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Diffraction from Various Surfaces with Modified Theory of Physical Optics

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Abstract: The scattering problem from the surfaces is introduced by method of Modified Theory of Physical Optics (MTPO). The method is based on the physical optics (PO) current, on the scaterers' and aperture surfaces, which assist in finding the integral equation and the evaluation is achieved asymptotically with the stationary phase method. The comparisons, on various surfaces, prove the agreement of the MTPO with respect to the exact solution of Helmholtz equation. MTPO can be applied to calculate not only the reflected fields but also the exact edge-diffracted fields contrary to Physical Optics which has some weak points in edge-diffraction calculation. It is edge-point technique which is used to find edge diffractions of various surfaces such as perfectly electrically conducting (PEC) plane, impedance parabolic surface and black half-plane surfaces. In some applications, the obtained results for edge diffractions are non-uniform expressions which has zero value in denominator. Fresnel integral is used in order to convert non-uniform expressions to uniform expression.

Keywords: Physical optics, Modified theory of physical Optics, diffracted field, Edge diffraction, Edge-point technique.

1. Introduction

Physical optics (PO) is one of the major methods to calculate the scattering fields from various geometries. Reflected fields are obtained accurately by this method. Nevertheless, it has some weak points while evaluating diffracted fields [1-3]. The innovative method of "Modified Theory of Physical Optics" (MTPO) has been derived to have an exact solution not for only the reflected fields but also for the edge diffraction. MTPO has some axioms for solution of scattering problems. By usage of MTPO axioms, which will be explained in this study, MTPO scattering integral is obtained. The result is in accord with the one found by the exact solution of the Helmholtz equation [4].

The previous applications of MTPO have been researched on perfectly electrically conducting (PEC) [5], impedance [6] half plane, PEC [7, 12], black [15] cylindrical reflector, impedance [9] and PEC parabolic reflector [13-14]. These applications are compiled in the articles [16]. In this study, the applications will be deeply examined after the analysis of this method.

2. Theory

A random geometry can be selected to explain the general rules of this method. In this article the Fig. 1

is to be considered for defining the variables [5]. According to the PO, S_1 is the surface on which a current is induced and the reflected and reflected diffracted fields are obtained. However the incident diffracted field is excluded by this method since the edge effect is not taken into account. An aperture surface (S_2) is imagined due to missing value of edge effect. By usage of Equivalent Source Theorem, equivalent surface currents are obtained on these surface

$$\vec{J}_{es} = \vec{n}_1 \times \vec{H}_t\big|_{S}, \tag{1a}$$

$$\vec{J}_{es} = \vec{n}_2 \times \vec{H}_i\big|_{S_2}, \tag{1b}$$

$$\vec{J}_{ms} = -\vec{n}_2 \times \vec{E}_i\big|_{S_2} \tag{1c}$$

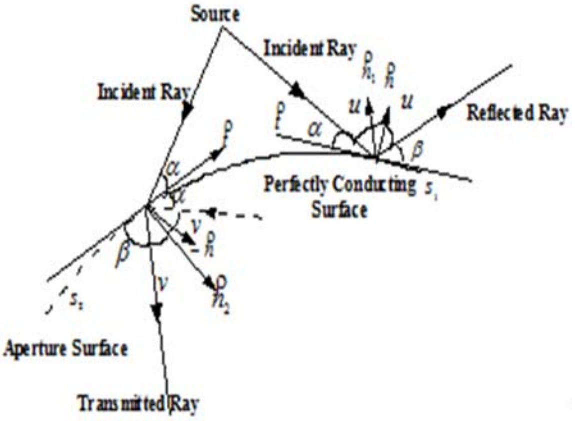

Fig. 1. The demonstration for explanation of MTPO [5].

