V_0(k+1) \text{ is the residual covariance matrix, can be calculated like this:}
\]

\[
V_0(k+1) = E[r(k+1)r^T(k+1)] \\
= \begin{cases} r(1)r^T(1), & k = 0 \\ \frac{\rho V_0(k)+r(k+1)r^T(k+1)}{1+\rho}, & k \geq 1 \end{cases} ,
\]

where \( \rho \), \( 0 < \rho \leq 1 \), is the forgetting factor, and usually \( \rho = 0.95 \). \( \beta \) is the weakening factor, and can be defined by experience. \( r(1) \) is the initial residual.

The calculation of system noisy covariance is:
\[
\sigma^2_a = \begin{cases} 4 - \pi \left[ a_{\max} - \bar{x}(k+1/k) \right]^2, & \bar{x}(k+1/k) > 0 \\ 4 - \pi \left[ a_{\max} + \bar{x}(k+1/k) \right]^2, & \bar{x}(k+1/k) < 0 \end{cases} ,
\]

2.2. The Modified Strong Tracking Filter Based on Current Statistical Model (CS-MSTF)

Although, in the above algorithm, STF has good performance in tracking maneuvering target, but it has unrealistic tracking accuracy in non-maneuvering time. Moreover, the approximate calculation method for innovation variance has defect, and calculation formula of estimation error covariance can neither ensure symmetry or positive semidefinite of \( P(k/k) \). Because of this, we propose new modified algorithm based on CS-STF. New algorithm make some modifies below.

Firstly, as mentioned before, when maneuver occur, the performance of STF is priori to KF because of introducing fading factor \( \lambda(k+1) \). But, in non-maneuvering situation, the tracking accuracy of STF is not ideal. In STF algorithm, we see that the second item of equation (4), covariance \( Q(k) \) is ignored. In fact, the maneuver of target state can be seen as model mismatch. So, during the recursive calculation of \( P(k+1/k) \), it is very reasonable to increase the weight of \( Q(k) \). Then, \( P(k+1/k) \) can be defined as follows:
\[
P(k+1/k) = \lambda(k+1)[F(k)P(k/k)F^T(k)+Q(k)] ,
\]

Thirdly, in order to satisfy the orthogonal principle, that is
\[
E\{r(k+1+j)r^T(k+1)\} = 0 ,
\]

So, the time-varying gain matrix should satisfy the following equation,
\[ P(k + 1 / k)H^T(k + 1) - K(k + 1)V_o(k + 1) = 0 \]  \hspace{1cm} (17)

Introducing equation (5) into equation (17), we can obtain,
\[ P(k + 1 / k)H^T(k + 1) - P(k + 1 / k) \times R(k + 1)]^{-1}V_o(k + 1) = 0 \]  \hspace{1cm} (18)

That is,
\[ P(k + 1 / k)H^T(k + 1)[I - (H(k + 1)P(k + 1 / k)] \times R(k + 1)]^{-1}V_o(k + 1) = 0 \]  \hspace{1cm} (19)

So, if we want equation (19) zero, it must satisfy equation (20)
\[ I - (H(k + 1)P(k + 1 / k)] \times R(k + 1)]^{-1}V_o(k + 1) = 0 \]  \hspace{1cm} (20)

That is,
\[ V_o(k + 1) - R(k + 1)H(k + 1)P(k + 1 / k)H^T(k + 1) = 0 \]  \hspace{1cm} (21)

Using equation (15) to compute the equation (21), and then we obtain,
\[ V_o(k + 1) - R(k + 1) = \lambda(k + 1)H(k + 1) \times [F(k)P(k / k)F^T(k) + Q(k)][H^T(k + 1)] \]  \hspace{1cm} (22)

Define:
\[ N(k + 1) = V_o(k + 1) - \beta R(k + 1) \]  \hspace{1cm} (23)

\[ M(k + 1) = H(k + 1)[F(k)P(k / k)F^T(k) + Q(k)][H^T(k + 1)] \]  \hspace{1cm} (24)

In order to make the state estimation smoother, we adopt the characteristics of square root function and make equation (20) and (21) be modified as follows,
\[ \lambda(k + 1) = \begin{cases} \lambda_0, & \lambda_0 \geq 1 \\ 1, & \lambda_0 < 1 \end{cases} \]  \hspace{1cm} (25)

\[ \lambda_0 = \frac{\text{tr}[N(k + 1)]}{\text{tr}[M(k + 1)]} \]  \hspace{1cm} (26)

