An Improved Music Algorithm for DOA Estimation of Coherent Signals

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Abstract: The estimation of direction of arrival (DOA) is the main study of the direction finding and location of radio signal, MUSIC algorithm is the most classic DOA estimation. MUSIC algorithm can estimate independent signals’ DOA effectively, but it is failure to coherent signals. Concerning the issue, the modified MUSIC algorithm works when the correlative signals exist. However, the estimation performance of modified MUSIC algorithm will deteriorate in the case that the signal-noise rate is low. This paper improves the modified MUSIC algorithm by using matrix decomposition. Finally, the computer simulation results prove that this proposed method has better direction finding performance than the modified MUSIC algorithm with low signal to noise ratio. Copyright © 2014 IFSA Publishing, S. L.

Keywords: Direction finding and location, MUSIC algorithm, Modified MUSIC algorithm, Improved modified MUSIC algorithm.

1. Introduction

Radio positioning is an important work of radio management. It is the necessary means to maintain the air radio waves order, safeguard the legitimate rights of users and combat the illegal occupation of frequency resources. In mobile communication, wireless location technology is a method that determines the physical position of the radio signal’s transmitting terminal in a certain radius by using the wireless signal. The estimation of direction of arrival (DOA) of multiple narrowband signals is an important problem in array signal processing including radar, sonar, radio astronomy, and mobile communications [1]. Direction of arrival estimation is to confirm simultaneously the spatial position of multiple signals in a certain space, which is the angle of the multiple signals that reach the reference array.

The far-field signals’ angle estimation is mainly using the array signal processing technology [2]. Array signal processing is also often referred to as the spatial signal processing, where the spatial spectrum estimation is a new spatial signal processing technology developed in the last 30 years. The MUSIC algorithm is the most classic spatial spectrum estimation algorithm, which is proposed by Schmidt [3]. MUSIC is an acronym which stands for Multiple Signal Classification. Its basic idea is eigenvalue decomposition about the covariance matrix of the arbitrary array’s output data, thereby we will get the signal subspace which is corresponding to the signal component and the noise subspace which is orthogonal to the noise component, then we could construct the spectral function by using the orthogonality of the two subspace, and estimate the signal incidence direction by means of searching the
spectrum peak, finally achieving the signal high resolution parameter estimation.

MUSIC algorithm has a good estimation performance. Its estimation’s complexity is moderate, but the MUSIC algorithm is just adjusted to the independent signals. It is ineffective to coherent signal [4, 5]. Therefore the direction finding of coherent signal has been the focus of research, and many modified MUSIC algorithm have been proposed to improve the performance of the estimation of the coherent signal, that is preprocessing the data to eliminate or weaken the correlation between sources.

This paper modifies the traditional MUSIC method utilizing the idea of spatial smoothing algorithm, and makes further improvement on the basic of the modified algorithm, compared with the modified method by computer simulating, last verified the improved algorithm of accuracy and effectiveness.

2. Music Algorithm

2.1. Array Signal Mathematical Model

In order to facilitate the analysis, Fig. 1 shows the ideal state of array signal mathematical model [6-8]:

![Array signal model](image)

**Fig. 1. Array signal model.**

1) The detected signal has the same unrelated polarization. Generally consider the signal is narrowband, and each signal source has the center frequency \( \omega_w \), and the number of the detected signal source is \( D \).

2) We consider the uniform linear array that composed of \( M \) isotropic sensors, and \( M > D \). \( d \) is the spacing between adjacent elements, and \( d \) is not more than half wavelength of the highest frequency.

3) Antenna array is the far field of each signal source, namely the signals received by the antenna array is the plane wave.

4) \( n_m(t) \) is the receiver noise component assumed to be Gaussian distributed with zero mean and \( \sigma^2 \) variance and the noise in each channel is uncorrelated.

5) Each of the receiving channel has the same properties.

