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Wave Optics by Fourier Transform. Diffractions by Identical 3D Objects

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Abstract: The aim of this work is to outline a method for studying wave optics in 3D space based on the well-known properties of the Fourier transform and the Heaviside, Dirac delta functions. Applications for obtaining the amplitudes of diffraction of plane waves by one then by many identical 3D objects utilizing convolution products are given. It results from this study that the forbidden (permitted) diffracted directions caused by an object is the same (altered) for many identical objects. Studies of diffractions of a plane wave $\vec{k}_0$ by sets of equidistant points, planes, prisms, spheres are given.

Keywords: Laws of wave optics, Object functions, Fourier transform, Fresnel formulae, Diffractions by prisms, Diffractions by spheres, Diffractions by sets of identical 3D objects.

1. Introduction

Consider the diffraction [1] or interaction of a plane wave $exp\, i(-\vec{k}_0\vec{r}-\omega t)$, where $\vec{k}_0$ is the wave vector and $k_0=\frac{2\pi}{\lambda}$ by a 3D object having some geometric form. In colleges' cursus geometric forms are represented by equations or inequalities such as $ax+by+cz=d$ for a plane, $x^2+y^2+z^2<R^2$ for a dense sphere. On the contrary in this study a geometric form of a 3D object occupying a domain D in space will be represented by a function $f_D(\vec{r})$ equal to unity for a point $\vec{r}$ inside D and to zero for a point outside, called object function. This function is similar to the function describing a 2D aperture in the Kirchhoff's diffraction theory [1]. From this definition of object functions we see that the well-known Heaviside function $H(z)$ and the Dirac delta function $\delta(z)$ together with the unity function $u(x)=1$ will be largely utilized. For example $H(x)H(y)H(z)$ represents the domain where all points have positive coordinates. We propose the readers to recognize the domain represented by $H(x-a)H(y-b)\delta(z-c)$ for familiarizing with the notion of object functions.

From the above considerations we may deduce the following results for wave optics.

2. Diffraction of a Wave $\vec{k}_0$ by a 3D Object

2.1. Objects Centered at the Origin of Coordinates

The advantage of the introduction of object functions consists in allowing us to interpret

$f_D(\vec{r})e^{-i\vec{k}_0\vec{r}-i\omega t}$ as the compound wave resulting from the diffraction of the incident wave $e^{-i\vec{k}_0\vec{r}-i\omega t}$ by the object represented by $f_D(\vec{r})$.

For calculating the amplitude the wave diffracted in the direction $\vec{k}$ of this compound wave as illustrated in Fig. 1 let us consider the Fourier transform

$$\tilde{f}(\vec{k}) = FTf(\vec{r}) = (2\pi)^{-\frac{3}{2}} \int_{R^3} e^{-i\vec{k}\vec{r}} f(\vec{r})d\vec{r} \qquad (2.1)$$

We know that

$$\int_{R^3} e^{-i(\vec{k}-\vec{k}')\vec{r}} d\vec{r} = (2\pi)^3 \delta(\vec{k}-\vec{k}') \qquad (2.2)$$

and that a simple function may be expanded into a series of functions $exp(i\vec{k}\vec{r})$

$$f_D(\vec{r})e^{i\vec{k}_0\vec{r}} = \sum_{\vec{k}'} c(\vec{k}')e^{i\vec{k}'\vec{r}}, \qquad (2.3)$$

where $c(\vec{k})$ is the amplitude of the component wave $exp(i\vec{k}\vec{r})$ in the compound wave $f_D(\vec{r})e^{i\vec{k}_0\vec{r}}$. By (2.2) we get

$$\int_{R^3} f_D(\vec{r})e^{i\vec{k}_0\vec{r}}e^{-i\vec{k}\vec{r}}d\vec{r} = \sum_{\vec{k}'} c(\vec{k}') \int_{R^3} e^{i\vec{k}'\vec{r}}e^{-i\vec{k}\vec{r}}d\vec{r} \qquad (2.4)$$

$$= (2\pi)^3 c(\vec{k}),$$

$$c(\vec{k}) = (2\pi)^{-3} \int_{R^3} e^{-i(\vec{k}-\vec{k}_0)\vec{r}} f_D(\vec{r})d\vec{r}$$

and obtain the fundamental theorem [2]:

The amplitude of the diffracted wave $\vec{k}$ *is within a constant factor equal to the Fourier transform of the object function* $f_D(\vec{r})$ *calculated for* $\Delta\vec{k} = \vec{k} - \vec{k}_0$

$$A_D(\vec{k}_0 \to \vec{k}) = \tilde{f}_D(\Delta\vec{k}) \qquad (2.6)$$

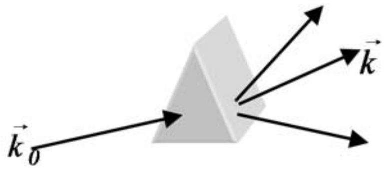

Fig. 1. Diffraction of a plane wave by a 3D object.

The Theorem (2.6) is applicable for diffractions by 2D apertures and is to be utilized for correcting the similar theorem for apertures found for example in [3] by changing in it $\vec{k}$ with $\Delta\vec{k} = \vec{k} - \vec{k}_0$.

2.2. Applying the fundamental theorem for the diffraction

As example, applying the fundamental theorem for the diffraction of $\vec{k}_0$ by the plane Oxy as illustrated by Fig. 2. Oxy is described by the object function $f_D(\vec{r}) = u(x)u(y)\delta(z)$. As

$$FT\delta(z) = \frac{1}{\sqrt{2\pi}}u(k_z)$$

$$FTu(x) = \sqrt{2\pi}\delta(k_x)$$

we see that

$$A_D(\vec{k}_0 \to \vec{k}) = \sqrt{2\pi}\delta(\Delta\vec{k}_x)\delta(\Delta\vec{k}_y)u(\Delta\vec{k}_z) \qquad (2.7)$$

so that there are diffracted waves only for $\Delta\vec{k}_x \equiv (\Delta\vec{k})_x = 0$ and $\Delta\vec{k}_y \equiv (\Delta\vec{k})_y = 0$.

By construction we get the Descartes law of reflection which implies the incident and the reflected rays $\vec{k}''$ to be symmetric as shown Fig. 2. Moreover if the diffracted wave $\vec{k}'$ is situated in a medium where the refractive index is n so that $k' = nk_0$ we get the Snell's law for refraction $sin\,i = n\,sin\,r$.

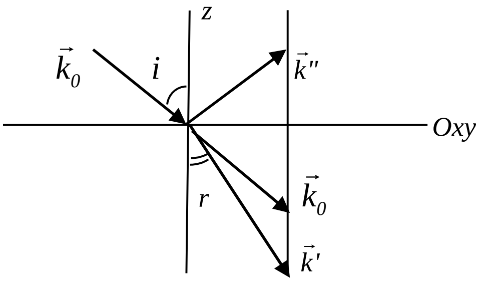

Fig. 2. Diffraction by the plane Oxy.

Now, let a, a', a'' denoted the amplitudes of the incident, the refracted and the reflected waves; n_1, n_2 the upper and lower semi-spaces refraction indices. a', a'' are proportional to a and respectively to $\tilde{f}_D(0,0,(\vec{k}'-\vec{k}_0)_z)$, $\tilde{f}_D(0,0,(\vec{k}''-\vec{k}_0)_z)$ so that

$$a' = \frac{\nu a}{(\Delta\vec{k}')_z} = \frac{\nu a}{k'\cos r - k_0\cos i} = \frac{\nu a \sin r}{k_0 \sin(i-r)}$$
$$a'' = \mu a \frac{1}{(\Delta\vec{k}'')_z} = -\frac{\mu a}{2k_0 \cos i} \qquad (2.8)$$

