Signal Processing of Vortex Flow with Noise Based on Wavelet Analysis

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Abstract: The vortex of low speed flows is difficult to measure resulted in that the output signal of vortex flowmeter appears the spectral line splitting and frequency offset in the low SNR and causes the frequency resolution to be decreased. In the paper, it first conducted the multi-scale wavelet decomposition to the vortex signal with noise, then removed the noise spectrum and carried on power spectrum analysis and frequency correction for the actual signal of stress type vortex flowmeter, and finally determined the vortex signal frequency to be as the basis for designing band-pass filter. By means of Matlab software, it took a signal with noise as an example, and reconstructed an effective sine signal by wavelet denoising algorithm. The simulation experiment shows that the presented method is effective.

Keywords: Vortex flow, Hydrometry, Signal processing, Wavelet analysis.

1. Introduction

With the speed development of industrial process, energy metering, city public utilities etc., the demand on flow measurement has got a greatly rapid growth since twentieth century, and many new type flowmeter come to the fore such as bamboo shoots after a spring rain like [1]. Vortex flow meter is a new type of flow rate, and because it has the advantages of relatively wide range [2-3], high accuracy and small pressure loss etc., it has got the rapid development and has been among the general flow meter column in recent years. But at present, the vortex flowmeter is still difficult to guarantee the measurement precision at low flow rates, so it is the main problem of vortex flowmeter signal processing for how to extract the useful signal of vortex at low frequencies from the superposition of noise signal [4]. Up to now, the processing method of vortex flow signal mainly are the cross-correlation method, spectrum analysis, FFT spectrum analysis method, and signal threshold detection method proposed Rosemount corporation and so on [5].

The ideal output of vortex flowmeter is sine periodic signal [1, 6], and its power is mainly concentrated in the low flow period. Therefore it can improve the precision of the measurement of low frequency signals by using the wavelet analysis method, and the wavelet analysis method can extract the main frequency of the useful signal from signal with noise.

2. Measurement Principle of Vortex Flowmeter

By using the principle of fluid natural oscillation and detecting formed Kaman vortex frequency after vortex occurs, the vortex flowmeter can find the
volume flow rate through the pipeline, and the basic relation expression is shown as in formula (1):

\[ f = s, \frac{v}{d}, f \in L^2(\mathbb{R}) \]  

(1)

In which \( v \) (m/s) is the mean velocity of fluid that is not affected by noise interference, \( d \) (m) is the effective diameter of the vortex occurring body, and \( f \) is the Kaman vortex frequency. In the measuring range (Re from \( 5 \times 10^3 \) to \( 1.5 \times 10^5 \)), \( St \) is a constant, and the volume flow of fluid flowing through the pipe is shown as in formula (2):

\[ Q = \frac{\pi D^2}{4S_s} \left(1-1.25 \cdot \frac{d}{D}\right) \]  

(2)

In which \( Q \) denotes the volume flow rate (m³/s), and \( D \) is the pipe diameter (m). The formula (2) is the measurement basis of a vortex flowmeter. From the equation (2), it can be seen that the key of determining flow is to measure vortex frequency \( f \). But due to the impact of factors such as change in velocity pulse by the fluid turbulent state, pipeline random vibration in industry field measurement and so on, the output signal of vortex frequency transmitter is not an ideal sinusoidal signal, but mixed with noise signals (mainly white noise), and therefore it must carry on filtering processing [7-8].

3. Signal Processing of Wavelet Denoising

The output of vortex flowmeter theoretically is a sine wave [9]. Although it contained different noise components in the output signal of actual vortex flow, but under the condition of the signal not to be drowned out by the interference, the main energy is still focused on the frequency of useful sine signal [10]. Based on the above theory, it can perform wavelet transform on the actual vortex signal, draw the diagram of signal in frequency domain, find the highest peak of spectrum and the biggest point of spectrum width to length ratio, and thereby extrapolate the point of the frequency for the measurement of vortex flow value [11].

The wavelet transform and the traditional Fourier transform (STFT) (namely windowed Fourier transform) come down in one continuous line [12], but the Fourier transform (STFT) has the advantages of less. The formula (3) is the signal \( f(x) \) wavelet transform under the wavelet function \( \Psi \).

\[ W_{\psi,f}(b,a) = \left|\frac{1}{a}\int_{-\infty}^{\infty} f(x) \Psi\left(\frac{x-b}{a}\right) dx\right| \]  

(3)

The scale parameter \( a \) and displacement parameter \( b \) in the wavelet transform can also change at the same time. For the high-frequency spectrum information, the time interval can be relatively reduced (scale a reduced) so as to make the frequency precision be better, and for the low frequency spectrum information, the time interval can be relatively increased (scale a increased) so as to make frequency information be summarized, and that is to say that this is called time-frequency localization. Based on the above characteristics, the wavelet analysis of signal processing is more fine and flexible, and it is more in time and effectively to remove the noise.

