

An Achievable Rate Region for Relay Multiple Access Channel Based on Decode-and-Forward

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Abstract: In this paper, we model a specific relay channel named relay multiple access channel (RMAC) aiming at providing a clue to select the relay node in multiple access channels such as the uplink of cellular communication system. In this channel, the relay node not only aids to send the information of the source but also sends its private message to the destination. We derive the achievable rate regions based on decode-and-forward strategy for the discrete memoryless RMAC (DM-RMAC) and the additive white Gaussian noise RMAC (AWGN-RMAC), respectively. Numerical results of the achievable rate region for the AWGN-RMAC show that the proposed scheme can improve the rate of user far from the base station with the help of a relay node near to the base station. Copyright © 2013 IFSA.

Keywords: Relay multiple access channels, Relay channels, Multiple access channels, Achievable rate region, Decode-and-forward.

1. Introduction

In wireless communication, the capacity of point-to-point link can be improved by cooperative users [1, 2]. The relay channel, first introduced in 1971 [3], is the most basic channel with cooperation. In this channel, one sender intends to communicate with one receiver with an idle relay to help the communication from the sender to the receiver. And then the capacities of the physically degraded relay channel, the reversely physically degraded relay channel and the relay channel with feedback are determined in [4]. In addition, it provides an achievable rate based on decode-and-forward strategy for the general relay channel. We call this relay channel as the general relay channel in order to differentiate from the channel discussed in this paper. In the general relay channel, the relay node has no private messages to communicate with other nodes and its all resources

are used to aid the other nodes. In fact, it is difficult to find such idle relay nodes in some practical communications. So some network nodes that have few communication traffic or low rate may be selected as the candidate relay node [5, 7, 8]. The article [5] introduces a specific relay channel with private messages, where the relay node has private messages to communicate with both the source and the destination nodes. However, this scenario is also rare in practical telecommunication systems. In [7, 8], the authors consider and provide a rate region of the relay broadcast channel, where the relay node needs receiving its private message from the source node.

Inspired by this, we considered another specific relay channel named relay multiple access channel (RMAC) in this paper. In the channel, the relay node not only aids to send the information of the source but also sends its private message to the destination node. In this paper, we derive the achievable rate

regions based on decode-and-forward strategy introduced in [4] for the discrete memoryless RMAC (DM-RMAC) and the additive white Gaussian noise RMAC (AWGN-RMAC), respectively. Considering the practical applications, we only give numerical results of the achievable rate region for the AWGN-RMAC. It shows that the node with low communication rate to the destination can be selected as the relay node to improve the rate from the source to the same destination.

This paper is organized as follows. In Section 2, the channel RMAC is defined. The achievable rate regions for the channels of DM-RMAC and AWGN-RMAC are respectively derived by the decode-and-forward strategy in Section 3. Numerical results of the achievable rate regions for the AWGN-RMAC are also provided in this section. Discussion and conclusion are given in section 4.

Before proceeding, we define some notations. Let X and \mathcal{X} denote a random variable and its range, respectively. Deterministic variable or the realization of a random variable and vector are denoted by lower case letter x and bold lower case letter \mathbf{x} , respectively. And define $X_t^i \triangleq (X_{t,1}, X_{t,2}, \dots, X_{t,i})$, $C(x) = \frac{1}{2} \log(1+x)$ and $\bar{x} = 1-x$. Throughout the paper, the logarithmic function is to the Base 2.

2. Channel Model

The channel model of RMAC is given in Fig. 1. And the definitions involved in the channel are illustrated as follows.

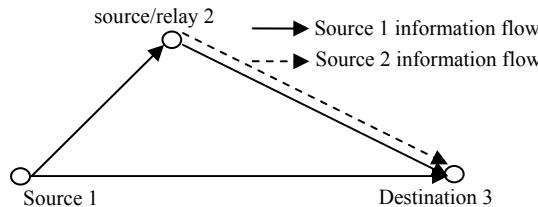


Fig. 1. The relay multiple access channel.

