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Decentralized Control for Time-Delay Interconnected Systems Based on T-S Fuzzy Bilinear Model

*** Guo ZHANG, Yunwang GE**
Electrical Engineering Department, Luoyang Institute of Science and Technology,
Luoyang, 471023, P. R. China
* E-mail: zhangguo163163@163.com

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Abstract: The paper presents the problem of decentralized state feedback control for nonlinear interconnected systems with time-varying delay in both states and inputs which is composed by a number of Takagi-Sugeno (T-S) fuzzy bilinear subsystems with interconnections. Based on the Lyapunov criterion and the parallel distribute compensation scheme, the delay-dependent stabilization sufficient conditions are derived for the whole close-loop fuzzy interconnected systems. The corresponding decentralized fuzzy controller design is converted into a convex optimization problem with linear matrix inequality (LMI) constraints. Finally, a simulation example shows the effectiveness of the proposed approach. *Copyright © 2014 IFSA Publishing, S. L.*

Keywords: Nonlinear interconnected system, Decentralized control, Delay-dependent, Linear matrix inequality (LMI), Fuzzy bilinear model.

1. Introduction

Large-scale interconnected systems can be found in many real-life practical applications such as electric power systems, nuclear reactors, economic systems, process control systems, computer networks, and urban traffic network, etc. [1]. On the other hand, there are few studies concerning with the stabilization control for the interconnected nonlinear systems [2]. Since linearization technique and linear robust control are used, these results are always conservative and only applicable to some special nonlinear interconnected systems. Due to the physical configuration and high dimensionality of interconnected systems, a centralized control is neither economically feasible nor even necessary [3]. Therefore, decentralized scheme is preferred in control design of the large-scale interconnected systems [4].

In recent years, T-S (Takagi- Sugeno) model-based fuzzy control has attracted wide attention, essentially because the fuzzy model is an effective and flexible tool for control of nonlinear systems [5-7]. In this approach, the T-S fuzzy model substitutes the consequent fuzzy sets in a fuzzy rule by a linear model. Local dynamics in different state-space regions are represented by linear models and the overall model of the system is represented as the fuzzy interpolation of these linear models. Just because of this, T-S fuzzy model has been paid considerable attention and is widely used to the control design of nonlinear interconnected systems. The problem of stabilization of nonlinear interconnected systems was studied in [7], while robust stabilization of a class of multiple time-delay nonlinear interconnected systems was investigated. It is noted that the above nonlinear interconnected systems are all based on T-S fuzzy linear model.

It is known that bilinear models can be described many physical systems and dynamical processes in engineering fields [8]. There are two main advantages of the bilinear system. One is that it provides a better approximation to a nonlinear system than a linear one. Another is that many real physical processes may be appropriately modeled as bilinear systems when the linear models are inadequate. A good example of a bilinear system is the population of biological species described by $d\theta / dt = \theta v$, where v is the birth rate minus death rate, and θ denotes the population. It is impossible to approximate the aforementioned equation by a linear model [8].

Considering the advantages of bilinear systems and T-S fuzzy control, the bilinear fuzzy system based on the T-S fuzzy model with bilinear rule consequence was attracted the interest of researchers [9-12]. The T-S FBS may be suitable for some classes of nonlinear plants. The robust stabilization for continuous-time fuzzy bilinear system (FBS) is studied in [9], then the result was extended to the FBS with time-delay only in the state [10]. However, the paper [11] is only considered the delay in the state and the derivatives of time-delay, $\dot{d}(t) < 1$, is required. So far, the decentralized control of nonlinear interconnected systems based T-S bilinear model has not been discussed.

In this paper, the purpose is to develop a state feedback controller and obtain some sufficient conditions for nonlinear interconnected systems, which is concerned about the time-varying delay both in state and input. For objective, T-S fuzzy bilinear model is employed for the nonlinear interconnected systems. Then, based on the parallel distribute compensation (PDC) scheme, the robust stabilization conditions can be established, and moreover, the decentralized controller design procedure can cast as solving a set of linear matrix inequalities (LMIs).