Assume the \( S_k(t) \) is the wavefront signal which is the \( k \) th signal source incident the array and \( S_k(t) \) is narrowband signal [9], so \( S_k(t) \) can be represented by the following form:

\[
S_k(t) = s_k(t) e^{j\omega_k t}, \quad (1)
\]

where \( s_k(t) \) for \( k=1,2,\cdots,D \) is the complex envelope of \( S_k(t) \), and \( \omega_k \) is the angular frequency of the signal \( S_k(t) \), we suppose the \( D \) signals has the same center frequency, so:

\[
\omega_k = \omega_b = \frac{2\pi c}{\lambda}, \quad (2)
\]

where \( c \) is the speed of electromagnetic wave, and \( \lambda \) is the common signal wavelength, \( t_1 \) indicates the time electromagnetic wave through the antenna array. Then according to the narrowband assumption [10], exists the following approximate:

\[
s_k(t-t_1) = s_k(t), \quad (3)
\]

Then the delayed wavefront signal is:

\[
\tilde{S}_k(t-t_1) = s_k(t-t_1) e^{j\omega_k (t-t_1)} = s_k(t) e^{j\omega_k (t-t_1)} = S_k(t) e^{j\omega_k t_1}, \quad (4)
\]

So, if the first array element as reference point, the complex received signal of the \( m \) th sensor for the \( k \) th signal at time can be expressed as:

\[
a_k S_k(t) e^{j\omega_k t_1} = a_k S_k(t) e^{j\omega_k t_1} \quad (5)
\]

For \( m=1,2,\cdots,M \) and \( k=1,2,\cdots,D \),

where \( a_k \) is the impact that the \( m \) th sensor for the \( k \) th signal. We assume each sensor is nondirectional, so define \( a_k = 1 \). The DOA of the \( k \) th source signal is \( \theta_k \), and \( (m-1) \frac{d \sin \theta_k}{c} \) is the propagation delay between the first sensor and the \( m \) th sensor.

Consider the measurement noise and all signal sources, the output signal of the \( m \) th sensor can be expressed as:

\[
x_m(t) = \sum_{k=1}^{D} a_k S_k(t) e^{j\omega_k t_1} + n_m(t), \quad (6)
\]
where \( n_m(t) \) is the measurement noise, the volume labeled \( m \) means the volume belongs to the \( m \)th sensor and the volume labeled \( k \) means the volume belongs to the \( k \)th signal source.

We set \( a_m(\theta_k) \) is the response function of the \( m \)th sensor for the \( k \)th signal. It can be summarized as follows:

\[
a_m(\theta_k) = \exp[-j(m-1) \frac{2\pi d \sin \theta_k}{\lambda}], \tag{7}
\]

Then the received signal \( x_m(t) \) of the \( m \)th sensor can be written as:

\[
x_m(t) = \sum_{k=1}^{D} a_m(\theta_k) S_k(t) + n_m(t), \tag{8}
\]

where \( S_k(t) \) is the signal strength of the \( k \)th signal.

Using vector notation for the received signals of \( M \) sensors, the data model can be presented as:

\[
X(n) = AS(n) + N(n), \tag{9}
\]

where

\[
X(t) = [x_1(t), x_2(t), \ldots, x_M(t)]^T, \tag{10}
\]

\[
S(t) = [s_1(t), s_2(t), \ldots, s_M(t)]^T, \tag{11}
\]

\[
A = [a(\theta_1), a(\theta_2), \ldots, a(\theta_D)] = \begin{bmatrix}
1 & 1 & \cdots & 1 \\
e^{-j\phi_1} & e^{-j\phi_2} & \cdots & e^{-j\phi_D} \\
\vdots & \vdots & \ddots & \vdots \\
e^{-j(M-1)\phi_1} & e^{-j(M-1)\phi_2} & \cdots & e^{-j(M-1)\phi_D}
\end{bmatrix}, \tag{12}
\]

\[
\phi_k = \frac{2\pi d}{\lambda} \sin \theta_k, \tag{13}
\]

\[
N(t) = [n_1(t), n_2(t), \ldots, n_M(t)]^T, \tag{14}
\]

where \( k = 1, 2, \ldots, D \) and \( T \) superscript denotes transpose of a matrix.