Thirdly, the estimation error covariance matrix for STF is:
\[ P(k + 1 / k + 1) = [I - K(k + 1)H(k + 1)] \times P(k + 1 / k) \times [I - K(k + 1)H(k + 1)]^T + K(k + 1)R(k + 1)K^T(k + 1) \]  \hspace{1cm} (27)

This equation can not ensure symmetry and positive semidefinite of \( P(k / k) \). So we use the following equation which is more robust in numerical value to recursive.
\[ P(k + 1 / k + 1) = [I - K(k + 1)H(k + 1)] \times \]  \hspace{1cm} (28)

\[ P(k + 1 / k) \times [I - K(k + 1)H(k + 1)]^T + K(k + 1)R(k + 1)K^T(k + 1) \]  \hspace{1cm} (29)

This equation is the famous Joseph form or Joseph steady form, which can ensure symmetry and positive semidefinite of the calculation result through the product of three matrices, and reinforce numerical value steady and robust of algorithm.

To sum up, the complete calculation steps of new algorithm are as follows:
\[ \dot{X}(k + 1 / k) = F(k)X(k / k) + U(k)\sigma(k) \]  \hspace{1cm} (30)

\[ r(k + 1) = Z(k + 1) - H(k + 1)X(k + 1 / k) \]  \hspace{1cm} (31)

\[ V_o(k + 1) = \begin{cases} r(1)r^T(1), & k = 0 \\ \rho V_o(k) + r(k + 1)r^T(k + 1) + 1 + \rho, & k \geq 1 \end{cases} \]  \hspace{1cm} (32)

\[ N(k + 1) = V_o(k + 1) - \beta R(k + 1) \]  \hspace{1cm} (33)

\[ M(k + 1) = H(k + 1)[F(k)P(k / k)F^T(k) + Q(k)][H^T(k + 1)] \]  \hspace{1cm} (34)

\[ \lambda(k + 1) = \begin{cases} \lambda_0, & \lambda_0 \geq 1 \\ 1, & \lambda_0 < 1 \end{cases} \]  \hspace{1cm} (35)

\[ P(k + 1 / k + 1) = \lambda(k + 1)[F(k)P(k / k)F^T(k) + Q(k)][H^T(k + 1)] \]  \hspace{1cm} (36)

\[ K(k + 1) = P(k + 1 / k)H^T(k + 1) \times \]  \hspace{1cm} (37)

\[ [I - K(k + 1)H(k + 1)] \times P(k + 1 / k) \times [I - K(k + 1)H(k + 1)]^T + K(k + 1)R(k + 1)K^T(k + 1) \]  \hspace{1cm} (38)

3. Simulation Examples and Results

In order to test the validity of the new algorithm, CS-MSTF, we will simulate the new algorithm and CS-STF by two examples.
3.1. One-dimensional Maneuvering Target

We suppose that the initial state of target is $X(0)$, and $X(0)=[0 \ 0 \ 5]^T$, target motion process lasts 200 s, the sampling period is $T=1s$, maneuvering frequency of the target $\alpha_0$ is equal to 1/20, the maximum maneuvering acceleration is set to $\pm 50m/s^2$. Moreover, we suppose that only the position component in state component can be measured directly, that is $H=[1 \ 0 \ 0]$. Measurement error is 20 m. The measurement noise satisfies the zero mean Gauss distribution.

The target motion state is as below, Table 1.

<table>
<thead>
<tr>
<th>Scan period(s)</th>
<th>Flight state</th>
<th>Acceleration(m/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1~50 s</td>
<td>Constant acceleration</td>
<td>5</td>
</tr>
<tr>
<td>51-100 s</td>
<td>Constant velocity</td>
<td>0</td>
</tr>
<tr>
<td>101-200 s</td>
<td>Constant acceleration</td>
<td>20</td>
</tr>
</tbody>
</table>

We will test and verify the performance of new algorithm by 200 Monte-Carlo simulations. The target state vector is $X(k)=\begin{bmatrix} x(k) & \dot{x}(k) & \ddot{x}(k) \end{bmatrix}^T$. Evaluation index is Root Mean Square Error (RMSE) for position, velocity and acceleration component. RMSE is defined as below.

Root mean square error for the position:

$$RMSE_{pos} = \sqrt{\frac{1}{M} \sum_{i=1}^{M} (x_i(k) - \hat{x}_i(k/k))^2}, \quad (40)$$

where $M$ is the number of Monte-Carlo simulation, 200; $x_i(k)$ is the true position at the $k$ moment; $\hat{x}_i(k/k)$ is the estimated position at the $k$ moment.