Reflection and transmission angle β is not equal to incidence angle and β has a dependancy on surfaces. Also $\vec{n}_1$ and $\vec{n}_2$ are not equal to normal vector but they are the vectors dividing the angle, which occurs between incident ray and reflecred ray, into 2 equal parts. By the geometry and equivalent surface currents, the incident and reflected scattering electrical fields & magnetic fields can be found as

$$\vec{E}_{is} = -\frac{j\omega\mu_0}{4\pi}\iint\limits_{S_2} \vec{n}_2 \times \vec{H}_i\big|_{S_2} \frac{e^{-jkR_2}}{R_2} dS' + \frac{1}{4\pi}\iint\limits_{S_2} \nabla \times (\vec{n}_2 \times \vec{E}_i\big|_{S_2} \frac{e^{-jkR_2}}{R_2}) dS', \tag{2a}$$

$$\vec{E}_{rs} = -\frac{j\omega\mu_0}{4\pi}\iint\limits_{S_1} \vec{n}_1 \times \vec{H}_t\big|_{S_1} \frac{e^{-jkR_1}}{R_1} dS' \tag{2b}$$

$$\vec{H}_{is} = \frac{1}{4\pi}\iint\limits_{S_2} \nabla \times (\vec{n}_2 \times \vec{H}_i\big|_{S_2} \frac{e^{-jkR_2}}{R_2}) dS' \tag{3a}$$

$$+\frac{j\omega\epsilon}{4\pi}\iint\limits_{S_2} \left(\vec{n}_2 \times \vec{E}_i\big|_{S_2} \frac{e^{-jkR_2}}{R_2}\right) dS',$$

$$\vec{H}_{rs} = \frac{1}{4\pi}\iint\limits_{S_1} \nabla \times (\vec{n}_1 \times \vec{H}_t\big|_{S_1} \frac{e^{-jkR_1}}{R_1}) dS' \tag{3b}$$

3. Applications of MTPO and Numerical Results

Since MTPO is an innovation method there have been studies on various surfaces.

a) The method was introduced with an application on ’perfectly conducting (PEC) half-plane’ surface by Umul [5]. Considering the incident plane waves, by the series of operations, the reflected scattered field and incident scattered field integrals are found to be as

$$\vec{E}_{rs} = -\vec{e}_z \frac{kE_i}{2}\int_{x'=0}^{\infty} e^{jkx'cos\emptyset_0} H_0{}^{(2)}(kR) \sin\left(\frac{\beta+\varphi_0}{2}\right) dx' \tag{4}$$

$$\vec{E}_{is} \approx \vec{e}_z \frac{kE_i}{\sqrt{2\pi}} e^{j\frac{\pi}{4}} \int_{x'=-\infty}^{0} e^{jkx'cos\emptyset_0} \frac{e^{-jkR_1}}{\sqrt{kR_1}} \sin(\frac{\beta+\varphi_0}{2}) dx' \tag{5}$$

respectively. The diffracted fields are obtained by using the amplitude and phase functions by stationary phase approach. The results are found as

$$\vec{E}_{rd} = \vec{e}_z \frac{E_i}{2\sqrt{2\pi}} \frac{e^{-jk\rho}}{\sqrt{k\rho}} \frac{\cos(\frac{\varphi-\varphi_0}{2})}{cos\varphi + cos\varphi_0} e^{-j\frac{\pi}{4}} \tag{6a}$$

$$\vec{E}_{id} = -\vec{e}_z \frac{E_i}{2\sqrt{2\pi}} \frac{e^{-jk\rho}}{\sqrt{k\rho}} \frac{\cos(\frac{\varphi+\varphi_0}{2})}{cos\varphi + cos\varphi_0} e^{-j\frac{\pi}{4}} \tag{6b}$$

The found data is compared by the exact solution [9] as it is shown in Fig. 2.

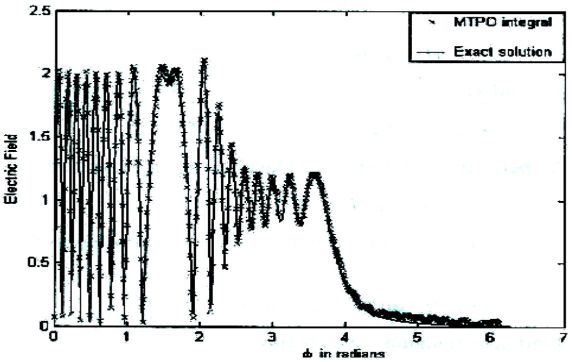

Fig. 2. Coherence between exact solution and MTPO [5].

b) Umul applied the method on ’Impedance Half Plane’ in 2006 [6].