The DOA problem is finding proper estimations of \( \theta_1, \theta_2, \ldots, \theta_D \) from a finite number \( N \) of data samples or snapshots of \( x_m(i) \) taken at times \( i, i = 1, 2, \ldots, N \) \[11\].

As a result, the array signal can be naturally regarded as the superposition of several space harmonic with the noise interference, thus the DOA estimation problem will be combined with the spectral estimation.

### 2.2. The Eigen-decomposition of Array Covariance Matrix

Make the processing about array output matrix \( R_0 \), then the covariance matrix can be received as:

\[
R_0 = E[XX^H], \tag{15}
\]

where \( H \) indicates the conjugate transpose of a matrix.

With the above assumptions, the noise and signal waveforms are uncorrelated, and the noise at each sensor is a stationary zero mean complex white Gaussian process. So, from (9) and (15), the \( M \times M \) covariance matrix \( R_0 \) of received signal becomes:

\[
R_0 = E[(AS + N)(AS + N)^H] = AE[SS^H]A^H + E[NN^H], \tag{16}
\]

\[
= AR_0A^H + R_N, \tag{17}
\]

where \( R_0 \) is the signal correlation matrix, \( R_N \) is the noise correlation matrix, \( \sigma^2 \) is the noise power at each sensor and \( I \) is an \( M \times M \) identity matrix.

In practice, the data covariance matrix \( R_0 \) is not available but a maximum likelihood estimate \( \hat{R}_0 \) based on a finite number \( N \) of data samples can be obtained as:

\[
\hat{R}_0 = \frac{1}{N} \sum_{i=1}^{N} X(i)X^H(i), \tag{18}
\]

And the estimation of direction of arrival of sources is based on this sample covariance matrix \( \hat{R}_0 \) [9, 12].

According to the theory of matrix eigendecomposition, the array covariance matrix \( R_0 \) can be decomposed. First the noise-free vector of received signals is given by:

\[
R_0 = AR_0A^H, \tag{19}
\]

For the uniform linear array, the matrix \( A \) in (12) is Vandermonde matrix, as long as the following condition is met:

\[
\theta_i \neq \theta_j, i \neq j \tag{20}
\]

Then, the columns of the matrix \( A \) are independent of each other, in addition, if the matrix \( R_0 \) is nonsingular matrix, and \( M > D \), then using the assumptions made in previous section, \( AR_0A^H \) in (19) is a rank \( D \) matrix and it can be shown as:

\[
\text{rank}(AR_0A^H) = D, \tag{21}
\]

From (15), we obtain:

\[
R_0^H = R_0, \tag{22}
\]
where $R_s$ is the Hermite matrix, and its eigenvalues are real number.

Besides, $R_s$ is positive definite matrix, so $AR_sA^H$ is positive semidefinite having $D$ positive eigenvalues and $M - D$ zero eigenvalues.

The signal-plus-noise model can be written as the following:

$$R_s = AR_sA^H + \sigma^2 I,$$  \hspace{1cm} (23)

where $\sigma^2 > 0$, $R_s$ is the full rank matrix, so $R_s$ has $M$ positive real eigenvalues ($\lambda_0, \lambda_1, \ldots, \lambda_M$) corresponding to the $M$ eigenvectors($v_0, v_1, \ldots, v_M$).

Moreover, $R_s$ is Hermite matrix, so its eigenvalues are orthogonal to each other:

$$v_i^H v_j = 0, \quad i \neq j$$ \hspace{1cm} (24)

The number of eigenvalues which associated with the signal is $D$. Its values equals to the $\sigma^2$ plus the eigenvalues of matrix $AR_sA^H$, and the $M - D$ eigenvalues is $\sigma^2$. That is to say, $\sigma^2$ is the minimum eigenvalues of the matrix $R_s$. In the next section, we can get the direction of arrival $\theta_i$ by exploiting the above properties of the eigendecomposition.

