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A Super-resolution Reconstruction Method of Remotely Sensed Image Based on Sparse Representation

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Abstract: The traditional method of image super-resolution reconstruction uses the sub-pixel displacement information between multi-frame low-resolution images to reconstruct a high-resolution image. Image super-resolution reconstruction is a typical mathematical inverse problem, and it is ill-posed problem [1]. To solve this problem, prior knowledge of data or question should be added. As the latest development achievements of signal priori or modeling, sparse representation of the signal has been studied in depth in the field of image processing. Super-resolution reconstruction based on sparse representation can improve the image quality and get richer image details [8]. Due to the sparse representation of image reconstruction has strong priority, this paper focuses on super-resolution reconstruction of the single frame remotely sensed image based on sparse representation. Compared with other algorithms, it is proved that the super-resolution reconstruction algorithm based on sparse representation has advantages in remotely sensed image reconstruction. *Copyright © 2013 IFSA.*

Keywords: Super-resolution reconstruction, Sparse representation, Training dictionary.

1. Introduction

The technology of super resolution image reconstruction is the process that one or several low resolution images of a scene are given and they are combined with some prior information to reconstruct high-resolution image of the scene. Super-resolution image can be divided into single image super-resolution and multiple image super-resolution according to the image source to be handled. Single image super-resolution refers to recovering high-resolution image from single low-resolution image, and recovering the information lost due to the image capture (mainly refers to the high frequency information). Multiple image super-resolution refers to recovering high-resolution image from low resolution image sequence. Compared with the method of single image super-resolution, it not only uses priori information of objects, but also makes full use of the complementary information between different images. However, single image super-resolution technology is particularly important in the case of unable to get the same scenario sequence images. At the same time, multiple image super-resolution methods are mostly based on single image super resolution. Only the more extensive and in-depth research is done on super resolution image of single images, the broader prospects of the multiple image super-resolution can be achieved [7, 8].

Based on the analysis of the super resolution reconstruction technology, this paper mainly focuses on the study of sparse representation and its

application in image super-resolution reconstruction. The main clue of the research is the sparse representation model of image data. In this paper, exploratory research on several key problems, such as the optimization algorithm of the sparse representation problem, super complete dictionary building of the sparse representation theory and its application in image reconstruction, etc. has been done. The following contents focus on the single frame remotely sensed image and study the method of super-resolution reconstruction based on the sparse representation. It introduces the algorithm process and steps of this method in details, and it gets the characteristics of the method through the experimental analysis and contrast. Experimental results show the superiority of sparse representation method in remotely sensed image processing.

2. Single Frame Image Super-Resolution Reconstruction Based on Sparse Representation

2.1. Algorithm Flow

The process of single frame image super-resolution reconstruction based on sparse representation usually includes the following three steps:

1) Extracting samples;
2) Constructing a Training dictionary;
3) Reconstructing high-resolution images

The relationship between them is shown in Fig. 1.

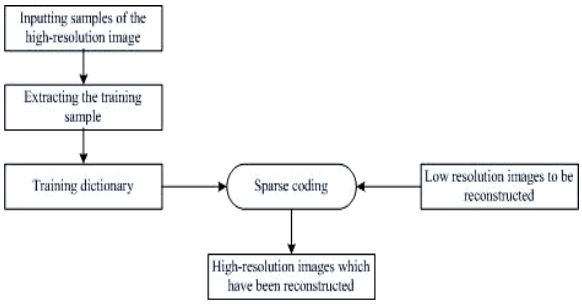


Fig. 1. The algorithm flow chart of Single frame image super-resolution reconstruction based on sparse representation.

2.2. Sample Set Selection

Selecting appropriate training sample is the first step in the process of constructing a learning dictionary [4]. In the process of reconstructing single frame image super-resolution based on sparse representation, according to the feature that natural images have similar structural motif, the high-resolution data set was randomly selected from other high-resolution images which is independent with images to be reconstructed, and low resolution samples were selected from low resolution images which is corresponding to the nature images of high-resolution [5, 6]. To construct a dictionary that has extensive ability of expression, we select training images as many as possible and the similarity of these images is as small as possible when obtaining samples.

