Application of Singular Value Energy Difference Spectrum in Axis Trace Refinement

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Abstract: Focusing on purification of axis trace of rotating machinery, a new refinement method was proposed by application of singular value decomposition. At first, the original vibration signal was reconstructed in phase space by Hankel matrix; then the attractor track matrix was decomposed with singular value decomposition. Then, in order to determine the reconstruction order number of singular values, the concept of energy difference spectrum of singular values was defined; then the reconstruction order number was determined according to the peak position of the energy difference spectrum. In the end, the effectiveness of the method was proved by simulation and successful purification of axis trace coming from the turbine generator units of power plant. Simulation and practical application show the proposed method can retain the original signal characteristic effectively and eliminate noise as much as possible. It will provide a new way for axis trace refinement.

Keywords: Energy difference spectrum of singular value, Singular value decomposition, Axis trace, Refinement, Rotating machinery.

1. Introduction

Vibration monitoring is a very important content for turbine generator unit. And the axis trace can express the vibration characteristic of generator unit. Different axis trace represents different rotor condition or fault information, such as ellipse represents unbalance fault, outer eight represents misalignment, and inner eight represents oil-film whirl, et al. [1]. Generally speaking, the original axis trace can’t be used due to the noise interference. How to eliminate noise and to obtain the clear axis trace is the main studying content for axis trace purification.

At present, more and more studies have been done in axis trace purification; the common used methods are digital/analog low-pass filtering and wavelet transformation. The wavelet transformation uses wavelet de-noising principle, its basic method is to decompose original signal into different frequency bands by wavelet transformation, then wave in time domain will be obtained in different frequency bands, and the purification of signal could be realized by signal reconstruction of some frequency bands. The wavelet transformation commonly uses binary way, the amount of data and detailed signal will be lost. And the results of wavelet package transformation have the problem of energy overlapping in different frequency bands [2].

In recent years, the morphological filtering technology has become an attractive non-linear filtering method; its application has spread from power system to mechanical engineering. It has been introduced morphological filter in signal de-noising [3] and axis trace purification [4].

2. Principle of Singular Value Decomposition

Singular value decomposition (SVD) is a powerful numerical technique for solving systems of linear equations, and is easy to implement for order matrix calculations. It also has the advantage that it can deal with sets of equations that are close to singular, which would happen if the measured angular data were redundant because of co-linearity of vectors [12].

It is well know from linear algebra that any matrix $A$ with $M$ rows and $N$ columns can be written in terms of its singular value decomposition, i.e., as the product of an $M\times M$ column-orthogonal matrix $U$, and an $M\times N$ diagonal matrix $D$, with nonnegative diagonal elements, and the transpose of an $N\times N$ orthogonal matrix $V$. That is,

$$A = U D V^T,$$  \hspace{1cm} (1)

where $D=\text{diag}(\sigma_1, \sigma_2, \ldots, \sigma_N)$ or its transposition, which depends on $M < N$ or $M > N$. And 0 refers to zero matrix. Moreover, $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_N > 0$, and the numbers $\sigma_i$ are called the singular values of $A$. If $M=N$, $D$ is an $M\times M$ diagonal matrix with the singular values in decreasing order on its main diagonal. If $M<N$, $D$ consists of a $M\times M$ diagonal matrix with the $M$ singular values on its main diagonal (in decreasing order) extended on the right-hand side with a $M\times (N-M)$ matrix of zeros. If $M>N$, $D$ consists of a $N\times N$ diagonal matrix with the $N$ singular values on its main diagonal (in decreasing order) on top of a $(M-N)\times N$ matrix of zeros. The de-noising principle of singular value decomposition is using the energy difference between useful signal and noises, the matrix constructed by noise interrupted signal is decomposed, then the useful singular values are obtained in reconstructed procedure, and the singular values corresponding with the noise are replaced by zero [13].

Let $x(i)$ as sample data, here $i=1, 2, \ldots, N$. The matrix $A$ is constructed by phase space reconstruction theory.

That is,

$$A = \begin{bmatrix} x_1 & x_2 & \ldots & x_n \\ x_2 & x_3 & \ldots & x_{n+1} \\ \vdots & \vdots & \ddots & \vdots \\ x_m & x_{m+1} & \ldots & x_{m+n-1} \end{bmatrix}.$$  \hspace{1cm} (2)

where $1<n<N$, and $m+n-1=N$, this matrix is called Hankel matrix.

The difficulty of using singular value decomposition to process the interrupted signal is how to determine the number of array for reconstructed matrix and the effective rank order. According to the conclusion from ZHAO [10] and simulation experiments, if $N$ is even number, the column $n=N/2$ and the row $m=N/2+1$; while $N$ is odd number, the column and the row may be the same as $(N+1)/2$.

In signal reconstruction procedure, determination of the effective rank order is very important. When the less singular values are selected, due to the low de-noising rank order, useful information may be lost; while the more singular values are selected, due to the high de-noising rank order, some noises will be included in reconstructed signal. Therefore, in this paper, the energy difference spectrum of singular value is defined, and the effective rank order is determined by the singular value energy between useful signal and noise interference.

Wang [13] said that signal energy could be defined by singular value as follows:

$$E = \sum_{i=1}^{N} \sigma_i^2$$  \hspace{1cm} (3)

Then, define the energy difference spectrum of singular value and normalized it as follows:

$$p(i) = \frac{\sigma_i^2 - \sigma_{i+1}^2}{E}$$  \hspace{1cm} (4)

The numbers of $p(i)$ are called energy difference spectrum of singular value. Due to energy of the useful signal is bigger than that of the noises, in the boundary of useful signal and noises,
the energy difference spectrum must have a peak. So we can search the peak position as the effective rank order for the reconstructed matrix. Zhao [9] divided signal into signal with direct-current component and signal without direct-current. When signal with direct-current component, the peak of energy difference spectrum will be gotten in the first position. If we use this peak position as the effective rank order, we will only get the direct-current component. So, in this paper, we only study the vibration signal without direct-current component.

3. Simulation

In order to test the good refinement performance of singular value decomposition, we simulate the common four axis traces by using the same parameters as that of Li [14]. And add the periodical spike pulse interference and white noises. Now use singular value decomposition for refinement of simulated axis trace. According to the definition of energy difference spectrum of singular value, the rank order of reconstructed matrix can be selected automatically. Fig. 1 to Fig. 4 show the simulation signal and the refinement results by the proposed method.
From the above figures, we can see that the noise interferences are eliminated successfully and the refinement of axis traces are obtained by using the proposed method.

4. Practical Axis Trace Refinement

In this section, the proposed method is applied to purify the practical axis trace. Fig. 5 shows two practical axis traces of the turbine generator units in the power plant. The speed of these rotors is near 3000 rpm. Let the sampling frequency equal to 6400 Hz. Due to the serious noise interference, these two original axis traces are too disordered to get any fault information. Now we use the proposed method to process these two signals and Fig. 6 show the refined axis traces.

Fig. 6 shows the refined axis traces by the proposed method. According to the common fault spectrum feature, we know that these elliptical axis traces mean unbalanced fault for the turbine generator units.
5. Conclusions

In this paper, a novel refinement method for axis trace is proposed by using singular value decomposition. The definition of energy difference spectrum of singular value is introduced and it has been used to determine the effective rank order of reconstructed matrix. When using the singular value decomposition to process the vibration signal, there is no need to consider the frequency-domain feature for the sample data, and it only needs to process the signal in time-domain. Simulation and practical application show the good performance of the proposed method in axis trace refinement. It will supply a new method for fault diagnosis of rotating machinery.

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References