2. System Description and Assumptions

Consider a time-delay interconnected system Ω composed of S subsystems $\Omega_i, i = 1, 2, \ldots, S$. Each fuzzy rule of the subsystem Ω_i can be represented by a T-S fuzzy bilinear model as follows

$$\begin{aligned}
&R_i^m \textit{ if } \xi_{i1}(t) \textit{ is } F_{i1}^m \textit{ and } \ldots\ \xi_{iv_i}(t) \textit{ is } F_{iv_i}^m \\
&\textit{then } \dot{x}_i(t) = A_{im}x_i(t) + B_{im}u_i(t) + N_{im}x_i(t)u_i(t) \\
&\qquad + A_{idm}x_i(t-d_i) + B_{idm}u_i(t-d_i) + N_{idm} \\
&\qquad \times x_i(t-d_i)u_i(t-d_i) + \sum\nolimits_{j=1,j\neq i}^{S} C_{jim}x_j(t) \\
&\quad x_i(t) = \phi_i(t) \qquad t = [-\tau_i \quad 0] \qquad m = 1, 2, \ldots, r_i
\end{aligned}, \quad (1)$$

where r_i is the number of the fuzzy rules. $\xi_{ij}(t)$ and $F_{ij}^m, j = 1, 2, \ldots, v_i$ are some measurable premise variables, and fuzzy sets. $x_i(t) \in R^{n_i}, u_i(t) \in R$ are the state vector and control input, respectively. $A_{im}, A_{idm}, N_{im}, N_{idm} \in R^{n_i \times n_i}, B_{im}, B_{idm} \in R^{n_i \times 1}$ denote the system matrices with appropriate dimensions. $C_{jim} \in R^{n_i \times n_j}$ represents the interconnection matrix between the i^{th} and the j^{th} subsystems. d_i is a time-varying differentiable function that satisfies $0 \le d_i \le \tau_i$, where τ is a real positive constant as the upper bound of the time-varying delay. It is also assumed that $\dot{d}_i \le \alpha_i < 1$ and a_i is a known constant.

By using singleton fuzzifier, product inferred, and weighted defuzzifier, the system can be expressed by the following globe model:

$$\begin{aligned}
&\dot{x}_i(t) = \sum\nolimits_{m=1}^{r_i} h_{im}(\xi_i(t))[A_{im}x_i(t) + B_{im}u_i(t) \\
&+ N_{im}x_i(t)u_i(t) + A_{idm}x_i(t-d_i) + N_{idm}x_i(t-d_i) \\
&\times u_i(t-d_i) + B_{idm}u_i(t-d_i) + \sum\nolimits_{j=1,j\neq i}^{S} C_{jim}x_j(t)]
\end{aligned}, \quad (2)$$

where $h_{im}(\xi_i(t)) = \dfrac{\omega_{im}(\xi_i(t))}{\sum_{m=1}^{r_i} \omega_{im}(\xi_i(t))}$,

$\omega_{im}(\xi_i(t)) = \prod_{j=1}^{v_i} \mu_{imj}(\xi_i(t))$. $\mu_{imj}(\xi_i(t))$ is the grade of membership of $\xi_{ij}(t)$ in F_{ij}^m. In this paper, it is assumed that $\omega_{im}(\xi_i(t)) \ge 0$ and $\sum_{m=1}^{r_i} \omega_{im}(\xi_i(t)) > 0$ for all t. Then we have the following conditions $h_{im}(\xi_i(t)) \ge 0$ and $\sum_{m=1}^{r_i} h_{im}(\xi_i(t)) = 1$ for all t. In the consequent, we use abbreviation $h_{im}, x_{id}(t), u_{id}(t)$ to replace $h_{im}(\xi_i(t)), x_i(t-d_i), u_i(t-d_i)$, respectively, for convenience.

Based on PDC, the fuzzy controller shares the same premise parts with (1); that is, the i^{th} fuzzy controller is formulated as follow

$$\begin{aligned}
&\textit{if } \ \xi_{i1}(t) \ \textit{ is } \ F_{i1}^m \ \textit{ and } \ldots\ \xi_{iv_i}(t) \ \textit{ is } \ F_{iv_i}^m \\
&\textit{then } \ u_i(t) = \frac{\rho_i K_{im} x_i(t)}{\sqrt{1 + x_i^{\mathrm{T}} K_{im}^{\mathrm{T}} K_{im} x_i}} \\
&\qquad = \rho_i \sin\theta_{im} = \rho_i \cos\theta_{im} K_{im} x_i(t)
\end{aligned}, \quad (1)$$

From (3), the $u_{di}(t)$ can be written as:

$$\begin{aligned}
u_{id}(t) &= \frac{\rho_i K_{im} x_{id}(t)}{\sqrt{1 + x_{id}^{\mathrm{T}} K_{im}^{\mathrm{T}} K_{im} x_{id}}} \\
&= \rho_i \sin\varphi_{im} = \rho_i \cos\varphi_{im} K_{im} x_{id}(t)
\end{aligned}, \quad (4)$$

where $K_{im} \in R^{1\times n_i}$ is the local controller gain to be determined and $\rho_i > 0$ is the scalar to be assigned.