The ideal sine signal is continuous, differentiable and smooth, and the emergence of the noise has randomness. It is almost everywhere singular, and when it makes multi-scale wavelet transform for the two signals, the wavelet spectrum owns different characteristic. Its wavelet spectrum of effective signal increases with the scale, the number of maximum (high frequency signal point) is more stable, and it has obvious inheritance characteristic. But the number of random noise spectrum maximum points reduced with the scale increases significantly, and the emergence moment of maximum point between adjacent scales has no inheritance characteristic.

From the view of functional analysis, because the wavelet operator \( \Psi \) is integrable on real line of Lebesgue square, it has the formula (4):

\[ \int_{-\infty}^{\infty} |\psi(x)|^2 dx < \infty \]  

(4)

It is also absolutely integrable, i.e.

\[ \int_{-\infty}^{\infty} |\psi(x)| dx < \infty \]  

(5)

The assumption that the signal \( x \) is absolutely integrable, then it has formula (6):

\[ \|\psi(x)\| \leq \|\psi\| \|x\| \]  

(6)

The white noise is the noise signal of expectation being as zero, and it is absolutely integrable obviously [11]. With the increasing of scale (even at smaller scales), the time interval of integral increases, and the norm \( \|x\| \) of noise signal \( x \) will tend to expected value zero. And from formula (6), it can be seen that its wavelet spectrum expectation \( \|\psi(x)\| \) will also tend to zero. Particularly important is that for wavelet spectrum of effective mutation signal in the signal increases with the scale (at smaller scales), the norm is instead increased, the polarity is more obvious, and it has stronger inheritance characteristic.

Let \( n(x) \) be the real and wide stationary white noise, and the variance is \( \sigma^2 \), then the expectations of wavelet transform \( W_n(s, x) \) for white noise is shown as in formula (7):

\[ E[|W_n(s, x)|^2] = \frac{|\Psi|^2 \cdot \sigma^2}{s} \]  

(7)
In which $\Psi$ is the wavelet operator, $s$ denotes the scale of wavelet transform, and it can be seen that the attenuation of $E(|W_n(s, x)|^2)$ is proportional to $1/s$, namely along with the increasing of scale, the amplitude on average of white noise wavelet transform reduces. If $n(x)$ is Gauss white noise, then in the scale $s$, the average density of the wavelet transform is shown as in formula (8):

$$d_s = \frac{1}{s^2 \pi} \left| \frac{\Psi(1)}{2} + \frac{\Psi(2)}{2} \right|$$

(8)

In which $\Psi(1)$ and $\Psi(2)$ are respectively the one-order and two-order derivative. From formula (8), it can be seen that $d_s$ is proportional to $1/s$, and if the scale increases then the density decreases.

From the contrastive analysis for an effective signal and white noise and their high frequency coefficient of wavelet transform, it can get the results shown in Fig. 1. The graphic in Fig. 1(a) and Fig. 1(b) is respectively the high frequency coefficient map of sine signal and white noise under condition of four scales db6 of wavelet transform. From Fig. 1(a), it can be seen that the original signal is composed of two sections of different frequency sine signal formed by splicing, and there are three mutation points. After making wavelet transform of four scales, the position of mutation signal point is highlighted, and with the increase of the scale of $j$, the number of extreme points is very stable. And the emergence of the approximate position is respectively as the following, if $j = 1$ then it is 0, 200 and 400, if $j = 2$ then it is 0, 100 and 200, if $j = 3$ then it is 0, 50 and 100, if $j = 4$ then it is 0, 25 and 50. The position of emergence is $1/2$ of maximum position number of previous scale, and it can be seen that it has very strong inheritance characteristic. From Fig. 1(b), it can be seen that the number and amplitude of extremum point of white noise wavelet spectrum decreases rapidly with increasing scale of $j$, and the emergence location is any relation with scale change.

![Fig. 1](image)

**Fig. 1.** High frequency coefficients of wavelet transform.

According to the above theory, as long as it removes the wavelet spectrum component generated by the noise on each scale, especially the noise wavelet spectrum component that the noise spectrum is dominated by scales, then what the wavelet spectrum preserved basically is the wavelet spectrum of effective signal. If it reuses the wavelet transform and reconstruction algorithm, then it can reconstruct the effective signal. The denoising process is as follows.