2.3. The MUSIC Algorithm Principle [13, 14]

After the eigendecomposition of the covariance matrix $R_s$, we received the following conclusion:

Compute the eigenvalues of $R_s$ that are in decreasing order, then its eigenvalues have the property:

$$\lambda_0 \geq \lambda_1 \geq \ldots \geq \lambda_D \geq \lambda_{D+1} = \cdots = \lambda_M = \sigma^2$$ \hspace{1cm} (25)

We can obtain from (25), being $D$ larger eigenvalues of $R_s$ associated with signals and $M - D$ smaller eigenvalues of $R_s$ associated with the noise.

Simultaneously, the eigenvectors of matrix $R_s$ belonging to these eigenvalues are also corresponding to the signal and the noise. Therefore, these eigenvalues (eigenvectors) can be divided into signal eigenvalues (eigenvectors) and noise eigenvalues (eigenvectors).

We set $\lambda_i$ is the $i^{th}$, $i = D+1, D+2, \ldots, M$ eigenvalue of matrix $R_s$, and its corresponding eigenvector is $v_i$, then we have:

$$R_s v_i = \lambda_i v_i,$$ \hspace{1cm} (26)

If $\lambda_i = \sigma^2$ is the minimum eigenvalue, then:

$$R_s v_i = \sigma^2 v_i,$$ \hspace{1cm} (27)

Equation (23) and (27) may be combined as:

$$\sigma^2 v_i = (AR_sA^H + \sigma^2 I)v_i,$$ \hspace{1cm} (28)

Expand the right side of (28), then compared with the left, (28) may be rewritten as:

$$AR_sA^H v_i = 0,$$ \hspace{1cm} (29)

For the $A^H A$ is the nonsingular matrix, and simultaneously, the matrix $(A^H A)^{-1}$ and $R_s^+$ exist. Then, both sides of (29) are multiplied by $R_s^+(A^H A)^{-1}A^H$. We can derive:

$$R_s^+(A^H A)^{-1}A^H AR_s A^H v_i = 0,$$ \hspace{1cm} (30)

and then (30) may be simplified to

$$A^H v_i = 0, \quad i = D+1, D+2, \ldots, M$$ \hspace{1cm} (31)

Equation (31) states that noise eigenvalues corresponding to the eigenvectors are orthogonal to each column of matrix $A$ that in fact is a steering vector corresponding to the angles of arrivals. The MUSIC algorithm uses this property to estimate direction of sources.

We can then construct the $M \times (M - D)$ noise subspace $E_n$ spanned by the noise eigenvectors such that:

$$E_n = [v_D, v_{D+1}, \ldots, v_M],$$ \hspace{1cm} (32)

We define the spectrum function of the MUSIC algorithm as:

$$P_n(\theta) = \frac{1}{a^H(\theta)E_n^H E_n a(\theta)} = \frac{1}{E_n^H a(\theta)},$$ \hspace{1cm} (33)

Equation (33) suggests that the denominator is the inner product of the steering vector and the noise subspace matrix. When the steering vector $a(\theta)$ is orthogonal to each column of matrix $E_n$, the denominator is zero. But due to the presence of noise, the denominator is a minimum value, which resulted in the spectrum defined in (33) having a sharp peak. Therefore, from the analysis above, it can be seen that we can estimate the angle of arrival by observing the peak in the case of the incident angle $\theta$ vary.