$$\sin\theta_{im} = \frac{K_{im}x_i(t)}{\sqrt{1+x_i^{\mathrm{T}}K_{im}^{\mathrm{T}}K_{im}x_i}}, \cos\theta_{im} = \frac{1}{\sqrt{1+x_i^{\mathrm{T}}K_{im}^{\mathrm{T}}K_{im}x_i}},$$

$$\sin\varphi_{im} = \frac{K_{im}x_{id}(t)}{\sqrt{1+x_{id}^{\mathrm{T}}K_{im}^{\mathrm{T}}K_{im}x_{id}}}, \cos\varphi_{im} = \frac{1}{\sqrt{1+x_{id}^{\mathrm{T}}K_{im}^{\mathrm{T}}K_{im}x_{id}}},$$

$i = 1,2,...,S; m = 1,2,...,r_i$.

The overall fuzzy control law can be represented by

$$\begin{aligned} u_i(t) &= \sum_{m=1}^{r_i} h_{im}\rho_i \sin\theta_{im} \\ &= \sum_{m=1}^{r_i} h_{im}\rho_i \cos\theta_{im}K_{im}x_i(t) \\ u_{id}(t) &= \sum_{m=1}^{r_i} h_{im}\rho_i \sin\varphi_{im} \\ &= \sum_{m=1}^{r_i} h_{im}\rho_i \cos\varphi_{im}K_{im}x_{id}(t) \end{aligned}, \quad (5)$$

By substituting (5) into (2), the i th closed-loop subsystem can be represented as

$$\begin{aligned} \dot{x}_i(t) = \sum_{m,n=1}^{r_i} h_{im}h_{in}[\Lambda_{i,mn}x_i(t) \\ + \Lambda_{i,dmn}x_{id}(t) + \sum_{j=1,j\neq i}^{S} C_{jim}x_j(t)] \end{aligned}, \quad (6)$$

where $\Lambda_{i,mn} = A_{im} + \rho_i \sin\theta_{in}N_{im} + \rho_i \cos\theta_{in}B_{im}K_{in}$, $\Lambda_{i,dmn} = A_{idm} + \rho_i \sin\varphi_{in}N_{idm} + \rho_i \cos\varphi_{in}B_{idm}K_{in}$. The objective of the paper is to design decentralized fuzzy state feedback controllers (5) such that the closed-loop systems (6) is decentralized robust stability.

3. Main Results

Before proceeding with the following theorems, we introduce the following lemmas which will be used in our results.

Lemma 1 [18]: Given any matrices M and N with appropriate dimensions such that $\varepsilon > 0$, we have $M^{\mathrm{T}}N + N^{\mathrm{T}}M \le \varepsilon M^{\mathrm{T}}M + \varepsilon^{-1}N^{\mathrm{T}}N$.

The following theorem gives the sufficient conditions for the existence of the fuzzy controller for the interconnected system (6).

Theorem 1 For given scalars $\rho_i > 0, a_i > 0, \varepsilon_{1i} > 0, \varepsilon_{2i} > 0, i = 1,2,...,S$, the interconnect system (6) is decentralized delay-dependent asymptotically stable if there exist matrices $P_i > 0, R_i > 0, i = 1,2,...,S$ and $K_{im}, i = 1,2,...,S; m = 1,2,...,r_i$ such that the following inequality (7) is satisfied.

$$\Phi_{i,mm} < 0, \quad i = 1,2,...,S; \quad m = 1,2,...,r_i, \quad (7a)$$

$$\begin{aligned} \Phi_{i,mn} + \Phi_{i,nm} < 0, \\ i = 1,2,...,S; \quad 1 \le m < n \le r_i \end{aligned}, \quad (7b)$$

where $\Phi_{i,mn} = \begin{bmatrix} \phi_{i,1mn} & P_iA_{idm} \\ * & \phi_{i,2mn} \end{bmatrix}$,

$$\begin{aligned} \phi_{i,1mn} &= A_{im}^{\mathrm{T}}P_i + P_iA_{im} + (\varepsilon_{1i} + \varepsilon_{2i})\rho_i^2P_iP_i \\ &+ \varepsilon_{1i}^{-1}N_{im}^{\mathrm{T}}N_{im} + \varepsilon_{1i}^{-1}(B_{im}K_{in})^{\mathrm{T}}(B_{im}K_{in}) \\ &+ \sum_{j=1,j\neq i}^{S} P_iC_{jim}C_{jim}^{\mathrm{T}}P_i + (S-1)I + R_i, \\ \phi_{i,2mn} &= \varepsilon_{2i}^{-1}N_{idm}^{\mathrm{T}}N_{idm} + \varepsilon_{2i}^{-1}(B_{idm}K_{in})^{\mathrm{T}}(B_{idm}K_{in}) \\ &- (1-\alpha_i)R_i. \end{aligned}$$