The key of selecting sample set is determining appropriate characteristics of images. Only when the selected image characteristics can fully express the details of high-resolution image, an effective dictionary can be established. Because the information lost when high-resolution images are transformed into low resolution images is mainly high frequency components, the high frequency characteristics, such as brims of images and texture structures, are usually chosen to construct the dictionary. Then determine the appropriate number and dimension of samples [12].

To design a dictionary with full expression ability, the number of samples is usually far more than the number of dictionary element, and the more the number of samples is, the more accurate the designed dictionary is. However it will also increase the amount of calculation. Therefore, the number of samples should be neither too small nor too big. We should not only keep good expression ability of the dictionary, but also maintain a moderate amount of calculation. The appropriate number of samples should be chosen according to different images and different dimensions of dictionary elements.

The dimension of the selected samples is another issue to consider when selecting sample set. In super resolution image reconstruction methods based on learning, image feature blocks (such as blocks of 3*3) are usually used as unit to do training and recovery. The algorithm will pull each block into a column vector, and sometimes in order to enhance the expressive power of the dictionary, it is necessary to extract two or more features (e.g., the first derivative and the second derivative of images) of images. A column vector, composed of the characteristic blocks corresponding to the feature images, is constructed to form a feature vector. The dimension of this feature vector is referred to as the dimension of the sample. If there is only one kind of image features are considered, the sample dimension is directly related to the size of characteristic block. If the block size selected is larger, it shows that the large scale features of the image are mainly considered, otherwise, the small scale features are mainly considered. We usually select the size of characteristic block according to the characteristics of the proposed image.

2.3. Training Dictionary

To learn a valid dictionary through the selected sample, we should create an optimization goal equation firstly, and then select an appropriate optimization algorithm to solve this equation. The

process of optimization usually includes two stages: sparse representations coefficient update and dictionary update. The dictionary and the corresponding sparse representation coefficients can be obtained by iterative steps. According to the difference of reconstruction methods and steps, two types of dictionaries can be learned. One is the study of high and low resolution dictionary, the corresponding reconstruction steps are as follows: Firstly, representation coefficients of low-resolution images characteristic blocks are calculated by using low resolution dictionary, then the corresponding high-resolution image characteristic blocks are obtained by using the above representation coefficients and high-resolution dictionary. Another is the study of only a high-resolution dictionary, the corresponding reconstruction steps are as follows: Firstly, optimal equation is established by using low-solution images, learning high-resolution dictionary and image degradation model. Secondly, the representation coefficients of high-resolution image are got by the iterative method. Finally, the high-resolution images are estimated.

It should be noted that, in order to maintain the consistency between the reconstruction image blocks in the high resolution image reconstruction, the overlapping portion is required in the block-by-block reconstruction. Finally, the pixel value of the overlap portion is the average.

In this paper, the study of sparse representation method focuses on high and low resolution dictionary pair based on image features.

In the reconstruction method of super-resolution images based on sparse representation, as proposed by yang et al, high and low resolution images are linked by a pair of dictionaries, established by learning the extracted image features. Therefore, it is called dictionary pair based on image features. The learning process of this dictionary pair will be introduced in details as following.

First, we select a group of high-resolution natural images which contains sufficient details, and simulate image degradation process to produce the corresponding low-resolution image, namely high-resolution image is fuzzy and down sampling. Because natural images usually have similar edge structure characteristics, the corresponding relations are established in the high-frequency spatial of low and high resolution image [2]. To reduce the dimension difference between high and low resolution samples, low-resolution image is interpolated to make high and low resolution images have the same size firstly [3]. Then the low-resolution samples were randomly extracted on the first and second derivative image of the low-resolution image in the form of block and the high-resolution samples were extracted on the corresponding position in the mean-removal high-resolution image. The size of the sample block is typically chosen to be 3×3.

Assuming the high-resolution samples extracted as mentioned above is X_h, and the low resolution samples is X_l. Our purpose is learning a dictionary pair to make the high and low resolution image block have the same coefficient of sparse representation. The corresponding high and low resolution dictionaries can be obtained one by one through the formula (1) and (2):

$$\left\{D_h^*, \Lambda^*\right\} = \arg\min_{D_h,\Lambda} \left\{\left\|X_h - D_h\Lambda\right\|_2^2 + \lambda\left\|\Lambda\right\|_1\right\}, \quad (1)$$

$$\left\{D_l^*, \Lambda^*\right\} = \arg\min_{D_l,\Lambda} \left\{\left\|X_l - D_l\Lambda\right\|_2^2 + \lambda\left\|\Lambda\right\|_1\right\}, \quad (2)$$