In order to calculate the coefficients μ, ν we will make use of the law of conservation of energies. The incoming density of energy at the interface Oxy is proportional to a^2, to the inclination $|\cos i_1|$ and the duration of time an incoming photon is in the vicinity of it, i.e. to v_1^{-1} or n_1. Similarly for the density of outgoing energies so that

$$n_1 a^2 \cos i = n_2 a'^2 \cos r + n_1 a''^2 \cos i \qquad (2.9)$$

The above equations and the formula $4\cos i \sin i \cos r \sin r = \sin^2(i+r) - \sin^2(i-r)$ lead by (2.8) to the following

$$\begin{gathered} 4k_1^2 n_1^2 \cos^2 i \sin^2(i-r) = \\ \mu^2 \sin^2(i-r) + \nu^2(\sin^2(i+r) - \sin^2(i-r)) \end{gathered} \qquad (2.10)$$

with

$$\mu = \nu = 2n_1 k_1 \cos i \sin(i-r) / \sin(i+r) \qquad (2.11)$$

we get the Fresnel formulae [4]

$$\begin{gathered} \frac{a'}{a} = \frac{2n_1 \cos i}{n_1 \cos i + n_2 \cos r} = \frac{2\cos i \sin r}{\sin(i+r)} \\ \frac{a''}{a} = \frac{n_1 \cos i - n_2 \cos r}{n_1 \cos i + n_2 \cos r} = -\frac{\sin(i-r)}{\sin(i+r)} \end{gathered} \qquad (2.12)$$

With $\mu = -\nu \cos(i+r)$ we get the second Fresnel formulae [4]

$$\begin{gathered} \frac{a'}{a} = \frac{2n_1 \cos i}{n_1 \cos i + n_2 \cos r} = \frac{2\cos i \sin r}{\sin(i+r)\cos(i-r)} \\ \frac{a''}{a} = \frac{\tan(i-r)}{\tan(i+r)} \end{gathered} \qquad (2.13)$$

From (2.13) we find again the Brewster's condition for total polarization $(i+r) = \frac{\pi}{2}$ [4].

2.3. Objects Centered at the Position $\vec{r}_j$

Now, for calculating the amplitude of diffraction of a plane wave by a 3D object centered at the position $\vec{r}_j$ instead of at the origin of coordinates and represented by $f_D(\vec{r} - \vec{r}_j)$ we utilize the convolution product property

$$\begin{aligned} f_D(\vec{r}) \otimes \delta(\vec{r} - \vec{r}_j) &= \int_{R^3} f_D(\vec{r}_0)\delta(\vec{r} - \vec{r}_j - \vec{r}_0) d\vec{r}_0 \\ &= f_D(\vec{r} - \vec{r}_j) \end{aligned} \qquad (2.14)$$

together with the Fourier transform property

$$FTf(z)g(z) = \frac{1}{\sqrt{2\pi}} \tilde{f}(k) \otimes \tilde{g}(k) \qquad (2.15)$$

to get the remarkable formula

$$FTf_D(\vec{r} - \vec{r}_j) = (2\pi)^{-\frac{3}{2}} \tilde{f}_D(\vec{k}) \exp(-i\vec{r}_j \vec{k}) \qquad (2.16)$$

From (2.16) we obtain the amplitudes of diffractions of $\vec{k}_0$ by many identical 3D objects

$$A_D(\vec{k}_0 \to \vec{k}) = \tilde{f}_D(\vec{k} - \vec{k}_0) \sum_j \exp(-i\vec{r}_j(\vec{k} - \vec{k}_0)) \qquad (2.17)$$

From the very important formula (2.17) we may emit a theorem on forbidden and allowed diffracted directions:

1) *If a direction* $\vec{k}$ *is forbidden in a diffraction by one object then it is also forbidden in a diffraction by many objects identical with it wherever they are.*

Applying this theorem for the case of Bragg's experience with many parallel planes we see that here all directions are forbidden except two.

2) *The amplitude of diffraction by a set of identical objects along a permitted direction* $\vec{k}$ *is proportional to the value of the sums over all the phases* $\sum_j e^{-i\vec{r}_j(\vec{k} - \vec{k}_0)}$.

This explains why in a diffraction by a material point all directions are equally allowed but in the diffractions by two or more points many directions may be forbidden or faint such as in the famous experience of Young on diffraction by two material points [3-4].