Proof: Take the Lyapunov function candidate as

$$\begin{aligned} V(t) = \sum_{i=1}^{s} V_i(t) = \sum_{i=1}^{s}[x_i^{\mathrm{T}}(t)P_ix_i(t) \\ + \int_{t-d_i(t)}^{t} x_i^{\mathrm{T}}(s)R_ix_i(s)ds] \end{aligned}, \quad (8)$$

The time derivatives of $V(t)$, along the trajectory of the system (6) is given by

$$\begin{aligned} \dot{V}(t) &= \sum_{i=1}^{S}\sum_{m,n=1}^{r_i} h_{im}h_{in}[\dot{x}_i^{\mathrm{T}}(t)P_ix_i(t) + x_i^{\mathrm{T}}(t)P_i\dot{x}_i(t) \\ &+ x_i^{\mathrm{T}}(t)R_ix_i(t) - (1-\dot{d}_i)x_{id}^{\mathrm{T}}(t)R_ix_{id}(t)] \\ &\le \sum_{i=1}^{S}\sum_{m,n=1}^{r_i} h_{im}h_{in}[x_i^{\mathrm{T}}(t)(\Lambda_{i,mn}^{\mathrm{T}}P_i + P_i\Lambda_{i,mn})x_i(t) \\ &+ x_{id}^{\mathrm{T}}(t)\Lambda_{i,dmn}^{\mathrm{T}}P_ix_i(t) + x_i^{\mathrm{T}}(t)P_i\Lambda_{i,dmn}x_{id}(t) \\ &+ \sum_{j=1,j\neq i}^{S} x_j^{\mathrm{T}}(t)C_{jim}^{\mathrm{T}}P_ix_i(t) + x_i^{\mathrm{T}}(t)P_i \\ &\times \sum_{j=1,j\neq i}^{S} C_{jim}x_j(t) + x_i^{\mathrm{T}}(t)R_ix_i(t) \\ &- (1-\alpha_i)x_{id}^{\mathrm{T}}(t)R_ix_{id}(t)] \end{aligned}, \quad (9)$$

Applying Lemma 1, we get

$$\begin{aligned} \Lambda_{i,mn}^{\mathrm{T}}P_i + P_i\Lambda_{i,mn} &= A_{im}^{\mathrm{T}}P_i + P_iA_{im} + (\rho_i \sin\theta_{in} \\ &\times N_{im})^{\mathrm{T}}P_i + P_i(\rho_i \sin\theta_{in}N_{im}) \\ &+ (\rho_i \cos\theta_{in}B_{im}K_{in})^{\mathrm{T}}P_i \\ &+ P_i(\rho_i \cos\theta_{in}B_{im}K_{in}) \\ &\le A_{im}^{\mathrm{T}}P_i + P_iA_{im} + \varepsilon_{1i}\rho_i^2P_iP_i + \varepsilon_{1i}^{-1}N_{im}^{\mathrm{T}}N_{im} \\ &+ \varepsilon_{1i}^{-1}(B_{im}K_{in})^{\mathrm{T}}(B_{im}K_{in}), \\ x_{id}^{\mathrm{T}}(t)\Lambda_{i,dmn}^{\mathrm{T}}P_ix_i(t) &+ x_i^{\mathrm{T}}(t)P_i\Lambda_{i,dmn}x_{id}(t) \\ &\le x_{id}^{\mathrm{T}}(t)A_{idm}^{\mathrm{T}}P_ix_i(t) + x_i^{\mathrm{T}}(t)P_iA_{idm}x_{id}(t) \\ &+ x_i^{\mathrm{T}}(t)\varepsilon_{2i}\rho_i^2P_iP_ix_i(t) + x_{id}^{\mathrm{T}}(t)\varepsilon_{2i}^{-1} \\ &\times N_{idm}^{\mathrm{T}}N_{idm}x_{id}(t) + x_{id}^{\mathrm{T}}(t)\varepsilon_{2i}^{-1}(B_{idm}K_{in})^{\mathrm{T}} \\ &\times (B_{idm}K_{in})x_{id}(t), \end{aligned}, \quad (10)$$