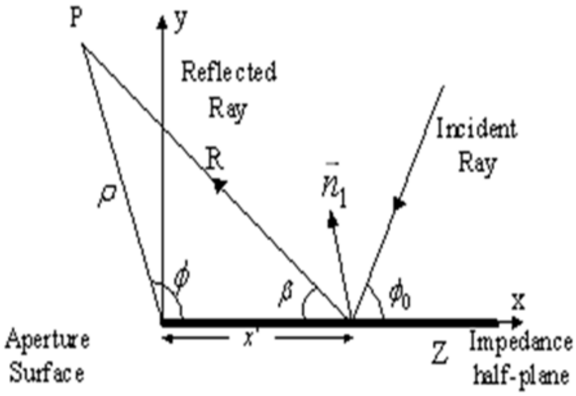

Fig. 3. Impedance Half-Plane Geometry [6].

The scattered reflected field has been calculated as

$$\vec{E}_{rs} = \vec{e}_z \frac{kE_i}{\sqrt{2\pi}} e^{j\frac{\pi}{4}} \int_{x'=0}^{\infty} \sin(\frac{\beta+\varphi_0}{2}) \times e^{jkx'cos\emptyset_0} \frac{Z\sin\left(\frac{\beta+\varphi_0}{2}\right)-Z_0}{Z\sin\left(\frac{\beta+\varphi_0}{2}\right)+Z_0}\frac{e^{-jkR}}{\sqrt{kR}}dx' \tag{7a}$$

and the incident scattered field is

$$\vec{E}_{is} = -\vec{e}_z \frac{kE_i}{\sqrt{2\pi}} e^{j\frac{\pi}{4}} \int_{x'=-\infty}^{0} \sin(\frac{\beta-\varphi_0}{2}) \, e^{jkx'cos\emptyset_0} \frac{Z\sin\left(\frac{\beta+\varphi_0}{2}\right)-Z_0}{Z\sin\left(\frac{\beta+\varphi_0}{2}\right)+Z_0}\frac{e^{-jkR}}{\sqrt{kR}}dx' \tag{7b}$$

The equations are reduced to the ones found in PEC plane if $Z = 0$. Moreover, the diffracted fields are obtained as:

$$\vec{E}_{rd} = -\vec{e}_z E_i \frac{e^{-j\frac{\pi}{4}}}{2\sqrt{2\pi}} \frac{1}{cos\frac{\varphi+\varphi_0}{2}} \frac{cos\frac{\varphi-\varphi_0}{2} - sin\theta}{cos\frac{\varphi-\varphi_0}{2} + sin\theta} \frac{e^{-jk\rho}}{\sqrt{k\rho}} \tag{8a}$$

for reflected diffracted field and

$$\vec{E}_{id} = \vec{e}_z E_i \frac{e^{-j\frac{\pi}{4}}}{2\sqrt{2\pi}} \frac{1}{cos\frac{\varphi-\varphi_0}{2}} \frac{cos\frac{\varphi-\varphi_0}{2} - sin\theta}{cos\frac{\varphi-\varphi_0}{2} + sin\theta} \frac{e^{-jk\rho}}{\sqrt{k\rho}} \tag{8b}$$

for incident reflected field, respectively. The results are compared with the ones found by exact solution [4] and there is a harmony between them according to Fig. 4.

c) Yalçın applied MTPO on ’PEC Cylindrical Reflector’ in 2007 [7]. The geometry is symbolized as in Fig. 5. Sarnık and Yalçın studied ”uniform” diffracted fields in 2013 [12].