3) By extension we think that *the amplitudes of the waves emitted in a direction* $\vec{k}$ *by a synchronized system of electromagnetic or acoustic sources are proportional to* $\sum_j \exp(-i\vec{r}_j \vec{k})$. This maybe the principle of stereo.

3. Diffraction of a Wave $\vec{k}_0$ by a System of Material Points on the Coordinates Axis

Let us consider as in Ref. [5] the following system of material points

$A_{-M}, ..., A_0, ..., A_M$ are the $(2M+1)$ equidistant points situated on Ox;

$B_{-N}, ..., B_0, ..., B_N$ the $(2N+1)$ equidistant points situated on Oy;

$C_{-L}, ..., C_0, ..., C_L$ the $(2L+1)$ equidistant points situated on Oz.

The object function of the system of points A_m is

$$f_A(\vec{r}) = \sum_{m=-M}^{M} \delta(x - ma)\delta(y)\delta(z) \qquad (3.1)$$

Its Fourier transform is

$$\tilde{f}_A(\vec{k})=(2\pi)^{-\frac{3}{2}}\sum_{m=-M}^{M} e^{-ima k_x}u(k_y)u(k_z) \qquad (3.2)$$

Because $\delta(x-ma)\delta(y-nb)\delta(z-lc)$ represented the point (ma,nb,lc), the object function of the systems of the points A_m and B_n on the plane Oxy is

$$f_{AB}(\vec{r})=\sum_{m=-M}^{M}\sum_{n=-N}^{N}\delta(x-ma)\delta(y-nb)\delta(z) \qquad (3.3)$$

so that

$$\tilde{f}_{AB}(\vec{k})=(2\pi)^{-\frac{3}{2}}\sum_{m=-M}^{M}\sum_{n=-N}^{N}e^{-imak_x}e^{-inbk_y}u(k_z) \qquad (3.4)$$

The sum of terms of a geometric progression is easy to obtain and gives

$$\begin{aligned}&\sum_{m=-M}^{M}e^{ima\,k_x}-e^{i0a\,k_x}=\sum_{m=-M}^{M}e^{ima\,k_x}-1\\&=2\cos\frac{(M+1)ak_x}{2}\sin\frac{Mak_x}{2}/\sin\frac{ak_x}{2}\end{aligned} \qquad (3.5)$$

so that

$$\begin{aligned}&\tilde{f}_{AB}(\vec{k})=2\cos\frac{(M+1)ak_x}{2}\sin\frac{Mak_x}{2}/\sin\frac{ak_x}{2}\\&\times(2\pi)^{-\frac{3}{2}}2\cos\frac{(N+1)bk_y}{2}\sin\frac{Nbk_y}{2}/\sin\frac{bk_y}{2}\end{aligned} \qquad (3.6)$$

We see then that in a diffraction of $\vec{k}_0$ by the system of points A_m and B_n without A_0 and B_0, the maximum amplitudes correspond to the diffracted $\vec{k}$ verifying

$$k_x-k_{0x}=\frac{2p\pi}{(M+1)a},p\in Z \qquad (3.7)$$

$$k_y-k_{0y}=\frac{2q\pi}{(N+1)b},q\in Z \qquad (3.8)$$

$$k_z^{\,2}+k_y^{\,2}+k_x^{\,2}=\left\|\vec{k}_0\right\|^2 \qquad (3.9)$$

In (3.7) the value of p is limited by the fact that $k_x-k_{0x}\le 2k_0=4\pi/\lambda$

i.e. $\quad p\le 2(M+1)a/\lambda \qquad (3.10)$

On the contrary there are extinctions of diffracted waves corresponding to

$$k_x-k_{0x}=\frac{\pi}{(M+1)a}p,\,p\in Z \qquad (3.11)$$

$$k_y-k_{0y}=\frac{\pi}{(N+1)b}q\,,q\in Z \qquad (3.12)$$

$$k_z^{\,2}+k_y^{\,2}+k_x^{\,2}=\left\|\vec{k}_0\right\|^2 \qquad (3.13)$$

Extension to diffractions by the system of points A_m, B_n, C_l without A_0, B_0, C_0 in the space O_{xyz} is not difficult to obtain.