1) Using Mallat algorithm [3], it can find the wavelet decomposition of noise signal $f(x)$ in each scale $j$ ($j < N$).

$$S_{2^j}f(x) = \sum_{k \in \mathbb{Z}} h_k S_{2^j} f \left( x - 2^j k \right)$$

(9)

$$W_{2^j}f(x) = \sum_{k \in \mathbb{Z}} g_k S_{2^j} f \left( x - 2^j k \right)$$

(10)
In which $h$, $g$ denotes respectively the corresponding low-pass filter and high pass filter coefficient of the wavelet function $\Psi$. The formula (9) is the smooth signal of wavelet decomposition, and the formula (10) is its high frequency signal.

2) Selecting threshold makes the soft threshold processing for high frequency coefficient from scale N to scale 1. If some modulus maximum points of wavelet transform amplitude decrease with the scale increase sharply, then it shows that these maximum points are almost controlled by white noise, and it should be removed. If some modulus maximum points of wavelet transform do not spread to larger scales, then it should also be removed. Generally, it sets a threshold for high frequency coefficient in the high scale to filter out the noise wavelet spectrum, keeps the wavelet spectrum of effective mutation signal, and then finds out the wavelet spectrum of effective mutation signal for low scale. The theory proposed in the literature can be found in Mallat algorithm [4].

3) From the low frequency coefficient of scale N (smooth signal) and high frequency coefficient through processing for from scale 1 to scale N, it can makes the reconstruction of the signal so as to get the denoising signal.

Many scholars have also put forward some wavelet denoising algorithms, for example, the iterative projection method between convex multiple Hilbert space (POCS) in reference [2], and simplified algorithm in reference [5] etc.

4. Simulation Result and Its Analysis

In the experiment, the signal with noise is still composed of two sine signals of different frequency after stitching and mixing white noise. The Fig. 2(a) is the noise free signal, and Fig. 2(b) is the signal with noise. It is visible, because of low in SNR, effective signal submerged in noise, and peak pulse has been replaced by the noise, and the vortex flowmeter is unable to correctly measure the flow.

Now it takes the db6 wavelet respectively to actualize the wavelet denoising of largest scale N from 3 to 8 for the signal with noise, and the simulation results are shown in Fig. 2.

From Fig. 2, it can be seen that with the increase of the scale of N the effect of noise filtering gradually becomes better. When it is the scale of $N = 8$ (the last figure), although the amplitude and the effective signal has also some differences, but the peak has come out prominently, and therefore the correct fluid frequency can be determined. Here what the scale needs to reach 8 is due to the SNR being small. It can be seen that for signals with different signal to noise ratio the selection of wavelet scale is different, and therefore it should be determined according to the effect. After considering the above some basic problems, it can actualize the signal processing for actual output signal of a stress type vortex flowmeter, and when the flow of vortex flowmeter is approximately to be as 0.50, 0.60, 0.70,
0.80, 0.90 and 1.00 m/s, the result of signal acquisition and processing is shown as in Fig. 3.

![Fig. 3. Approximate numerical analysis of periodic wavelet.](image)

Transverse coordinate in Fig. 3 is the flow velocity for each sampling point, and the unit is m/s. The ordinate represents the average value of the periodic wavelet calculation. Through the analysis of power spectrum for the signal, it can find out the following conclusions.

1) The average value of the periodic wavelet at different frequency calculated decreases with decreasing flow rate, and there is a linear relation.

2) Using linear prediction method, it can get the standard output of vortex signal for each point above, calculate the analysis error spectrum, and all the error for each the point is less than 6%.

So, the Wavelet calculation has certain superiority in measuring low flow rate, but when the vortex flowmeter is lower flow rate (flow rate < 0.5 m/s), it is difficult to extract the useful signal by using wavelet analysis method. This is caused by defects of wavelet analysis method itself.

5. Conclusions

For simulation of the vortex signal, it can improve the signal measurement precision of low flow rate in a certain extent by wavelet analysis method, and effectively extract the actual vortex signal produced by the flow rate. Due to defects of wavelet analysis method itself, there are still many problems in the extended range of vortex flowmeter with low flow rate, and it can be considered that some artificial intelligence methods are combined with spectrum analysis method so as to solve the problem of signal processing for the vortex flow signal of low frequency.

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