The total scattered field has been found as

$$\vec{E}_s = -\vec{e}_z \frac{k^2 Z_0 I\alpha}{2}\left\{\int_{\varphi'=-\varphi_0}^{\varphi_0} cos(\frac{\beta+\varphi_0}{2}) H_0{}^{(2)}(k\rho_2) H_0{}^{(2)}(k\rho_1) d\varphi + \int_{\varphi'=\varphi_0}^{-\varphi_0} sin(\frac{\varphi-\beta}{2}) H_0{}^{(2)}(k\rho_2) H_0{}^{(2)}(k\rho_1) d\varphi'\right\} \tag{9}$$

and diffracted field has been obtained as

$$\vec{E}_d = \vec{e}_z \frac{Z_0 I}{4\pi} \frac{\cos(\frac{\alpha_e+\beta_e}{2}) - sin(\frac{\alpha_e-\beta_e}{2})\theta}{sin\alpha_e - sin\beta_e} \frac{e^{-jk(l_0+l_1)}}{\sqrt{l_0 l_1}} \tag{10}$$

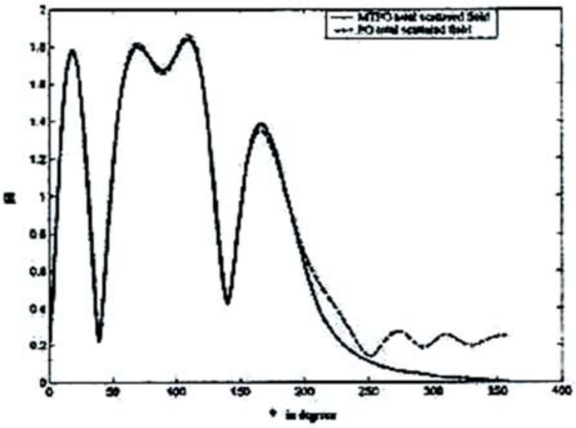

Fig. 4. The comparison between exact and MTPO solutions [6].

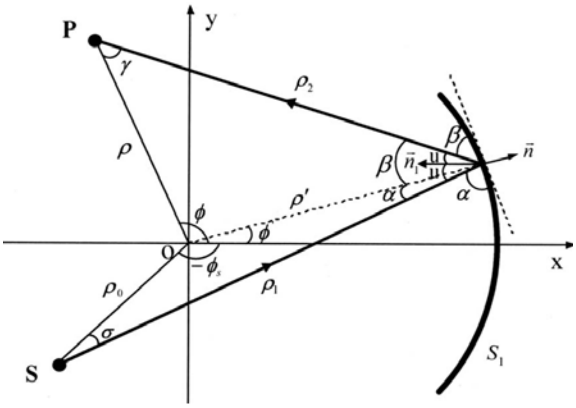

Fig. 5. Reflection on PEC Cylindrical Surface.

The graphic comparing PO, MTPO and exact solutions [11-12] is shown in the Fig. 6. Sarnık and Yalçın studied ’uniform’ diffracted fields in 2013 [12].

d) Umul applied the method on 'Cylindrical Parabolic Impedance Surface' in 2008 [8]. The total field equation

$$\vec{E}_{rs} = \vec{e}_z E_0 \frac{k e^{j(\pi/4)}}{\sqrt{2\pi}} \int_{-\varphi_0}^{\varphi_0} sin\alpha \frac{e^{-jk\rho'}}{\sqrt{k\rho'}} \frac{e^{-jkR}\rho'}{\sqrt{kR}\cos(\varphi'/2)} d\varphi' \qquad (11)$$

has been obtained by MTPO, where α denotes the angle between reflection point and the incident ray.

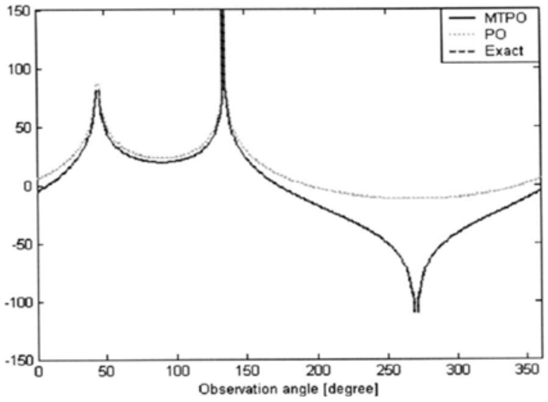

Fig. 6. PEC Cylindrical Reflector comparison between PO and MTPO solutions.

