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Application of Bragg Light Diffraction for Determination of Acoustic Wave Frequency

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Abstract: The possibility of determining the frequency of an acoustically active acoustic wave has been studied. The studies were carried out using the Bragg diffraction method of light on a high-frequency acoustic wave in a lanthanum gallosilicate crystal oriented with high accuracy along the third-order crystallographic axis. Transverse acoustic waves were excited in the range of 0.8-1.6 GHz using lithium niobate piezoelectric transducers. The attenuation coefficient of acoustic waves was previously determined by the pulse ultrasonic method. An expression is obtained for the magnitude of the specific rotation of the plane of polarization of an acoustic wave depending on the effective constant of the acoustic activity tensor and the effective elastic constant. It has been established that specific acoustic activity depends on the frequency of the acoustic wave according to a quadratic law. The results of the study showed that the proposed method makes it possible to determine the frequency of an acoustically active acoustic wave in a dynamic mode with an accuracy of up to 1 %.

Keywords: Acoustic wave, Acoustical activity, Bragg light diffraction, Frequency, Diffracted light intensity, Lanthanum gallosilicate crystal.

1. Introduction

In this work, to determine the frequency of an acoustic wave, it is proposed to use the phenomenon of acoustic activity, which is observed in crystals belonging to one of 21 non-centrosymmetric point symmetry groups. To understand the proposed technology, the phenomenon of acoustic activity is briefly described below. This phenomenon, associated with the spatial dispersion of elastic moduli, was first considered by Andronov [1] and then studied in a number of theoretical and experimental works [2-14], in which some of its regularities and features were revealed, in comparison with optical gyrotropy.

In general, acoustic activity provides useful information about the physical properties of crystals associated with the periodic structure. In addition, spatial dispersion leads to interesting acoustic phenomena, in particular to the dependence of the phase velocity of an acoustic wave on frequency and other effects [2, 9, 10]. However, in contrast to optical measurements, polarization measurements in crystal acoustics are extremely complex, and therefore this phenomenon has been studied experimentally in a limited number of crystals [8-14].

As in the case of optical activity, acoustic activity manifests itself in the rotation of the plane of polarization of a transverse acoustic wave propagating in a gyrotropic crystal in some special directions [9-12]. In particular, in crystals of point symmetry group 32, including quartz (SiO_2) and lanthanum gallosilicate ($La_3Ga_5SiO_{14}$), this direction is the crystallophysical axis [001], which is an acoustic as well as an optical axis [8, 9, 13, 14].

Thus, acoustic gyrotropy, like optical gyrotropy, can be considered as one of the types of anisotropy of the medium. Mathematically, it manifests itself in taking into account the first derivative in the expansion of the elastic modulus tensor with respect to the wave vector and is described by a fifth-rank tensor. The equation describing the contribution of spatial dispersion to elastic moduli can be written as [2, 9]:

$$c_{ijkl}(\omega,\vec{q}) = c_{ijkl}(\omega) + i\gamma_{ijklm}(\omega)q_m \qquad (1)$$

Here ω, q are the frequency and wave vector of the acoustic wave; c_{ijkl} are the components of the elasticity tensor. γ_{ijklm} are the components of the acoustic activity tensor. It can be shown that the components of the tensor γ_{ijklm} are equal in order of magnitude to the lattice constant [1, 8]. It follows that the influence of spatial dispersion of elastic constants on the propagation of acoustic waves will manifest itself at frequencies approximately equal to or greater than 1 GHz [8, 9]. Taking spatial dispersion into account leads to the following entry for Hooke's law [9]:

$$\sigma_{ij} = (c_{ijkl} + iq_m\gamma_{ijklm}\kappa_m)u_{ke}, \qquad (2)$$

where u_{kl} are the components of the strain tensor, κ_m are the components of the wave normal. From here, in the approximation of plane harmonic waves, and using the wave equation for acoustic waves, it can be obtained the following Green-Christoffel equations taking into account spatial dispersion:

$$(\Gamma_{ik} + iqG_{ik} - \rho V^2\delta_{ik})u_k = 0, \qquad (3)$$

where δ_{ik} is the Kronecker tensor, Γ_{ik} are the components of the Green-Christoffel tensor:

$$\Gamma_{ik} = c_{ijkl}\kappa_j\kappa_l \qquad (4)$$

The introduced second-rank tensor G_{ik} is written as [2, 3]:

$$G_{ik} = \gamma_{ijklm}\kappa_j\kappa_l\kappa_m \qquad (5)$$

Expression (5) can be conveniently written in terms of the rank 4 pseudotensor G_{mpjl}

$$G_{ik} = \delta_{ikp}G_{mpjl}\kappa_m\kappa_j\kappa_l, \qquad (6)$$

where δ_{ikp} is the Levi-Civita tensor.

It should be taken into account that due to the symmetry of the acoustic activity tensor:

$$G_{ik} = -G_{ki} \qquad (7)$$

It follows that the diagonal components of the tensor G_{ik} are equal to zero, i.e. $G_{ii} = 0$.

As a result, spatial dispersion leads to the fact that not many transverse waves with the same phase velocity V_0 propagate along the acoustic axis, but two circularly polarized waves with polarization vectors rotating in opposite directions and with slightly different velocities V_r and V_l.

$$V_{r,l} = V_0 \pm q\frac{G}{\rho}, \qquad (8)$$

where ρ is the crystal density, q is the wave number, G is the effective acoustic gyrotropy pseudotensor constant.

Thus, if a plane-polarized transverse wave propagates from the point z = 0 along the acoustic axis, then such a wave can be represented as a superposition of two waves, right- and left-polarized. Moreover, at the point z = 0 the phases of these waves can be taken equal to zero:

$$\varphi_l = \varphi_r = 0,$$

and after moving the waves to point z = L, their phases will change, respectively, by the amount:

$$\varphi_l = \frac{\omega L}{V_l}, \qquad (9)$$

and

$$\varphi_r = \frac{\omega L}{V_r} \qquad (10)$$

As a result, the resulting vector will rotate by an angle:

$$\varphi = \frac{\varphi_r - \varphi_l}{2} \qquad (11)$$

This angle of rotation of the polarization plane can be calculated through the wave velocities of two circular polarizations [2, 9]:

$$\varphi = \frac{\omega L}{2}\left(\frac{1}{V_r} - \frac{1}{V_l}\right) = \frac{\omega L}{2}\cdot\frac{(V_l - V_r)}{V_r \cdot V_l} \qquad (12)$$

After some transformations, taking into account relation (8) and that $V_r \cong V_l = V_0$, we obtain an expression for the specific rotation of the plane of polarization δ:

$$\delta = \frac{\varphi}{L} = \frac{\omega^2}{2V_0^2}\cdot\frac{G}{c_{44}} \qquad (13)$$

It can be seen that the value of δ is proportional to the square of the frequency of the acoustic wave. Note that in relation (13) for the lanthanum gallosilicate crystal we used, the effective pseudotensor constant G is equal to the component of the acoustic activity tensor γ_{543}.

The influence of spatial dispersion on the angles characterizing the deviation of the energy flow from the acoustic axis in the (100) and (010) planes in crystals of trigonal symmetry has been studied in [10] on the basis of the phenomenological theory of acoustical activity. The cumbersome relations obtained in [10] are significantly simplified for lanthanum gallosilicate crystals. As a result, the expression for determining the angle of deviation of the energy flow from the acoustic axis has the form:

$$\psi = \pm arctg\left(\frac{c_{14}}{c_{44}\sqrt{2}}\right) \quad (14)$$

Relationship (14) helps to determine the limitation imposed by the refraction of acoustic waves on the experimental conditions.

2. Samples and Experimental Methods

To experimentally study acoustic activity, the Bragg method of light diffraction by acoustic waves was used. The research sample was prepared from an optically pure single crystal. To exclude the influence of elastic anisotropy on the manifestation of spatial dispersion, the parallelepiped-shaped sample was oriented with an accuracy of no worse than 10′ along the [001] crystallographic direction, coinciding with the acoustic axis of the crystal.