To optimize the code and improve the accuracy, it is necessary to solve the two dictionaries jointly, namely, to minimize the following objective equation:

$$\left\{D_h^*, D_l^*, \Lambda^*\right\} = \arg\min_{D_h,D_l,\Lambda} \left\{\left\|X_h - D_h\Lambda\right\|_2^2 + \left\|X_l - D_l\Lambda\right\|_2^2 + \lambda\left\|\Lambda\right\|_1\right\} \quad (3)$$

This paper adopts the method that updates the two dictionaries and the representation coefficient one by one. The solving process is as follows:

1) Choose high-resolution sample X_h and low resolution sample X_l, and each column of them is normalized respectively by the corresponding columns' module values of X_i;;

2) Dictionary initialization;

Initialize D_h and D_l respectively by exacting samples of X_h and X_l randomly.

3) Fix D_h and D_l, solve the following equation by optimization algorithm, and update coefficient of sparse representation Λ per column.

$$\arg\min_{\{\Lambda\}} \left\{\left\|X_h - D_h\Lambda\right\|_2^2 + \left\|X_l - D_l\Lambda\right\|_2^2 + \lambda\left\|\Lambda\right\|_1^1\right\},$$

4) Fix coefficient Λ, update dictionary pair D_h and D_l respectively.

i). Update D_h, and the optimized target equation is simplified as

$$\arg\min_{\{D_h\}} \left\{\left\|X_h - D_h\Lambda\right\|_2^2\right\},$$

Update each column of the dictionary in turn, and the other columns are set to the known when updating a column.

ii). Update D_l, and the optimized target equation is similar to step i.

$$\arg\min_{\{D_l\}} \left\{\left\|X_l - D_l\Lambda\right\|_2^2\right\},$$

The updating step is also similar to step i.

5) Return to step (3) until the convergence.

6) Output the dictionary pair D_h and D_l.

2.4. To Reconstruct High-Resolution Images

2.4.1. Basic Principle of Reconstruction Algorithm [8, 20]

As mentioned above, signal $x \in R^n$ can be represented through a linear combination of over-complete dictionary D and a sparse matric, namely, $x = D\alpha_0$. $\alpha_0 \in R^K$ and it is a handful vector whose elements are nonzero. Actually:

$$y = Lx = LD\alpha_0, \tag{4}$$

where $L \in R^{k \times n}$, and it is a projection matrix, $k<n$, x is a high-resolution image block, y is a low-resolution image block.

Because D is the over complete dictionary, in the formula $x = D\alpha$, α is unknown coefficient, and $y = LD\alpha$ is underdetermined. However, in certain circumstances, the solution of sparse α_0 is the only. If dictionary D satisfies proper conditions, for various forms of matrix L, the high-resolution image block x can be fully sparse linear represented from a low resolution image y. In order to calculate conveniently, is no longer a direct calculation of the high-resolution image block sparse representation. In order to calculate conveniently, we no longer calculate sparse representation of the high-resolution image block directly, but through training high and low resolution dictionaries D_h and D_l. The sparse matrix α can be calculated by low resolution dictionary D_l and the input low resolution image y. High-resolution images can be obtained by linear combination of high-resolution dictionary D_h and sparse matrix α. The high-resolution dictionary D_h and the low resolution dictionary D_l share a sparse matrix.

High-resolution image recovery from single images in the same scene is an ill posed problem, sparse super resolution method involves two constraints: (1) Reconstruction constraints: it requires high and low resolution images meet the corresponding relationship of observation model; (2) Sparse prior constraints: It is thought that the high-resolution image block can be sparse represented by appropriate over complete dictionary, and the sparse representation matrix can be obtained from a low resolution image input, which has the same sparse representation in this redundant dictionary.

Reconstruction constraints: According to the observation model of single frame image, it is observed that low-resolution image Y is obtained from fuzzy and down sampled high-resolution images X:

$$Y = SHX, \tag{5}$$

where S is the down sampling matrix, H is equivalent to fuzzy filter. Super resolution has been an extremely ill posed problem. Normalization of this underdetermined problem can be achieved by norm sparsity of the image block x in high-resolution image X.