The Fig. 7 indicates the comparison of the total scattered fields found on the conducting reflector by PO and MTPO. The extreme points (maximum and minimum angles) are obtained in the same angle for each method.

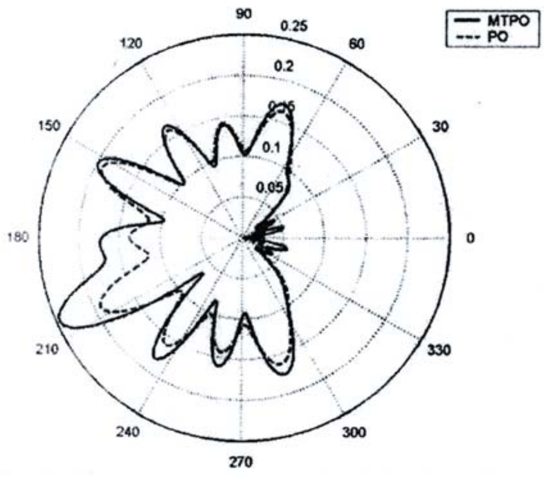

Fig. 7. Cylindrical Parabolic Impedance comparison between PO and MTPO solutions [8].

e) Sarnık and Yalçın studied non-uniform solution for 'PEC parabolic reflector' in 2016 [13] and reflection is shown in Fig. 8. Diffracted field was found as

$$\vec{E}_d = \vec{e}_z \frac{Z_0 I_0}{8\pi} \left[\frac{1}{\sin\frac{\beta_e - \alpha_e}{2}} - \frac{1}{\cos\frac{\beta_e + \alpha_e}{2}} \right] \frac{e^{-jk(\rho_e + l_e)}}{\sqrt{\rho_e l_e}} \qquad (12)$$

and this result is consistent with the ones found for PEC cylindrical reflector

$$\vec{E}_d = \vec{e}_z \frac{Z_0 I_0}{8\pi} \left[\frac{1}{\sin\frac{\beta_e - \alpha_e}{2}} - \frac{1}{\cos\frac{\beta_e + \alpha_e}{2}} \right] \frac{e^{-jk(l_0 + l_1)}}{\sqrt{l_0 l_1}} \qquad (13)$$

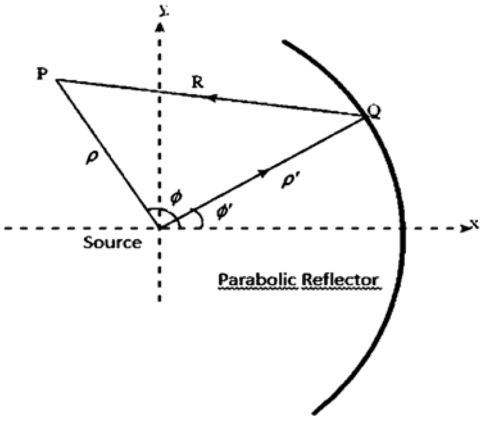

Fig. 8. Scattering from a parabolic reflector.

As it is observed that the terms ρ_e and l_e in parabolic reflector have an analogy with l_0 and l_1 in cylindrical reflector [7], it can be seen that the results found for cylindrical and parabolic surfaces are coherent.