Root mean square error for the velocity:

$$RMSE_{vel} = \sqrt{\frac{1}{M} \sum_{i=1}^{M} (\dot{x}_i(k) - \hat{\dot{x}}_i(k/k))^2}, \quad (41)$$

where $M$ is the number of Monte-Carlo simulation; $\dot{x}_i(k)$ is the true velocity at the $k$ moment; $\hat{\dot{x}}_i(k/k)$ is the estimated velocity at the $k$ moment.

Root mean square error for the acceleration:

$$RMSE_{acc} = \sqrt{\frac{1}{M} \sum_{i=1}^{M} (\ddot{x}_i(k) - \hat{\ddot{x}}_i(k/k))^2}, \quad (42)$$

where $M$ is the number of Monte-Carlo simulation; $\ddot{x}_i(k)$ is the true acceleration at the $k$ moment; $\hat{\ddot{x}}_i(k/k)$ is the estimated acceleration at the $k$ moment.

Fig. 1 gives the true velocity trajectory of the target. The comparison of RMSE for position, velocity and acceleration component is given in Fig. 2, Fig. 3 and Fig. 4.
From Fig. 2 to Fig. 4, we can see that the position, velocity and acceleration RMSE of CS-MSTF algorithm are lower than that of CS-STF algorithm no matter in maneuver time or non maneuver time, which illustrate that the new algorithm improves the tracking accuracy and adapts to track maneuvering target. Moreover, because the computation of fading factor in the new algorithm adopts square-root form, the estimated curve is smoother.

### 3.2. Two-dimensional Maneuvering Target

Using the experimental scheme in literature [6], we still compare the new algorithm with CS-STF algorithm by Monte Carlo simulation. The number of simulation is set to 500.

We suppose that the initial state of target is \( X(0) = [2000 m, 15 m/s, 10000 m, 0 m/s]^T \), target motion process lasts 200s, the sampling period \( T = 1 s \), maneuvering frequency of the target \( \alpha_0 = 1/60 \), the maximum maneuvering acceleration is set to \( a_{x,\text{max}} = \pm 50m/s^2 \), the ranging error is 50 m. The measurement noise satisfies the zero-mean Gauss distribution. The maneuver of target is shown in Table 2.

<table>
<thead>
<tr>
<th>Maneuvering moment (s)</th>
<th>Acceleration in X direction (m/s²)</th>
<th>Acceleration in Y direction (m/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T=31s )</td>
<td>5</td>
<td>-10</td>
</tr>
<tr>
<td>( T=38s )</td>
<td>-8</td>
<td>18</td>
</tr>
<tr>
<td>( T=49s )</td>
<td>10</td>
<td>-20</td>
</tr>
<tr>
<td>( T=61s )</td>
<td>0</td>
<td>30</td>
</tr>
<tr>
<td>( T=65s )</td>
<td>-10</td>
<td>-8</td>
</tr>
<tr>
<td>( T=66s )</td>
<td>-5</td>
<td>0</td>
</tr>
<tr>
<td>( T=81s )</td>
<td>5</td>
<td>-10</td>
</tr>
</tbody>
</table>

From the acceleration data in Table 2, we can see that acceleration maximum difference of adjacent time is 18 m/s² in the direction of X axis, compared to the maximum maneuver acceleration 50 m/s², that almost no maneuvering. While acceleration maximum difference of adjacent time is 50 m/s² in the direction of Y axis, which can be seen strong maneuvering.

Fig. 5 to Fig. 7 is the motion trajectory and the target velocity and acceleration curve of the target.

From the position RMSE curve in Fig. 8, we can see that the tracking accuracy of CS-MSTF is prior to CS-STF in strong maneuvering and weak maneuvering. From the velocity RMSE curve in Fig. 9, we can see that the tracking accuracy of CS-MSTF is obviously prior to CS-STF in both maneuvering and non maneuvering. From the acceleration RMSE curve in Fig. 10, the tracking accuracy of CS-MSTF is slightly prior to CS-STF in maneuvering, obviously prior to CS-STF in non-maneuvering.
Lastly, the simulation shows that CS-MSTF enhances the tracking accuracy for maneuvering and non-maneuvering target compared to CS-STF from the above two examples.

4. Conclusion

The paper propose a new adaptive filtering algorithm (CS-MSTF). CS-MSTF makes modifications as such: First, STF has the defect that it achieves the perfect performance in maneuvering segment at a cost of the precision in non-maneuvering segment, so the new algorithm modified the prediction error covariance matrix and the fading factor to improve the tracking precision both of the maneuvering segment and non-maneuvering segment; The estimation error covariance matrix was calculated using the Joseph form, which is more stable and robust in numerical. The Monte-Carlo simulation shows that the CS-MSTF algorithm has a more excellent performance than CS-STF and can estimate efficiently.

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