Definition 1: The DM-RMAC consists of an channel input alphabet \mathcal{X}_1 , a relay input alphabet \mathcal{X}_2 , two channel output alphabets \mathcal{Y}_2 and \mathcal{Y}_3 , and a probability transition function $p(y_2, y_3 | x_1, x_2)$, where x_1, x_2 denote source and relay inputs, respectively, while y_2 and y_3 denote the outputs at the relay and destination nodes, respectively.

Definition 2: A $((2^{nR_{13}}, 2^{nR_{23}}), n)$ code for a DM-RMAC consists of the following components:

Two index sets,

$$\mathcal{W}_{23} = \{1, 2, \dots, 2^{nR_{23}}\} \text{ and } \mathcal{W}_{13} = \{1, 2, \dots, 2^{nR_{13}}\}.$$

An encoder at the source terminal,

$$X_1 : \mathcal{W}_{13} \rightarrow \mathcal{X}_1^n.$$

A set of relay functions $\{f_i\}_{i=1}^n$,

$$x_{2,i} = f_i(y_{2,1}, \dots, y_{2,i-1}, w_{23}), \quad 1 \leq i \leq n.$$

A decoder at the destination terminal,

$$d : \mathcal{Y}_3^n \rightarrow \mathcal{W}_{13} \times \mathcal{W}_{23}.$$

In this paper, the relay node is assumed to operate in full duplex and to be causal. Its encoding and decoding structure is illustrated in Fig. 2.

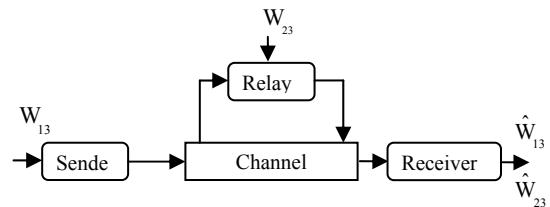


Fig. 2. The encoding and decoding structure for DM-RMAC.

Definition 3: The RMAC is said to be degraded if its transition probability satisfies

$$p(y_2, y_3 | x_1, x_2) = p(y_2 | x_1, x_2)p(y_3 | y_2, x_2),$$

i.e., $X_1 - [Y_2, X_2] - Y_3$ forms a Markov chain.

Definition 4: The AWGN-RMAC has a continuous input and output alphabets and independent, additive white Gaussian noise. The channel outputs of the relay and destination are

$$Y_2 = X_1 + Z_2, \quad (1)$$

$$Y_3 = X_1 + X_2 + Z_3, \quad (2)$$

where $Z_2 \sim \mathcal{N}(0, N_2)$ and $Z_3 \sim \mathcal{N}(0, N_3)$ are the independent Gaussian noise. There are power constraints on the input sequences \mathbf{x}_1 and \mathbf{x}_2 , namely

$$\mathbb{E}[\mathbf{x}_i^2] < P_i, \quad i = 1, 2.$$

The channel model of AWGN-RMAC is illustrated in Fig. 3.

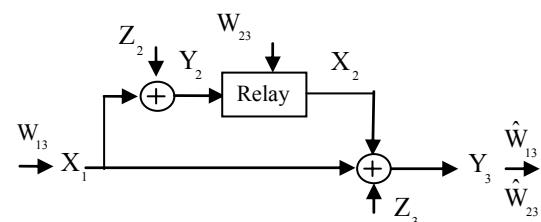


Fig. 3. The channel model of AWGN-RMAC.

Definition 5: A degraded AWGN-RMAC has a continuous input and output alphabets and independent additive white Gaussian noise given the input and the output of the relay node. The channel outputs of the relay and the destination are written as

$$Y_2 = X_1 + Z_2, \quad (3)$$

$$Y_3 = X_1 + X_2 + Z_2 + Z', \quad (4)$$

where $Z_2 \sim \mathcal{N}(0, N_2)$ and $Z' \sim \mathcal{N}(0, N_3 - N_2)$ are the independent Gaussian noise, and $N_2 < N_3$.