Considering the following inequality and applying Lemma 1 yield

$$
\begin{aligned}
&\sum_{i=1}^{S}\sum_{m=1}^{r_i} h_{im}[\sum_{j=1,j\neq i}^{S} x_j^{\mathrm{T}}(t)C_{jim}^{\mathrm{T}}P_i x_i(t) + x_i^{\mathrm{T}}(t) \\
&\qquad \times P_i \sum_{j=1,j\neq i}^{s} C_{jim}x_j(t)] \\
&= \sum_{i=1}^{S}\sum_{m=1}^{r_i} h_{im}[x_1^{\mathrm{T}}(t)C_{1im}^{\mathrm{T}}P_i x_i(t) + \ldots + x_{i-1}^{\mathrm{T}}(t) \\
&\quad \times C_{i-1im}^{\mathrm{T}}P_i x_i(t) + x_{i+1}^{\mathrm{T}}(t)C_{i+1im}^{\mathrm{T}}P_i x_i(t) + \ldots \\
&\quad + x_S^{\mathrm{T}}(t)C_{Sim}^{\mathrm{T}}P_i x_i(t) + x_i^{\mathrm{T}}(t)P_iC_{1im}x_1(t) + \ldots \\
&\quad + x_i^{\mathrm{T}}(t)P_iC_{i-1im}x_{i-1}(t) + x_i^{\mathrm{T}}(t)P_iC_{i+1im}x_{i+1}(t) \\
&\quad + \ldots x_i^{\mathrm{T}}(t)P_iC_{Sim}x_S(t)] \\
&\le \sum_{i=1}^{S}\sum_{m=1}^{r_i} h_{im}[x_i^{\mathrm{T}}(t)P_i\sum_{j=1,j\neq i}^{S} C_{jim}C_{jim}^{\mathrm{T}}P_i x_i(t) \\
&\qquad + \sum_{j=1,j\neq i}^{S} x_j^{\mathrm{T}}(t)x_j(t)] \\
&= \sum_{i=1}^{S}\sum_{m=1}^{r_i} h_{im}[x_i^{\mathrm{T}}(t)P_i\sum_{j=1,j\neq i}^{S} C_{jim}C_{jim}^{\mathrm{T}}P_i x_i(t) \\
&\qquad + (S-1)x_i^{\mathrm{T}}(t)x_i(t)]
\end{aligned}
\tag{11}
$$

Substituting (9) - (11) into (8) results in

$$
\begin{aligned}
\dot V(t) &\le \sum_{i=1}^{S}\sum_{m,n=1}^{r_i} h_{im}h_{in}\eta_i^{\mathrm{T}}(t)\Phi_{i,mn}x_i(t)\eta_i(t) \\
&= \sum_{i=1}^{S}[\sum_{m=1}^{r_i} h_{im}^2\eta_i^{\mathrm{T}}(t)\Phi_{i,mm}\eta_i(t) \\
&\quad + \sum_{1=m<n}^{r_i} h_{im}h_{in}\eta_i^{\mathrm{T}}(t)(\Phi_{i,mn}+\Phi_{i,nm})\eta_i(t)]
\end{aligned}
\tag{12}
$$

where $\eta_i^{\mathrm{T}}(t) = \begin{bmatrix} x_i^{\mathrm{T}}(t) & x_{id}^{\mathrm{T}}(t)\end{bmatrix}$.

Therefore, it is noted that (7) implies $\dot V(t) < 0$, so the interconnected system (6) is asymptotically stable. Thus, we complete the proof.

The matrix inequality (7) leads to bilinear matrix inequality (BMI) optimization, a non-convex programming problem. Non-convexity implies the existence of local minima and the BMI problems are NP-hard. In the following theorem, we will derive a sufficient condition such that the matrix inequality (7) can be transformed into an LMI problem.