To correctly set up the experiments, the maximum distance L_{lim} from the beginning of the crystal at which rotation of the plane of polarization can be observed was initially determined. According to [10], this distance is determined by the expression:

$$L_{lim} = 0.5 \cdot S \cdot ctg|\psi| \quad (15)$$

Here S is the aperture of the piezoelectric transducer, ψ is the largest angle of deviation of the energy flow from the acoustic axis in the (100) and (010) planes. To determine the distance L_{lim}, first, using expression (14), the angle of deviation of the energy flow from the acoustic axis was calculated using the values of the elastic constants c_{14} and c_{44} from [13, 15], which turned out to be equal to 10.5 degrees. Then, using expression (14) and (15) with the aperture of the piezoelectric transducer $S = 4$ mm, it was determined that the effective value of L_{lim} is equal to 1.1 cm.

Thus, measurements of the intensity of diffracted light had to be carried out at distances from the piezoelectric transducer no more than the calculated limit value. As a result, a sample with a length of 1.05 cm and transverse dimensions of 5×5 mm was prepared for the study.

To excite plane-polarized transverse acoustic waves with frequencies from 0.3 to 1.6 GHz, piezoelectric transducers were used in the form of lithium niobate X-cut plates with an aperture of 4×4 mm^2, with which measurements could be carried out. at the indicated frequencies. The piezoelectric transducer was oriented so that the initial polarization of the transverse wave was directed along the X axis.

Previously, the attenuation coefficient of transverse acoustic waves α in the lanthanum gallosilicate sample was determined using an ultrasonic method in a pulsed mode. The measurements were carried out at frequencies of 0.36, 0.42 and 0.72 GHz. In the ultrasonic system, echo pulses arrived at an amplitude-time selector introduced into the measuring circuit, which opens at a specified time interval, selecting a specific pulse, the amplitude of which is measured by a digital voltmeter. As a result, the attenuation coefficient of acoustic waves is determined from the measured amplitudes of neighboring pulses A_1 and A_2 [16]:

$$\alpha = \frac{ln\left(\frac{A_1}{A_2}\right)}{2L_0}, \quad (16)$$

where L_0 is the length of the sample. The accuracy of determining the attenuation coefficient due to multiple measurements of pulse amplitudes and subsequent averaging was ~5 %.

The diagram of the acousto-optic cell used to determine the intensity of diffracted light is shown in Fig. 1. Optical waves with a wavelength of 632.8 nm were generated by a helium-neon laser. Pulses of diffracted light were received, amplified, and converted into electrical pulses using a photomultiplier tube. As a result, it was possible to determine the intensities of diffracted light I_1 and I_2 at different points of the lanthanum gallosilicate sample along the direction of propagation of the acoustic wave, by moving the sample perpendicular to the direction of incidence of the laser beam. The intensities of diffracted light were measured automatically using a computer controlled by specially developed software.

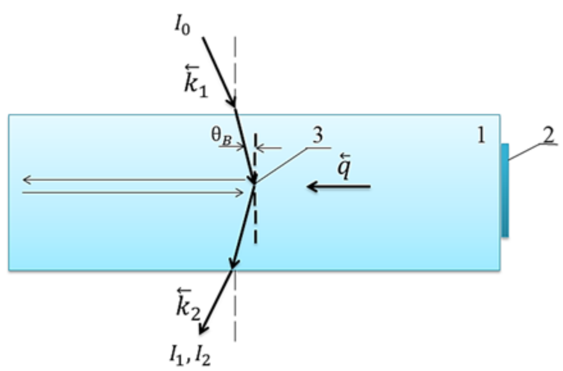

Fig. 1. Diagram of an acoustic-optical cell. The geometry of Bragg diffraction is shown at one of the transmission points along the direction of propagation of an acoustically active wave.

In Fig. 1, the following notations are introduced: 1 is $La_3Ga_5SiO_{14}$ sample, 3 is acoustic wave front, 2 is piezoelectric transducer, θ_B is the internal Bragg angle of incidence of the light beam. In the experiment, the magnitude of the specific rotation of the plane of polarization and the attenuation coefficient of transverse acoustic waves propagating along the acoustic axis were determined from measurements of the dependence of the intensity of diffracted light (*I*) on the distance (z) from the piezoelectric transducer along the direction of propagation of the acoustic wave. Note that the geometry of Bragg light diffraction, presented in Fig. 1, corresponds to the so-called isotropic diffraction of light by sound [17].