Definition 6: The average probability of the error is defined as the probability that the decoded messages are not equal to the transmitted messages, i.e.,

$$P_e^{(n)} = P\left(\hat{W}_{13}, \hat{W}_{23} \neq (W_{13}, W_{23})\right),$$

where \hat{W} denotes an estimation of W , and (W_{13}, W_{23}) are assumed to be uniformly distributed over $\mathcal{W}'_{13} \times \mathcal{W}'_{23}$.

Definition 7: A rate pair (R_{13}, R_{23}) is said to be achievable for the RMAC if there exists a sequence of $((2^{nR_{13}}, 2^{nR_{23}}), n)$ codes with $P_e^{(n)} \rightarrow 0$ as $n \rightarrow \infty$.

3. Main Result

3.1. Discrete Memoryless Channels

In this subsection, we derive an achievable rate region for the DM-RMAC based on decode-and-forward strategy.

Theorem 1: A rate pair (R_{13}, R_{23}) is said to be achievable for the DM-RMAC if the following inequalities hold

$$R_{23} < I(X_2; Y_3 | U, X_1), \quad (5)$$

$$R_{13} < \min\{I(U, X_1; Y_3), I(X_1; Y_2 | U, X_2)\}, \quad (6)$$

for

$$p(u)p(x_1|u)p(x_2|u)p(y_2, y_3|x_1, x_2)$$

Proof: The regular block Markov encoding and backward decoding technique [4] is used in this paper. We consider a transmission over B blocks, each of n symbols. A sequence of $B-1$ messages $W_{13}(b)$ and $W_{23}(b)$ will be sent over the channel in nB transmissions, where b denotes the block index, $b=1, 2, \dots, B-1$. The rate pair $(R_{13} \frac{B-1}{B}, R_{23} \frac{B-1}{B})$ approaches (R_{13}, R_{23}) as $B \rightarrow \infty$. An outline of the proof is as follows.

Fix $p(u, x_1, x_2) = p(u)p(x_1|u)p(x_2|u)$.

Random codebook construction:

- 1) Generate $2^{nR_{13}}$ sequences $\mathbf{u} = (u_1, u_2, \dots, u_n)$ according to $p(\mathbf{u}) = \prod_{i=1}^n p(u_i)$, label them $\mathbf{u}(w'_{13})$.
- 2) For each $\mathbf{u}(w'_{13})$, generate $2^{nR_{13}}$ sequences \mathbf{x}_1 according to $p(\mathbf{x}_1) = \prod_{i=1}^n p(x_{1,i}|u_i(w'_{13}))$, denote them as $\mathbf{x}_1(w'_{13}, w_{13})$.
- 3) For each $\mathbf{u}(w'_{13})$, generate $2^{nR_{23}}$ sequences \mathbf{x}_2 according to $p(\mathbf{x}_2) = \prod_{i=1}^n p(x_{2,i}|u_i(w'_{13}))$, label them $\mathbf{x}_2(w'_{13}, w_{23})$.

Encoding:

- 1) The source node sends the codeword $\mathbf{x}_1(w_{13,b-1}, w_{13,b})$ in block b , where $w_{13,b-1}$ denotes w'_{13} .
- 2) Assuming the relay node has determined $w_{13,b-1}$ in the previous block, and then sends $\mathbf{x}_2(w_{13,b-1}, w_{23,b})$ in block b . So the transmitted codeword pair can be written as

$$\mathbf{x}_1(1, w_{13,1}), \mathbf{x}_2(1, w_{23,1}) \quad b=1,$$

$$\mathbf{x}_1(w_{13,b-1}, w_{13,b}), \mathbf{x}_2(w_{13,b-1}, w_{23,b}) \quad b=2, \dots, B-1,$$

$$\mathbf{x}_1(w_{13,B-1}, 1), \mathbf{x}_2(w_{23,B-1}, 1) \quad b=B$$

Decoding:

- 1) Assuming the relay node has determined $w_{13,b-1}$ correctly. Then it can determine the index $w_{13,b}$ by finding a unique $\hat{w}_{13,b}$ such that $\mathbf{u}(w_{13,b-1})$, $\mathbf{x}_1(w_{13,b-1}, \hat{w}_{13,b})$, $\mathbf{x}_2(w_{13,b-1}, w_{23,b})$ and $\mathbf{y}_2(b)$ are jointly typical. Because the relay node knows $w_{23,b}$, the index $w_{13,b}$ will be decoded correctly if the inequality (7) holds.

$$R_{13} < I(X_1; Y_2 | U, X_2) \quad (7)$$

- 2) The destination node starts to decode using backward decoding technique after the transmission of block b is completed. From the last block, it can determine $w_{13,b}$. And it can decode $w_{13,b-1}$ by looking for a unique $\hat{w}_{13,b-1}$ such that $\mathbf{u}(\hat{w}_{13,b-1})$, $\mathbf{x}_1(\hat{w}_{13,b-1}, w_{13,b})$ and $\mathbf{y}_3(b)$ are jointly typical. Reliable communication requires satisfying

$$R_{13} < I(U, X_1; Y_3) \quad (8)$$

- 3) The destination node has determined $w_{13,b}$ and $w_{13,b-1}$, then it tries to find a unique $\hat{w}_{23,b}$ such that $\mathbf{u}(w_{13,b-1})$, $\mathbf{x}_1(w_{13,b-1}, w_{13,b})$, $\mathbf{x}_2(w_{13,b-1}, \hat{w}_{23,b})$, $\mathbf{y}_3(b)$ are

jointly typical. The decoding can be made reliable if the inequality (9) holds.

$$R_{23} < I(X_2; Y_3 | U, X_1) \quad (9)$$

The achievable rate region in Theorem 1 can be directly concluded from (7), (8) and (9).

3.2. Gaussian Channels

In this subsection, we apply the results in discrete memoryless channels to AWGN channels. For simplicity, we assume the input distributions of the AWGN-RMAC to be Gaussian.

Corollary 1: An achievable rate region for the AWGN-RMAC is the convex hull of the rate pairs (R_{13}, R_{23}) satisfying

$$R_{13} < \min \left\{ \mathcal{C} \left(\frac{P_1 + \bar{\gamma} P_2 + 2\sqrt{\alpha\bar{\gamma}P_1P_2}}{\gamma P_2 + N_3} \right), \mathcal{C} \left(\frac{\alpha P_1}{N_2} \right) \right\}, \quad (10)$$

$$R_{23} < \mathcal{C} \left(\frac{\gamma P_2}{N_3} \right), \quad (11)$$

where $\alpha, \gamma \in [0, 1]$, α controls the power allocated to the source-to-destination message in the previous block and in the current block, γ indicates the fraction of the relay's power allocated to convey its private message to the destination.

Proof: We choose the Gaussian variables as follows.

$U \sim \mathcal{N}(0, P_u)$, $V' \sim \mathcal{N}(0, \alpha P_1)$, $X'_2 \sim \mathcal{N}(0, \gamma P_2)$, where U, V' and X'_2 are independent.

Furthermore, we let

$$X_1 = \sqrt{\frac{\bar{\alpha}P_1}{P_u}}U + V', \quad X_2 = \sqrt{\frac{\bar{\gamma}P_2}{P_u}}U + X'_2.$$

Now, we estimate R_{23} and R_{13} .

From (5),

$$\begin{aligned} R_{23} &< I(X_2; Y_3 | U, X_1) \\ &= h(Y_3 | U, X_1) - h(Y_3 | U, X_1, X_2) \\ &= h(X'_2 + Z_3) - h(Z_3) \\ &= \frac{1}{2} \log \left(\frac{\gamma P_2 + N_3}{N_3} \right) \\ &= \mathcal{C} \left(\frac{\gamma P_2}{N_3} \right) \end{aligned} \quad (12)$$

From (6),

$$R_{13} < \min \{ I(U, X_1; Y_3), I(X_1; Y_2 | U, X_2) \}$$

We compute $I(U, X_1; Y_3)$ and $I(X_1; Y_2 | U, X_2)$, respectively.