4. Diffraction by a System of Identical 3D Objects

According to (2.14) we obtain the object functions for the system of identical objects centered at the system of points $\vec{r}_j$ by the formula

$$\sum_j f_D(\vec{r}-\vec{r}_j)=f_D(\vec{r})\otimes\sum_j\delta(\vec{r}-\vec{r}_j) \qquad (4.1)$$

The Fourier transform of this system of objects is

$$FT\sum_j f_D(\vec{r}-\vec{r}_j)=\tilde{f}_D(\vec{k})FT\sum_j\delta(\vec{r}-\vec{r}_j) \qquad (4.2)$$

$$=(2\pi)^{-\frac{3}{2}}\tilde{f}_D(\vec{k})\sum_j e^{-i\vec{k}\vec{r}_j} \qquad (4.3)$$

As example in a diffraction by a system of parallel planes perpendicular to the Oz axis as shown in Fig. 3 there are two permitted diffracted waves $\vec{k}$ corresponding to $\Delta\vec{k}=0$ or $\Delta\vec{k}\,//Oz$ as in the case of diffraction by Oxy. As for the amplitude of these allowed diffracted wave it is optimum for

$$\Delta k_z=\frac{2\pi}{(L+1)c}p,\;p\in Z \qquad (4.4)$$

According to Fig. 3

$$(\Delta\vec{k})_z=2k_0\sin\theta=2(2\pi/\lambda)\sin\theta \qquad (4.5)$$

so that from (4.7), (4.8) we may write down

$$(\Delta\vec{k})_z c=2c(2\pi/\lambda)\sin\theta=\frac{p}{L+1}2\pi \qquad (4.6)$$

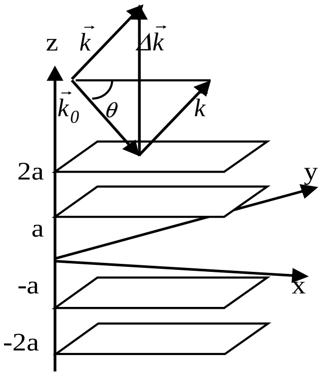

Fig. 3. Diffraction by parallel planes.

Changing the notation for the integer p into $(L+1)n$ we find again the famous Braag's law [3-4]

$$2c\sin\theta = n\lambda \tag{4.7}$$

4.1. Diffraction by a Set of Identical Spheres Centered at the Set of Points (ma, nb, lc)

Utilizing the Heaviside function we get the object function of a dense sphere illustrated in Fig. 4.

$$\begin{aligned} S(x,y,z) = H(R^2 - z^2)H(R^2 - y^2 - z^2) \\ \times H(R^2 - x^2 - y^2 - z^2) \end{aligned} \tag{4.8}$$

Its Fourier transform is invariant in any rotation around the origin O so that

$$\begin{aligned} \tilde{S}(k_x,k_y,k_z) = \tilde{S}(0,0,k) = 2(2\pi)^{-3/2}\int_{-R}^{R} e^{-ikz}dz \\ \times\int_{-\sqrt{(R^2-z^2}}^{\sqrt{(R^2-z^2}} dy\sqrt{((R^2 - z^2) - y^2)} \end{aligned} \tag{4.9}$$

Because

$$\begin{aligned} &\int_{-\sqrt{(R^2-z^2}}^{\sqrt{(R^2-z^2}} dy\sqrt{((R^2 - z^2) - y^2)} \\ &= (R^2 - z^2)\int_{-\pi/2}^{\pi/2}\sqrt{1 - \sin^2\varphi}\, d(\sin\varphi) = \\ &\frac{1}{2}\pi(R^2 - z^2), \end{aligned} \tag{4.10}$$

$$\begin{aligned} \int_{-R}^{R} dze^{-ikz}(R^2 - z^2) = \frac{2}{-ik}\left(\frac{1}{-ik}(Re^{-ikR} + Re^{ikR})\right) \\ - \frac{2}{-ik}\frac{1}{-ik}\frac{-1}{-ik}2i\sin kR \end{aligned} \tag{4.11}$$

we get

$$\tilde{S}(\vec{k}) = (2\pi)^{-\frac{1}{2}}\frac{2R}{k^2}\left(\frac{\sin Rk}{Rk} - \cos Rk\right) \tag{4.12}$$