When using Bragg light diffraction to determine the characteristics of acoustic waves, an important question is the intensity of the diffracted light, which is determined by the acoustic-optical quality factor M_2. This coefficient was introduced by Dixon as a characteristic of the efficiency of Bragg diffraction by acoustic waves [18].

The expression for this coefficient in the case of an anisotropic medium has the form [18]:

$$M_2 = \frac{n_1^3 n_2^3 p_{eff}^2}{\rho V^3}, \tag{17}$$

where n_1 and n_2 are the refractive indices of incident and diffracted light, respectively, ρ is the density, V is the velocity of the acoustic wave, p_{eff} is the effective photoelastic constant. The larger the M_2 value, the greater the intensity of the diffracted light and the higher the sensitivity of the acoustic-optical method compared to other methods of recording acoustic waves.

The effective photoelastic constant in relation (17) is a convolution of the values of the photoelastic constants of the crystal under consideration according to the normalized polarization vectors of the incident and diffracted light μ, β and the direction and polarization of the acoustic wave κ and η [18]:

$$p_{eff} = p_{ijkl}\mu_i\beta_j\eta_k\kappa_l \tag{18}$$

In the considered Bragg diffraction geometry, the wave vector of the transverse acoustic wave is directed along the [001] axis; the polarization of this wave is perpendicular to the wave vector and makes a certain angle ψ with the [100] axis in the plane (001). In turn, the wave vector of light is directed along the [010] axis (up to a small Bragg angle) and the polarization of the incident light is set parallel to the wave vector of the acoustic wave.

It is easy to show that in such a light diffraction geometry, the effective photoelastic constant is determined by the photoelastic tensor component p_{44} and the angle ψ using the relation:

$$p_{eff} = p_{44}\cos\psi , \tag{19}$$

where, in turn, due to the rotation of the plane of polarization of the transverse acoustic wave, the value of the angle ψ is a variable value depending on the specific rotation of the plane of polarization δ, namely $\psi = \delta \cdot z$.

To correctly set up experiments on the use of Bragg diffraction of light to determine the characteristics of the propagation of acoustic waves, it is also necessary to take into account the following. Crystals of lanthanum gallosilicate are optically uniaxial, therefore, during the diffraction of light on transverse acoustic waves with a rotation of the plane of polarization, features should be observed in these crystals depending on the angles of incidence θ_1 and diffraction θ_2 on the frequency of acoustic waves [19, 20]. This is due to the fact that in an optically anisotropic medium, the refractive indices of incident and diffracted light are generally different.

As a result, the geometry of anisotropic diffraction differs from conventional Bragg geometry in that the angles of incidence and diffraction are not equal [19]. In the geometry we are considering, the plane of light diffraction is the crystallophysical plane (100). In such a geometry, an acoustically active transverse wave propagates strictly along the [001] direction, and its polarization rotates in the (001) plane. The laser beam is incident and diffracted by this acoustic wave at small angles to the [010] direction.

Cross-sections of the surfaces of wave vectors for anisotropic diffraction in optically uniaxial crystals and expressions for calculating the angles of incidence and diffraction of light are considered in detail in [19]. The dependence of the angles of incidence θ_1 and diffraction θ_2 on the frequency of transverse acoustic waves propagating along [001] was determined using the calculation formulas given in [19, 20], but taking into account that in lanthanum gallosilicate crystals ne>n0 and that the incident light is extraordinary.

The calculated dependences of the angles of incidence θ_1 and diffraction θ_2 on the frequency are presented in the Fig. 2, which shows the pattern of changes in these angles in the studied frequency range.