And

$$\begin{aligned} I(U, X_1; Y_3) &= h(Y_3) - h(Y_3 | U, X_1) \\ &= h(Y_3) - h(X'_2 + Z_3) \\ &= \mathcal{C} \left(\frac{\alpha P_1}{N_2} \right) \end{aligned} \quad (13)$$

While

$$\begin{aligned} I(X_1; Y_2 | U, X_2) &= h(Y_2 | U, X_2) - h(Y_2 | U, X_1, X_2) \\ &= h(V' + Z_2) - h(Z_2) \\ &= \frac{1}{2} \log \left(\frac{\alpha P_1 + N_2}{N_2} \right) \\ &= \mathcal{C} \left(\frac{P_1 + \bar{\gamma} P_2 + 2\sqrt{\alpha\bar{\gamma}P_1P_2}}{\gamma P_2 + N_3} \right) \end{aligned} \quad (14)$$

Combining (12), (13) and (14), we have the result of *Corollary 1*.

3.3. Numerical Results

The rate regions of AWGN-RMAC, AWGN multiple access channel (AWGN-MAC) and AWGN relay channel (AWGN-RC) are respectively given in Fig. 4 for $P_1 = P_2 = 5$, $N_2 = 0.5$ and $N_3 = 1$.

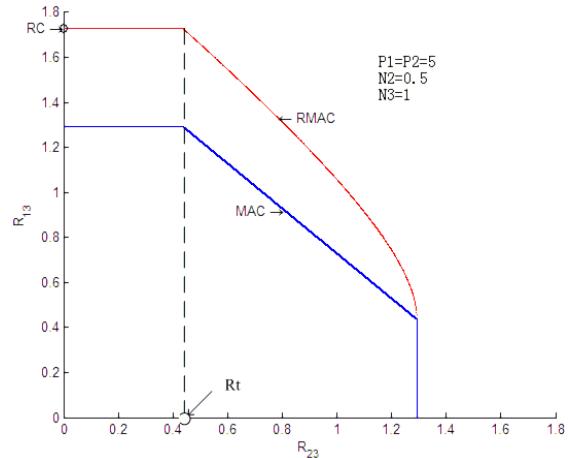


Fig.4. The rate regions of AWGN-RC, AWGN-MAC and AWGN-RMAC.

In Fig. 4, the rate R_{13} from Source 1 to destination increases when the other Source 2 provides some resources to help the communication between Source 1 and destination, i.e., the rate region improves indeed when the collaboration between the two sources is allowed. Moreover, the maximum R_{13}

is obtained when all the resources of the Source 2 are used to help the communication between Source 1 and destination. That is just the rate of relay channel based on decode-and-forward strategy, which is depicted by a small circle on the vertical axis in the figure. On the other hand, the maximal rate R_{23} is denoted by R_t when R_{13} achieves its maximum value. So a part of resources of the relay node can be used to help the communication from source to destination when its rate is not greater than the threshold value R_t , which is depicted by a small circle on the horizontal axis in the figure. From the figure, we can also conclude that the relay channel is a special case of the RMAC.

4. Discussion and Conclusion

In this paper, we propose a RMAC model inspired by the idea that some source nodes can exploit the other sources with few communications traffic or low rate to help their own communications in the multiple access channels. We develop the achievable rate regions for the DM-RMAC and the AWGN-RMAC based on decode-and-forward strategy, respectively. The numerical results show that the proposed scheme can improve the rate of user with the help of a relay node with private communication to the destination. And they also illustrate that the general relay channel is a special case of the RMAC under some circumstances. More important, the paper provides a way to select available relay node in some communication environment, especially the uplinks of cellular communications. The works to derive the achievable

rate regions based on partial decode-and-forward and compress-and-forward are under way.

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