As shown on Fig. 9 the reflected fields are found for $\varphi_0 = \pi/4$ in an interval between $\varphi_1 = \arcsin(\frac{-\rho_e' \sin(\varphi_0)}{\rho})$ and $\varphi_2 = \arcsin(\frac{-\rho_e' \sin(-\varphi_0)}{\rho})$

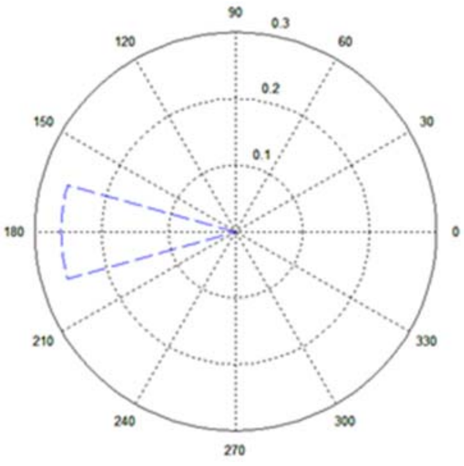

Fig. 9. Reflected fields $\varphi_0 = \pi/4$.

Moreover for $\varphi_0 = {}^{\pi}/_{6}$ the reflected is found as shown Fig. 10.

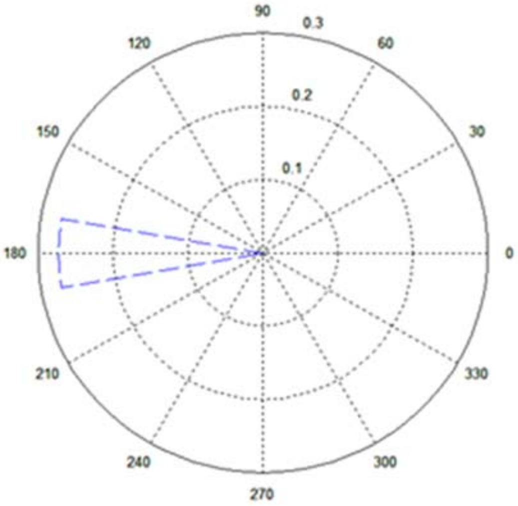

Fig. 10. Reflected fields $\varphi_0 = {}^{\pi}/_{6}$.

f) For diffracted field, non-uniform results were obtained and uniform solution was also studied in 2017 [14] by Sarnık and Umul. In order to eliminate the values diverging to infinity, detour parameter was used. Detoru parameter is formulised as

$$\xi_{i,r} = \sqrt{2\pi\rho_e}\sin(\frac{\beta_e \mp \alpha_e}{2}) \tag{14}$$

Fresnel integral uses that detour parameter variable

$$\hat{F}(\xi_{i,r}) = \frac{e^{-j(\xi_{i,r}{}^{2}+{}^{\pi}/_{4})}}{2\sqrt{\pi}\xi_{i,r}} \tag{15}$$

and diffracted field is obtained as a uniform expression

$$\vec{E}_d(\varphi_0) = \vec{e}_z\big[\hat{F}(\xi_i)e^{\cos(\beta_e-\alpha_e)} - \hat{F}(\xi_r)e^{\cos(\beta_e\mp)}\big]\frac{E_0e^{-jk(\rho_e+l_e)}}{\sqrt{\rho_e l_e}} \tag{16}$$

The scattered fields from a PEC parabolic reflector (for $\varphi_e = {}^{\pi}/_{3}$) which are claculated by PO and MTPO are compared and results are shown in Fig 11. It can be seen that the reflected fields are coherent for both methods whereas diffracted fields are different.

g) Sarnık and Yalçın applied the method for 'black cylindrical reflector' in 2017 [15]. According to the geometry in Fig. 12, the diffracted fields were found for non-uniform and uniform solutions and According to the Fig. 13, it can be observed that for $\varphi_e = {}^{\pi}/_{3}$ the uniform solution has a maximum value at the same angle at which non-uniform solution has infinite value. When a comparison is done for a different angle such as $\varphi_e = {}^{\pi}/_{7}$, shown in Fig. 14 the maximum and infinite values are observed at the same angle $(\pi - \varphi_e)$.