2.4. Implementation Steps of the MUSIC Algorithm [15]

1) According to the received signal vectors, we obtain the estimation of the following covariance matrix:

$$R_s v_i = \sigma^2 v_i,$$ \hspace{1cm} (27)
\[ R_x = \frac{1}{N} \sum_{i=1}^{N} X(i)X^H(i), \] (34)

Then the covariance matrix \( R_x \) can be expressed as:
\[ R_x = AR_x A^H + \sigma^2 I, \] (35)

2) By order of the eigenvalues of the matrix \( R_x \), we can acquire the signal subspace which is composed of the eigenvectors corresponding to the \( D \) eigenvalues that equals with the signal source and the noise subspace which is composed of the eigenvectors corresponding to the \( M-D \) eigenvalues, then we obtain the noise matrix \( E_n \):
\[ E_n = [v_{D+1}, v_{D+2}, \ldots, v_M], \] (36)

3) The DOAs of the incident signal sources can be estimated by locating the largest peaks of the function
\[ P_m(\theta) = \frac{1}{a^H(\theta)E_nE_n^H a(\theta)}, \] (37)

Estimates source directions from the spectrum obtained by the superposition of narrowband MUSIC spectra evaluated across several bins [16].

3. Proposed Method for DOA Estimation

3.1. Modified MUSIC Algorithm (MMUSIC)

The MUSIC algorithm is only for the spatial spectrum estimation of incoherent signals. When signal sources are coherent, the coherent signals will unite into one signal, then the independent signal sources that received by the array will decrease, which lead to the array covariance matrix rank reduce and the number of larger eigenvalues less than the incoming signal. Spatial spectral curve does not present the peak, thus cannot obtain the correct signal DOA estimation.

Hence, if we want to estimate the coherent signal DOA accurately, we must remove the correlation between the signal [17].

In MUSIC algorithm, the received signals model is given by:
\[ X(n) = AS(n) + N(n), \] (38)

where \( X'(n) \) denotes the complex conjugate of \( X(n) \), \( I_M \) is an \( M \times M \) identity matrix, and \( J_M \) is the \( M \times M \) exchange matrix. It can be shown as:
\[ J_M = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 1 & \cdots & 0 \\ 1 & 0 & \cdots & 0 \end{bmatrix}_{M \times M}, \] (40)

Obviously, \( J_MJ_M = I_M \). The covariance matrix \( R_x \) is expressed as:
\[ R_x = E[\{Y(n)Y^H(n)\}] = J_MAR_x^H(A^H)^HJ_M + \sigma^2 I_M, \] (41)

The matrix \( D \) is assumed such that:
\[ D = \text{diag}[e^{-j2\pi d \sin \theta_1}, e^{-j2\pi d \sin \theta_2}, \ldots, e^{-j2\pi d \sin \theta_D}], \] (42)

where \( \varphi_k = \frac{2\pi d}{\lambda} \sin \theta_k \), for \( k = 1, 2, \ldots, D \), and we obtain:
\[ J_MA' = AD', \] (43)

Using (41) and (43), the utilizing of the order of diagonal matrix product is exchangeable, so the covariance matrix \( R_x \) can be rewritten as:
\[ R_x = AR_x A^H + \delta^2 I = R_x, \] (44)

Then the covariance matrix \( R \) can be finally defined as follows:
\[ R = (R_x + R_y) / 2 = (R_x + J_M R_x J_M) / 2, \] (45)

It can be seen from the formula derivation process, the essence of the modified music algorithm is the special situation of the spatial before and after smoothing algorithm, which the length of subarray equals with the number of array element [18].

3.2. Improved Modified MUSIC Algorithm (IMMUSIC)

The spatial smoothing algorithm has \( M \) array elements in the uniform linear array. The \( M \) sensors divided into \( P \) \((P = M - q + 1)\) subarrays with \( q \) array elements, and \( R \) is the covariance matrix of the received data. Besides, the principle of matrix decomposition algorithm [19] is as follows:
From the \( l + 1 \) row to \( l + p \) row of the covariance matrix \( R \), we define a new matrix \( R^{(l)} \) as follows:

\[
R^{(l)} = \begin{bmatrix}
\gamma_{l+1,1} & \gamma_{l+1,2} & \cdots & \gamma_{l+1,M} \\
\vdots & \vdots & & \vdots \\
\gamma_{l+p,1} & \gamma_{l+p,2} & \cdots & \gamma_{l+p,M}
\end{bmatrix},
\]