Theorem 2 For given scalars $\rho_i > 0, a_i > 0,$ $\varepsilon_{2i} > 0, \varepsilon_{3i} > 0, i = 1,2,\ldots,S$, the interconnect system (6) is decentralized asymptotically stable if there exist matrices $Z_i > 0, \bar R_i > 0, i = 1,2,\ldots,S$ and $G_{im}, i = 1,2,\ldots,S;\ m = 1,2,\ldots,r_i$ such that the matrix inequality (13) is satisfied. Moreover, the feedback gains are given by $K_{im} = G_{im}Z^{-1}, i = 1,2,\ldots,S; m = 1,2,\ldots,r_i$.

$$
\begin{bmatrix}
T_{i,m} & * & * & * & * & * & * \\
Z_iA_{idm}^{\mathrm{T}} & -(1-\alpha_i)\bar R_i & * & * & * & * & * \\
Z_i & 0 & -\frac{I}{s-1} & * & * & * & * \\
N_{im}Z_i & 0 & 0 & -\varepsilon_{1i}I & * & * & * \\
B_{im}G_{im} & 0 & 0 & 0 & -\varepsilon_{1i}I & * & * \\
0 & N_{idm}Z_i & 0 & 0 & 0 & -\varepsilon_{2i}I & * \\
0 & B_{idm}G_{im} & 0 & 0 & 0 & 0 & -\varepsilon_{2i}I
\end{bmatrix} < 0, \quad i = 1,2,\ldots,S; m = 1,2,\ldots,r_i
\tag{13a}
$$

$$
\begin{bmatrix}
T_{i,m}+T_{i,n} & * & * & * & * & * & * \\
Z_iA_{idm}^{\mathrm{T}} + Z_iA_{idn}^{\mathrm{T}} & -2(1-\alpha_i)\bar R_i & * & * & * & * & * \\
2Z_i & 0 & -\frac{2I}{s-1} & * & * & * & * \\
\overline{(NZ)}_{i,mn} & 0 & 0 & -t_{55} & * & * & * \\
\overline{(BG)}_{i,mn} & 0 & 0 & 0 & -t_{66} & * & * \\
0 & \overline{(N_dZ)}_{i,mn} & 0 & 0 & 0 & -t_{77} & * \\
0 & \overline{(B_dG)}_{i,mn} & 0 & 0 & 0 & 0 & -t_{88}
\end{bmatrix} < 0, \quad i = 1,2,\ldots,S;\ 1 \le m < n \le r_i
\tag{13b}
$$

where

$$
T_{i,m} = Z_iA_{im}^{\mathrm{T}} + A_{im}Z + \sum_{j=1,j\neq i}^{S} C_{jim}C_{jim}^{\mathrm{T}} + (\varepsilon_{1i}+\varepsilon_{2i})\rho_i^2 I + \bar R_i,
$$

$$
\overline{(NZ)}_{i,mn} = \begin{bmatrix} N_{im}Z_i \\ N_{in}Z_i \end{bmatrix}, \overline{(BG)}_{i,mn} = \begin{bmatrix} B_{im}G_{in} \\ B_{in}G_{im} \end{bmatrix},
$$

$$
\overline{(N_dZ)}_{i,mn} = \begin{bmatrix} N_{idm}Z_i \\ N_{idn}Z_i \end{bmatrix}, \overline{(B_dG)}_{i,mn} = \begin{bmatrix} B_{idm}G_{in} \\ B_{idn}G_{im} \end{bmatrix},
$$

$$
t_{44} = t_{55} = diag\{\varepsilon_{1i}I, \varepsilon_{1i}I\}, t_{66} = t_{77} = diag\{\varepsilon_{2i}I, \varepsilon_{2i}I\}.
$$

Proof: letting $P_i = Z_i^{-1}, R_i = P_i\bar R_iP_i$ and noting $M_{im} = K_{im}Z$. Then, pre-multiplying and post-multiplying $diag\{P_i, P_i, I, I, I, I, I\}$ to (13a) results in

$$
\begin{bmatrix}
P_iT_{i,m}P_i & * & * & * & * & * & * \\
A_{idm}^{\mathrm{T}}P_i & -(1-\alpha_i)R_i & * & * & * & * & * \\
I & 0 & -\frac{I}{s-1} & * & * & * & * \\
N_{im} & 0 & 0 & -\varepsilon_{1i}I & * & * & * \\
B_{im}K_{im} & 0 & 0 & 0 & -\varepsilon_{1i}I & * & * \\
0 & N_{idm} & 0 & 0 & 0 & -\varepsilon_{2i}I & * \\
0 & B_{idm}K_{im} & 0 & 0 & 0 & 0 & -\varepsilon_{2i}I
\end{bmatrix} < 0, \quad i = 1,2,\ldots,S;\ m = 1,2,\ldots,r_i
\tag{14a}
$$

Applying the Schur complement to (14 a) results in the condition (7a). Similar, the (13 b) is equivalent to (7 b). According to Theorem 1, the interconnected system (6) is asymptotically stable. Thus the proof is completed.