Sparse prior constraints: Image block x of high-resolution image X can be represented by sparse linear combination of trained high-resolution dictionary D_h.

$$x \approx D_h\alpha, \ \alpha \in R^K, \ \|\alpha\|_0 \ll K, \tag{6}$$

Sparse representation is generally divided into two steps. The first step, we consider the space compatibility between the neighborhoods to find sparse representation of each image block; the second step, according to the reconstruction constraints and the sparse representation from the previous step, the whole picture should be further standardized and restrained. In this method, the sparse prior partial model is used to restore the lost high frequency information. The global model of reconstruction constraints can be used to remove the block effect in image processing, and to make the image more consistent and natural.

2.4.2. The Reconstruction Algorithm Description

We use the two high and low resolution dictionaries D_h and D_l which have the same sparse representation, and the average pixel value of each image block is subtracted. Therefore, what the dictionary says is image characteristics, but not absolute intensity. In the process of reconstruction, the mean value of every high-resolution image block is got by mean prediction of low resolution image blocks.

For each input low resolution image block y, we can find a sparse representation from low-resolution dictionary D_l, and high-resolution image block x is constituted by linear combination of corresponding high-resolution dictionary D_h and sparse representation. The process of finding y's sparse representation can be expressed as the following:

$$\min_{\alpha} \|\alpha\|_0, \ \text{s. t.} \ \|FD_l\alpha - Fy\|_2^2 \le \varepsilon, \tag{7}$$

where F is the (linear) feature extraction factor. Its main role is to provide constraints to illustrate the approximate degree between the sparse representation coefficient α and the input image y.

Although the optimization problem (7) is a NP-hard problem, it is generally believed that as long as the sparse coefficient α is sufficiently sparse, we can minimum norm l_1 instead of norm l_0.

$$\min_{\alpha}\|\alpha\|_1, \text{ s. t. } \|FD_l\alpha - Fy\|_2^2 \le \varepsilon \quad (8)$$

According to the constraint, equation (8) is equivalent to the following:

$$\min_{\alpha}\|FD_l\alpha - Fy\|_2^2 + \lambda\|\alpha\|_1, \quad (9)$$

where the regularization parameter λ can keep balance between the sparsity of results and y's approximation.

According to the formula (9), a separate solution for each image block does not guarantee the compatibility between adjacent blocks. We modify the formula (8), so that the high-resolution image block y which is reconstructed through $D_h\alpha$ is closer with the previously calculated adjacent blocks. The resulting optimization problem is as follows:

$$\min_{\alpha}\|\alpha\|_1, \text{ s. t. } \|FD_l\alpha - Fy\|_2^2 \le \varepsilon_1, \ \|PD_h\alpha - w\|_2^2 \le \varepsilon_2, \quad (10)$$

where the matrix P extracts the overlap region of target image blocks and the previously reconstructed high-resolution image, w contains the values of the previously reconstructed high-resolution images. Therefore formula (10) can be rewritten as follows:

$$\min_{\alpha}\|\tilde{D}\alpha - \tilde{y}\|_2^2 + \lambda\|\alpha\|_1, \quad (11)$$

Where $\tilde{D} = \begin{bmatrix} FD_l \\ \beta PD_h \end{bmatrix}$, $\tilde{y} = \begin{bmatrix} Fy \\ \beta w \end{bmatrix}$. Generally, $\beta = 1$.we can get the optimization α^* of formula (10). The high-resolution image block can be reconstructed as $x = D_h a^*$, and the high-resolution image of the whole image is X_0.

Then the sparse super-resolution results are optimized by constrained reconstruction. Norm l_1 optimization problem of formula (8) and (10) doesn't require that low resolution image block y should be strict equal with $D_l\alpha$, and image has noise, therefore it is inevitable that the sparse super-resolution reconstruction method will introduce false information. That is, High-resolution image X_0 obtained by the former steps may not accurately meet the reconstruction constraints $Y = SHX$. We consider that project image X_0 which is obtained by the sparse super resolution reconstruction to the space of reconstruction constraints through the following calculation:

$$X^* = \arg\min_X \|SHX - Y\|_2^2 + c\|X - X_0\|_2^2, \quad (12)$$

Actually the optimization of the norm l_2 needn't to be directly solved, but through the iterative methods. The expression is as follows:

$$X_{t+1} = X_t + \nu\left[H^T S^T (Y - SHX_t) + c(X - X_0)\right], \quad (13)$$

where X_t is the image after t times of iteration, ν is the step size of gradient descent. X^* is the image projected into the reconstructed constraints space. It is close enough to X_0, obtained by sparse super resolution, and meets the requirement of reconstruction constraints [11].