(46)

According to the matrix \( R^{(l)} \), the modified matrix \( M_R \) is given by:

\[
M_R = \left[ R^{(0)} \quad R^{(1)} \quad \cdots \quad R^{(M-p)} \right],
\]

(47)

It is clearly that the matrix \( M_R \) is \( P \times [(M - P + 1) \times M] \) dimension. Therefore, as long as meeting certain conditions \([20, 21]\), the matrix \( M_R \) rank equals with the signal source \( D \). Obviously, the matrix decomposition algorithm written in (47) is the matrix reconstruction under the ideal condition.

Similar with the forward-backward spatial smoothing algorithm \([22]\), we add reverse smooth item in the modified matrix \( M_R \) in order to improve the performance of coherent solutions. Then the improved matrix \( M_{R1} \) is represented as:

\[
M_{R1} = \left[ R^{(0)} \quad \cdots \quad R^{(M-p)} \quad JR^{(0)} \quad \cdots \quad JR^{(M-p)} \right],
\]

(48)

Based on the above IMMUSIC algorithm, the \( q = M, P = 1 \) is the special circumstance of the spatial smoothing algorithm. We improve the covariance matrix \( M_{R1} \) in (48), it can be rewritten as:

\[
M_{R1} = \left[ R_s \quad JR_s \right],
\]

(49)

Using (41), (49) can be simplified as:

\[
R = \left[ R_s \quad R_s \right],
\]

(50)

Through singular value decomposition (SVD) on the matrix \( R \), we will obtain the signal subspace, noise subspace and singular value. Thus using the MUSIC algorithm, we can estimate the signal source DOA.

The IMMUSIC algorithm is combined which the spatial smoothing algorithm and the matrix decomposition technology. Compared to the matrix decomposition algorithm, the IMMUSIC method has less computational load in resolving the uncorrelated and correlated sources.

4. Simulation Results

In all simulations, a uniform linear array of eight sensors is used as shown in Fig. 1. Spacing between elements is assumed to be \( 0.5 \lambda \).

In the first experiment, we consider three uncorrelated sources \( (p = 3) \) with direction of arrivals \(-20^\circ, 40^\circ, 60^\circ\). Fig. 2 shows the classic MUSIC and the modified MUSIC algorithm with SNR=10 dB. It can be verified from Fig. 2 that in all successful simulations of the classic MUSIC method the MMUSIC algorithm has also been successful.

In the second experiment, we consider three sources \( (p = 3) \) with direction of arrivals \(-20^\circ, 40^\circ, 60^\circ\). The first two sources are coherent which irrelevant with the third source. The SNR of both sources is 10 dB. We have preformed the classic MUSIC and the modified music algorithm in Fig. 3. This figure shows the conventional MUSIC is failure when the sources are coherent. By comparison, the MMUSIC can estimate the signal DOA accurately.

In the third experiment, we consider three sources \( (p = 3) \) with direction of arrivals \(-20^\circ, 40^\circ, 60^\circ\). The first two sources are coherent, but irrelevant with the third source. The SNR of both sources is -5 dB. We have preformed the modified MUSIC and the improved modified MUSIC algorithm in Fig. 4. This figure shows the estimation performance of the MMUSIC degrades when the signal-to-noise is low. On the contrary, the improved modified MUSIC spectrum takes the form of sharper peak in which angular resolution is improved.
5. Conclusion

In this paper, a new algorithm which has less computational load than the matrix decomposition and has better resolution capability than the spatial smoothing algorithm is presented for the case of low SNR. Its superiority over MUSIC method was shown through simulations. Simulations also proved the advantage of proposed method in resolving coherent sources. Simulation results have confirmed the theoretical analysis and have showed the effectiveness of the proposed method in the case of low SNR.

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References

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