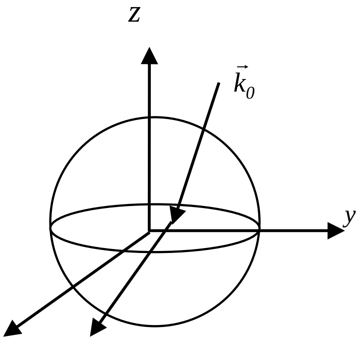

Fig. 4. Diffraction by a dense sphere.

From (4.12) we see that in a diffraction by a dense sphere the amplitude of diffraction is inversely proportional to

$$(\Delta k)^2 \text{ with } \Delta k = \left\|\Delta\vec{k}\right\|$$

and there is extinction if

$$\tan R\Delta k = R\Delta k \Rightarrow R\Delta k = 0.02 \tag{4.13}$$

Eq. (4.12) is useful for studying hemoglobins with X rays and noticeably the deflection of light by the sun, observed in 1919 [6].

The Fourier transform of a set of spheres centered at the points (ma, nb, lc) except the origin O is

$$\begin{aligned} F(\vec{k}) = \tilde{S}(\vec{k})2^3\,\frac{\cos\frac{(M+1)ak_x}{2}\sin\frac{Mak_x}{2}}{\sin(ak_x/2)} \times \\ \frac{\cos\frac{(N+1)bk_y}{2}\sin\frac{Nbk_y}{2}\cos\frac{(L+1)ck_z}{2}\sin\frac{Lck_z}{2}}{\sin(bk_y/2)\sin(ck_z/2)} \end{aligned} \tag{4.14}$$

The forbidden and allowed diffracted waves are obtainable from (4.13), (4.14) and the theorem (2.17).

For identical 3D objects situated in a system of oblique coordinates axis $(\vec{u}_1, \vec{u}_2, \vec{u}_3)$ having reciprocal vectors $(\vec{v}_1, \vec{v}_2, \vec{v}_3)$ we have to use the formulae

$$\vec{v}_i\vec{u}_j = \delta_{ij} \quad i, j = 1,2,3$$

$$\begin{aligned} \vec{r} &= (\bar{r}\vec{v}_1)\vec{u}_1 + (\bar{r}\vec{v}_2)\vec{u}_2 + (\bar{r}\vec{v}_3)\vec{u}_3 \\ &= (\bar{r}\vec{u}_1)\vec{v}_1 + (\bar{r}\vec{u}_2)\vec{v}_2 + (\bar{r}\vec{u}_3)\vec{v}_3 \end{aligned}$$

$$\begin{aligned} r^2 = \vec{r}\vec{r} = (\bar{r}\vec{v}_1)(\vec{r}\vec{u}_1) + (\bar{r}\vec{v}_2)(\vec{r}\vec{u}_2) + (\bar{r}\vec{v}_3)(\vec{r}\vec{u}_3) \\ \bar{k}\vec{r} = (\bar{k}\vec{v}_1)(\vec{r}\vec{u}_1) + (\bar{k}\vec{v}_2)(\vec{r}\vec{u}_2) + (\bar{k}\vec{v}_3)(\vec{r}\vec{u}_3) \end{aligned}$$

and the very useful one [2]

$$FTf(\vec{r}\vec{u}_1, \vec{r}\vec{u}_2, \vec{r}\vec{u}_3) = \left|det(\vec{v}_1, \vec{v}_2, \vec{v}_3)\right|\tilde{f}(\vec{k}\vec{v}_1, \vec{k}\vec{v}_2, \vec{k}\vec{v}_3) \tag{4.15}$$