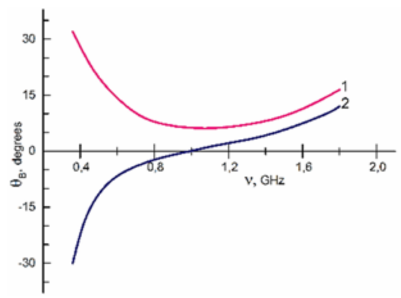

Fig. 2. Dependence of the angles of incidence θ_1 (1) and diffraction θ_2 (2) on the frequency of transverse acoustic waves along the [001] direction at the Bragg diffraction of light in lanthanum gallosilicate crystals.

It can be seen that in the frequency range 0.8-1.6 GHz, the diffraction angle θ_2 changes by approximately 10° with a practically unchanged angle of incidence. Using the values of the refractive indices of lanthanum gallosilicate crystals at the light wavelength λ_0 = 632.8 nm, equal to n_e = 1.9107 and n_0 = 1.8993 [15], we obtained the value of the characteristic frequency ν_m^* when the diffraction angle θ_2 is equal to zero:

$$\nu_m^* = \frac{V \cdot n_0}{\lambda_0 \cdot n_e} \cdot (n_e^2 - n_0^2)^{\frac{1}{2}} = 1.02\ GHz \quad (20)$$

At this frequency, the angle of incidence θ_1 is determined by the expression [8]:

$$\theta_{1m}^* = \arcsin\left(\frac{n_e^2 - n_0^2}{2n_e^2 - n_0^2}\right)^{1/2} \quad (21)$$

Calculation using relation (21) gives the value of the angle of incidence θ_{1m}^* equal to $6^0 13'$. Thus, in our experiments, the light fell at such small angle to the wave vector of the acoustic wave, changing slightly in the frequency region of 1 GHz, and diffracted at angles not exceeding ~7°. The considered geometry of light diffraction ensures minimal influence of the diffraction divergence of the acoustic wave, since in this case in the light diffraction does not participate the part of the acoustic beam, whose polarization does not experience rotation due to a large deviation from the acoustic axis.

3. Results and Discussion

In the first series of experiments, the dependence of the intensity of diffracted light (*I*) on the distance (z) from the piezoelectric transducer along the direction of propagation of an acoustic wave with a frequency of 0.8 GHz was determined. Then the same measurements were carried out at other, higher frequencies. The values of these frequencies changed in steps equal to twice the natural frequency of the piezoelectric transducer. In our studies, this step was approximately 0.06 GHz.

The results of the experiment at frequencies of 1.0 and 1.06 GHz are shown in Fig. 3.

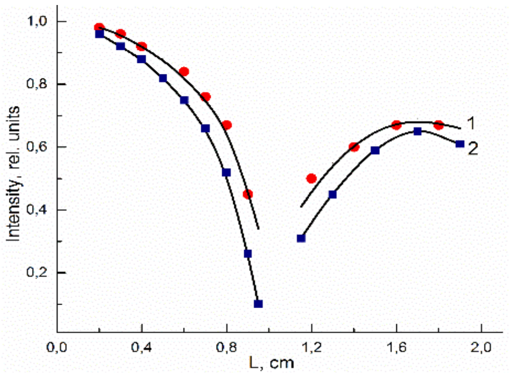

Fig. 3. Dependence of the intensity of diffracted light on the distance *L* from the piezoelectric transducer at frequencies of 1.01 GHz (1) and 1.06 GHz (2). The dots are the results of the experiment; curves – calculation according to equation (22).

It can be seen that the direction of change in the intensity of the diffracted light changes at the sample boundary (the length of the sample was 1.05 cm). This effect can be easily explained by the fact that the acoustic wave is reflected from the free boundary of the sample and, accordingly, the direction of rotation of the plane of polarization changes to the opposite direction.

The results obtained on changes in the relative intensity of diffracted light at various points along the direction of propagation of an acoustically active wave can be used to find the characteristics of this wave. These include the initial phase of the displacement vector φ_0, the attenuation coefficient α and the specific rotation of the wave polarization plane δ. To do this, you can use a set of k experimental values of diffracted light intensities for k independent measurements at different points of the sample.