4. Simulation Example

In this section, the proposed approach is applied to the following example to verify its effectiveness. We consider a fuzzy bilinear interconnected system, which is composed of two subsystems as follows

$$
\begin{aligned}
&Subsystem\ 1: \\
&R_1^1:\ if\ \ x_{11}\ \ is\ \ F_{11}^1 \\
&\quad then\ \ \dot x_1(t) = A_{11}x_1(t) + A_{1d1}x_{1d}(t) \\
&\quad + N_{11}x_1(t)u_1(t) + N_{1d1}x_{1d}(t)u_{1d}(t) \\
&\quad + B_{11}u_{1d}(t) + B_{1d1}u_{1d}(t) + C_{211}x_2(t);
\end{aligned}
$$

$$R_1^2: \ if \quad x_{11} \quad is \quad F_{11}^2$$
$$then \ \ \dot{x}_1(t) = A_{12}x_1(t) + A_{1d2}x_{1d}(t)$$
$$+ N_{12}x_1(t)u_1(t) + N_{1d2}x_{1d}(t)u_{1d}(t)$$
$$+ B_{12}u_{1d}(t) + B_{1d2}u_{1d}(t) + C_{212}x_2(t);$$

$$Subsystem \ 2:$$

$$R_2^1: \ if \quad x_{21} \quad is \quad F_{21}^1$$
$$then \ \ \dot{x}_2(t) = A_{21}x_2(t) + A_{2d1}x_{2d}(t)$$
$$+ N_{21}x_2(t)u_2(t) + N_{2d1}x_{2d}(t)u_{2d}(t)$$
$$+ B_{21}u_{2d}(t) + B_{2d1}u_{2d}(t) + C_{121}x_1(t);$$

$$R_2^2: \ if \quad x_{21} \quad is \quad F_{21}^2$$
$$then \ \ \dot{x}_2(t) = A_{22}x_2(t) + A_{2d2}x_{2d}(t)$$
$$+ N_{22}x_2(t)u_2(t) + N_{2d2}x_{2d}(t)u_{2d}(t)$$
$$+ B_{22}u_{2d}(t) + B_{2d2}u_{2d}(t) + C_{122}x_1(t);$$

where

$$A_{11} = \begin{bmatrix} -95 & 7 \\ -35 & -97 \end{bmatrix}, A_{12} = \begin{bmatrix} -82 & 9 \\ -30 & -90 \end{bmatrix}, A_{21} = \begin{bmatrix} -110 & 15 \\ -10 & -100 \end{bmatrix},$$

$$A_{22} = \begin{bmatrix} -102 & 9 \\ -5 & -90 \end{bmatrix}; N_{11} = \begin{bmatrix} -3 & 0 \\ 0 & 0 \end{bmatrix}, N_{12} = \begin{bmatrix} -3 & -3 \\ 1 & -3 \end{bmatrix},$$

$$N_{21} = \begin{bmatrix} 0 & 0 \\ 0 & -4 \end{bmatrix}, N_{22} = \begin{bmatrix} 0 & 0 \\ 4 & -1 \end{bmatrix}; B_{11} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}, B_{12} = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

$$B_{21} = B_{22} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}; A_{1d1} = \begin{bmatrix} -10 & 0 \\ 5 & 2 \end{bmatrix}, A_{1d2} = \begin{bmatrix} -15 & 0 \\ 0 & 7 \end{bmatrix},$$

$$A_{2d1} = \begin{bmatrix} -12 & 0 \\ 10 & 24 \end{bmatrix}, A_{2d2} = \begin{bmatrix} 10 & 0 \\ 14 & 7 \end{bmatrix}; N_{1d1} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.2 \end{bmatrix},$$

$$N_{1d2} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.5 \end{bmatrix}, N_{2d1} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.5 \end{bmatrix}, N_{2d2} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.5 \end{bmatrix};$$

$$B_{1d1} = B_{1d2} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, B_{2d1} = B_{2d2} = \begin{bmatrix} -1 \\ 0 \end{bmatrix};$$

$$C_{211} = C_{212} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}, C_{121} = C_{122} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}.$$

The membership functions are chosen as

$$\mu_{F_{11}^1}(x_{11}) = \frac{1}{1+\exp(-2x_{11})}, \mu_{F_{11}^2}(x_{11}) = 1 - \mu_{F_{11}^1}(x_{11})$$