For example for a set of equidistant points on $(\vec{u}_1, \vec{u}_2, \vec{u}_3)$ whose coordinates are $(\vec{r}\vec{v}_1, \vec{r}\vec{v}_2, \vec{r}\vec{v}_3)$ we see that within the factor $(2\pi)^{-\frac{3}{2}} det(\vec{u}_1, \vec{u}_2, \vec{u}_3)$

$$\tilde{f}_D(\vec{k}) = \sum_{m=-M}^{M} e^{-ima\vec{k}\vec{u}_1} \sum_{n=-N}^{N} e^{-inb\vec{k}\vec{u}_2} \sum_{l=-L}^{L} e^{-ilc\vec{k}v\vec{u}_3} \quad (4.16)$$

As example, consider the diffraction by a trihedron that has all the components of $\vec{r}$ on $\vec{u}_0, \vec{u}_1, \vec{u}_2$ positive as shown Fig. 5. It may simply be represented by the function

$$T^{(3)}{}_{\vec{u}_0,\vec{u}_1,\vec{u}_2}(\vec{r}) = H(\vec{v}_0.\vec{r})H(\vec{v}_1.\vec{r})H(\vec{v}_2\vec{r}) \quad (4.17)$$

$$\begin{aligned}\tilde{T}^{(3)}{}_{\vec{u}_0,\vec{u}_1,\vec{u}_2}(\vec{k}) &= \left|det(\vec{u}_0,\vec{u}_1,\vec{u}_2)\right| \\ &\times \tilde{H}(\vec{u}_0.\vec{k})\tilde{H}(\vec{u}_1.\vec{k})\tilde{H}(\vec{u}_2.\vec{k})\end{aligned} \quad (4.18)$$

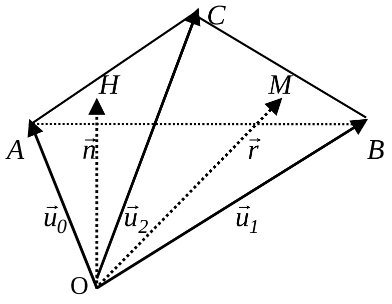

Fig. 5. Diffraction by a trihedron.

The Fourier transform of the Heaviside function may be calculate straightforwardly and is found to be

$$\tilde{H}(k) = (2\pi)^{-\frac{1}{2}}\left(\frac{1}{ik} + \pi\delta(k)\right) \quad (4.19)$$

and we may conclude that the amplitude of diffraction of a plane wave $\vec{k}_0$ into a plane wave $\vec{k}$ by a trihedron is maximum whenever

$$\begin{aligned} & \vec{u}_0\Delta\vec{k} = \vec{u}_1\Delta\vec{k} = 0 \\ \text{or} \quad & \vec{u}_0\Delta\vec{k} = \vec{u}_2\Delta\vec{k} = 0 \quad (4.20) \\ \text{or} \quad & \vec{u}_2\Delta\vec{k} = \vec{u}_1\Delta\vec{k} = 0 \end{aligned}$$

i.e. whenever $\Delta\vec{k}$ is perpendicular to two vectors of the set $\vec{u}_0, \vec{u}_1, \vec{u}_2$, i.e. to each one of its faces.

For diffractions by the tetrahedron or prism $OABC$ we remark that the points A, B, C correspond respectively to $\vec{u}_0, \vec{u}_1, \vec{u}_2$ so that they and the plane ABC verifies the equation

$$\vec{v}_0\vec{r} + \vec{v}_1\vec{r} + \vec{v}_2\vec{r} = 1 \quad (4.21)$$

A point under the plane ABC obeys the inequality

$$\vec{v}_0\vec{r} + \vec{v}_1\vec{r} + \vec{v}_2\vec{r} < 1 \quad (4.22)$$

The object function of the semi space under ABC is

$$S^{-}{}_{ABC}(\vec{r}) \equiv u(\vec{v}_0\vec{r})u(\vec{v}_1\vec{r})H(1-(\vec{v}_0+\vec{v}_1+\vec{v}_2)\vec{r}) \quad (4.23)$$