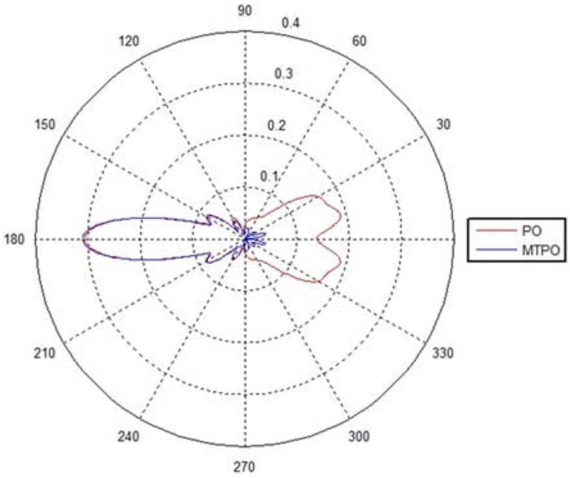

Fig. 11. Scattered field for PO & MTPO $\varphi_e = {}^{\pi}/_{3}$ [14].

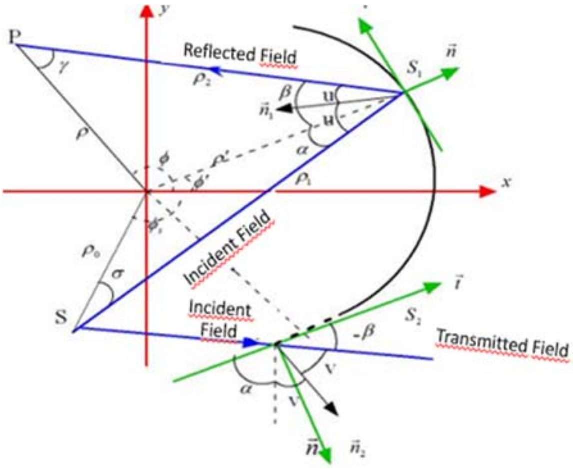

Fig. 12. Scattering geometry from the cylindrical surface.

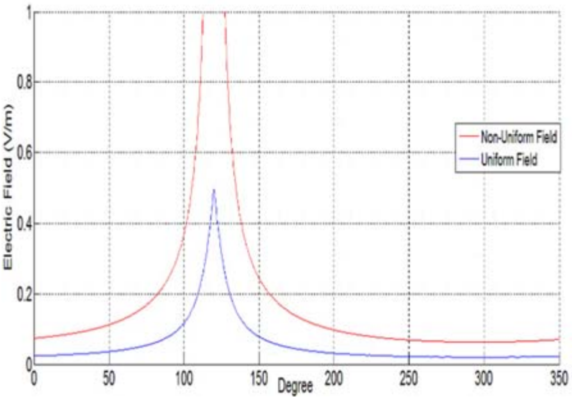

Fig. 13. Uniform and non-uniform fields $\varphi_e = {}^{\pi}/_{3}$.

4. Conclusion

In this study, firstly MTPO method, which leads to evaluate the results for various geometries, has been analyzed and discussed. Secondly, the applications, which have been studied via this method, have been examined. The applied surfaces are perfectly electrically conducting (PEC), impedance and black half plane. According to the comparison; coherence of exact solution of Helmholtz equation with MTPO solution is proved. Moreover, MTPO is advantageous than exact solution with its competence to find the exact result by an integral which is mathematically more preferable to compute. Furthermore, in order to evaluate by any simulation software, integral equation of MTPO is more suitable to trace. Finally, since this is an innovative method, there is also an opportunity for additional studies on different surfaces.

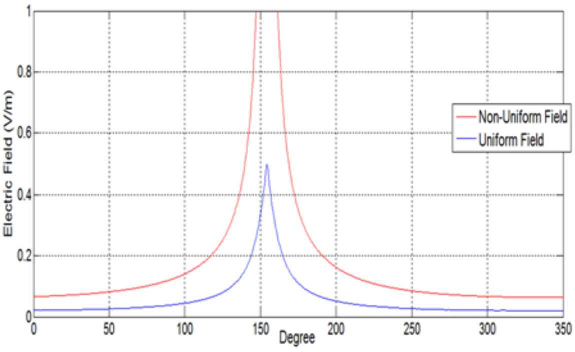

Fig. 14. Uniform and non-uniform fields $\varphi_e = \pi/7$.

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