And

$$\mu_{F_{21}^1}(x_{21}) = \frac{1}{1+\exp(-2x_{21})}, \ \mu_{F_{21}^2}(x_{21}) = 1 - \mu_{F_{21}^1}(x_{21})$$

By letting $\rho_1 = 0.75, \rho_2 = 0.63,$ $\varepsilon_{11} = \varepsilon_{12} = 0.13, \varepsilon_{21} = \varepsilon_{22} = 0.075, a_1 = a_2 = 0$, solving LMIs (13) gives the following feasible solution:

$$K_{11} = [-1.1233 \quad -1.0443];$$
$$K_{12} = [-1.8542 \quad -1.0211];$$
$$K_{21} = [-1.4735 \quad -1.2671];$$
$$K_{22} = [-1.8013 \quad -1.2024].$$

Initial condition is assumed to be $x_{10} = [-2.8 \quad 1.7]^{\mathrm{T}}$, $x_{20} = [3.0 \quad -1.6]^{\mathrm{T}}$.

The simulation results are shown in Fig. 1, Fig. 2 and Fig. 3. Fig. 1 and Fig. 2 show the state responses of two subsystems and control trajectory is shown in Fig. 3. It can be seen that with the decentralized fuzzy control law the closed-loop system is asymptotically stable. The simulation results show that the fuzzy controller proposed in this paper is effective.

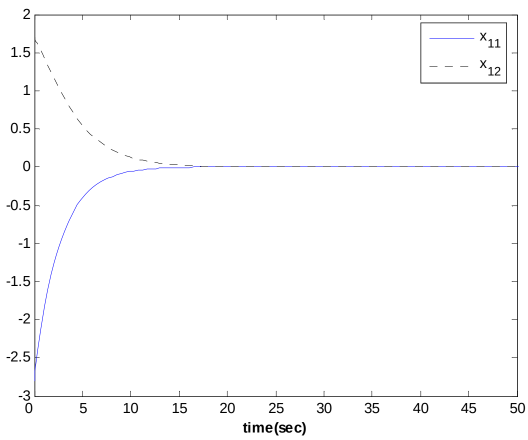

Fig. 1. State responses of subsystem 1.

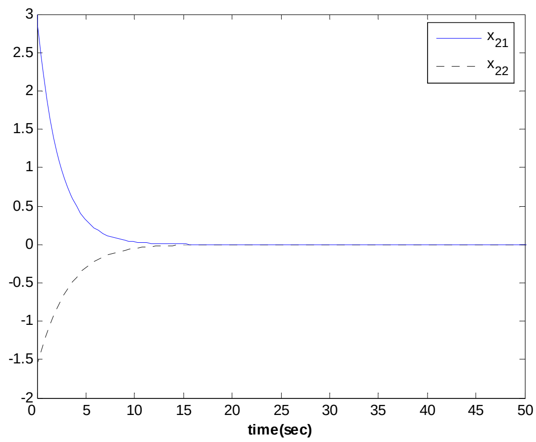

Fig. 2. State responses of subsystem 2.

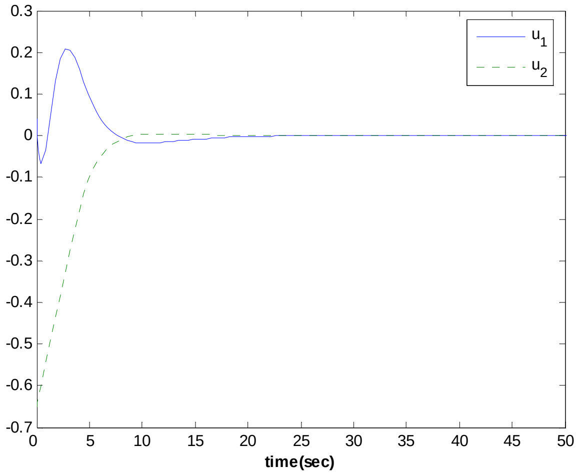

Fig. 3. Control trajectory.

5. Conclusion

In this paper, a T-S fuzzy bilinear model is proposed to study the control problems for time-delay nonlinear interconnected systems using fuzzy decentralized control. Based on the Lyapunov criterion, the sufficient conditions for delay-dependent stabilization of the interconnected system are presented.

The decentralized controllers designing problems can be formulated as a convex optimization problem with LMI constraints. A simulation example is included to show the effectiveness of the proposed approach.

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