For calculating the Fourier transform of $S^{-}{}_{ABC}(\vec{r})$ we remark that $(\vec{v}_0, \vec{v}_1, -(\vec{v}_0+\vec{v}_1+\vec{v}_2))$ and $(\vec{u}_0-\vec{u}_2, \vec{u}_1-\vec{u}_2, -\vec{u}_2)$ are reciprocal vectors so that

$$\begin{aligned}\tilde{S}^{-}{}_{ABC}(\vec{k}) &= 2\pi\, det(\vec{u}_0,\vec{u}_1,\vec{u}_2) \\ &\times \delta((\vec{u}_0-\vec{u}_2)\vec{k})\delta((\vec{u}_1-\vec{u}_2)\vec{k}) \\ &e^{-i\vec{u}_2\vec{k}}\tilde{H}(-\vec{u}_2\vec{k})\end{aligned} \quad (4.24)$$

It is cumbersome but possible to calculate in details the Fourier transform of the tetrahedron by convolution

$$\tilde{T}^{(4)}{}_{\vec{u}_0,\vec{u}_1,\vec{u}_2}(\vec{k}) = \tilde{T}^{(3)}{}_{\vec{u}_0,\vec{u}_1,\vec{u}_2}(\vec{k}) * \tilde{S}^{-}{}_{ABC}(\vec{k}) \quad (4.25)$$

A part of the results is with $z_j = \vec{u}_j\vec{k} \neq 0 \quad \forall j$

$$\begin{aligned}\tilde{T}^{(4)}{}_{\vec{u}_0,\vec{u}_1,\vec{u}_2}(\vec{k}) &= -i(2\pi)^{-\frac{3}{2}} det(\vec{u}_0,\vec{u}_1,\vec{u}_2) \\ &\times \left(\sum_{j=0}^{2} \frac{e^{-iz_j}}{(z_{j-1}-z_j)(z_{j+1}-z_j)z_j} - \frac{1}{z_0 z_1 z_2}\right)\end{aligned} \quad (4.26)$$

On the other hand for one or two $z_j = 0$, it is simply or doubly infinite [2].

By juxtapositions of prisms we may obtain oblique pyramids with polygonal bases, diamonds, etc.

5. Remarks and Conclusions

The main remark of this work is that waves and the Fourier transform utilize the same function $e^{-i\vec{k}\vec{r}}$ so that we may utilize the properties of the Fourier transform to clarify and resolve many problems in wave optics.

Secondly it is realized that the use of Heaviside and Dirac delta functions for representing the functions describing the geometric forms of objects are very advantageous. With these functions we may calculate the specters of diffractions by systems of points, planes, spheres and so all. By convolution we think that it is possible to get also specters of diffractions of unions and intersections of them.

From these remarks and the results given in this work we may conclude that wave optics may be constructed by combining the use of $\exp i(-\vec{k}_0\vec{r} - \omega t)$

to designate a plane wave and the works of Fourier, Heaviside, Dirac.

The main interest of this work, we suppose, is providing to researchers in chemistry, biology, acoustics, optics a tool for studying the structures of crystals, molecules, systems of hemoglobins, etc.

To be more useful we have to list the object functions of simple objects such as:

A disk in the $Oyz\ plane$:

$$\delta(x)H(R^2-z^z)H(R^2-y^2-z^z)$$

A ring in the $Oyz\ plane$:

$$\delta(x)\delta(R^2-z^z)\delta(R^2-y^2-z^z)$$

A thin film:

$$u(x)u(y)H(z-a)H(b-z)$$

A symmetric segment of length $2d$ on the straight line $\frac{x}{a}=\frac{y}{b}=\frac{z}{c}$:

$$\delta(\frac{x}{a}-\frac{y}{b})\delta(\frac{y}{b}-\frac{z}{c})H(d^2-x^2-y^2-z^2)$$

A truncated cylinder in the z -direction:

$$H(R^2-x^2-y^z)H(R^2-y^z)H(z-a)H(b-z)$$

Lastly we may state from (2.3), (2.5) with $\vec{k}_0=0$ that for any material object (Yang element) corresponds a wave (Ying element) of which the $\vec{k}$ component is its Fourier transform.

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