The intensity of diffracted light can be represented by the following functional dependence of the relative intensity of diffracted light relative to the intensity of incident light (I/I_0) on the distance (z) over which the acoustic wave moves along the [001] axis [9]:

$$I/I_0 \sim p_{44}\, exp(-\alpha \cdot z) cos^2(\delta \cdot z + \varphi_0) \quad (22)$$

Thus, the problem of finding three independent characteristics of an acoustically active wave is reduced to solving a system of k nonlinear equations of the form (22), when k>3. For k = 3, the problem is purely algebraic, but its solution significantly depends on the choice of experimental points at which the intensity was measured, and the result will be noticeably affected by random measurement errors. On the contrary, the more measurement points, that is, the more the system is overdetermined, the more the random errors of individual measurements compensate each other and the solution becomes more reliable.

As is known, in nonlinear problems the solution can be obtained using various methods of linearizing the problem and using least squares estimates. The requirement for the set of required parameters is that, according to Legendre's principle, it minimizes the sum of squared residuals for the redundant system of equations. The described approach was implemented using an application program created in the Pascal language, with a special subprogram that minimizes the sum of squares of the difference function and works using the gradient descent method [21, 22].

The described procedure for determining the specific rotation of the polarization vector and the attenuation coefficient of a transverse acoustic wave propagating along the [001] direction was carried out in the frequency range from 0.8 to 1.6 GHz. The calculation results are shown in Fig. 4, which also shows the values of the attenuation coefficient measured by the ultrasonic method. Solid lines are plotted assuming a quadratic dependence of these characteristics on frequency. It can be seen that the results follow these dependencies well.

In the second series of experiments, the dependence of the intensity of diffracted light on the frequency of acoustically active transverse waves at a certain point of a crystalline sample used as a measuring transducer was studied. The results of the study are presented in Fig. 5, which shows the change in the frequency of the acoustic wave with a change in the intensity of the diffracted light. It can be seen that by observing a change in the intensity of diffracted light at a certain point of the sample, one can estimate the change in the frequency of the acoustic wave.

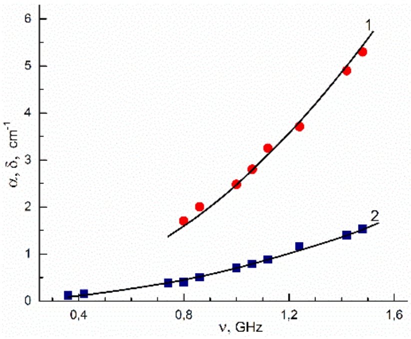

Fig. 4. Frequency dispersion of the specific rotation of the plane of polarization (1) and the attenuation coefficient (2) of transverse acoustic waves along the [001] direction in $La_3Ga_5SiO_{14}$ crystals.

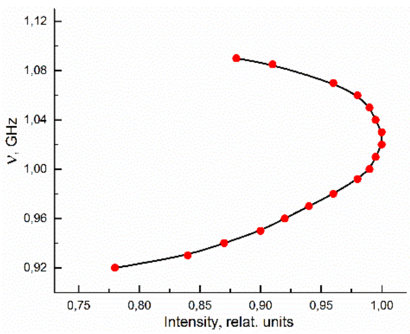

Fig. 5. Dependence of the frequency of transverse acoustic waves along the [001] axis on the intensity of diffracted light at a fixed point of a lanthanum gallosilicate sample.

Having previously calibrated the dependence of the intensity of diffracted light on frequency at a selected point of a crystalline sample used as a meter-sensor, this sample can be used to control the frequency of a transverse acoustic wave.

4. Conclusions

A comparison of the measured frequency values with the frequencies of the radio generator used showed that the proposed method makes it possible to determine the frequency of the acoustic wave with an accuracy of 1 %. Such a relatively large error of the proposed method is due to the influence of the internal conical refraction and the relatively low accuracy of determining the intensity of diffracted light. However, using the special comparison circuits for measuring light intensity, it is possible to achieve a fairly high accuracy in determining the frequency of the acoustic wave.

The method can be useful for monitoring the stability or deviation of the frequency of the modulating signal in acousto-optic signal processing devices that use gyrotropic crystals. It should also be noted that it is necessary to ensure the broadband operation of the piezoelectric transducer in the controlled frequency range and, of course, the linear mode of operation of the photodetector.

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