In summary, after obtaining high and low resolution dictionary pair based on image characteristics, the reconstruction of high-resolution image involves two steps: Sparse representation coefficients of low resolution image block are got by sparse coding and using low resolution dictionary; High-resolution image block is obtained by using representation coefficients and high-resolution dictionary. The details are the following:

1) Input: High and low resolution dictionary D_h, D_l, low resolution image Y.

2) Y is divided into image blocks y whose size is 3*3.

i. Calculating the average value m of every image block y;

ii. Solving the optimization equation (11) from dictionary $\tilde{D}$ and image block $\tilde{y}$ to obtain optimal sparse representation coefficients α^*.

iii. According to the formula $x = D_h a^*$, high-resolution image block is got, and the average value m of low resolution image block has the following equation: $x_0 = D_h\alpha + m$. Therefore, high-resolution image X_0 is reconstructed.

3) To meet reconstruction constraints

$$X^* = \arg\min_X \|SHX - Y\|_2^2 + c\|X - X_0\|$$

4) Output: high-resolution image X^*.

3. Experimental Results and Analysis of Remote Sensing Image

Based on the method proposed above, we carry out experiments on the two single frame remote sensing images. The results of the method based on sparse representation and Bicubic spline surface interpolation method and MSE, SNR, PSNR index value are calculated in order to compare fitness between the two methods [9].

Fig. 2 is the Super-resolution reconstruction result of image 1. Fig. 2(a) is low-resolution image, which was sampled from high-resolution image. Fig. 2(b) is the result of Bicubic spline surface interpolation method. Fig. 2(c) is the result of the method based on

sparse representation. Fig. 2 (d) is the high - resolution image. Compare with Fig. 2 (d), Fig. 2 (b) lost a lot of details, and it is comparatively fuzzy. Compare with Fig. 2 (b), Fig. 2 (c) is clearer and details are significant.

(a) Low-resolution Image

(b) Reconstruction Result of Bicubic Interpolation

(c) Super-resolution Reconstruction Result based on Sparse Representation

(d) The Original High-resolution Image.

Fig. 2. Super-resolution Reconstruction Result of Remotely Sensed Image 1.

Table 1 is MSE, SNR, and PSNR index value of the two methods. Signal-to-noise ratio (SNR) and peak signal-to-noise ratio (PSNR) of the method based on sparse representation are higher than that of Bicubic spline surface interpolation method. Mean square error (MSE) is opposite. This means that the method based on sparse representation can obtain better results.

Table 1. MSE, SNR, PSNR results of Remote Sensing Image 1 based on different methods.

Methods	MSE	SNR	PSNR
Bicubic interpolation	575.208	15.811	20.533
The method based on sparse representation	555.440	15.963	20.684

Fig. 3 is the Super-resolution reconstruction result of image 2. The details of image 2 are more than image 1. Fig. 3(c) is clearer than Fig. 3(b).

Table 2 illustrates MSE, SNR and PSNR value of the two methods. SNR and PSNR value of the method based on sparse representation are higher than the value of Bicubic spline surface interpolation method and the MSE value is lower than that of Bicubic spline surface interpolation method. This means that the method based on sparse representation is more effective in Super-resolution reconstruction.

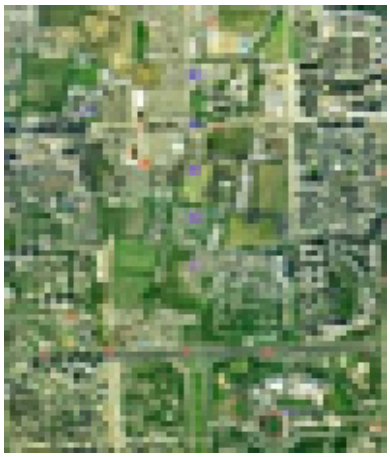
Table 2. MSE, SNR, PSNR results of Remote Sensing Image 2 based on different methods.

Methods	MSE	SNR	PSNR
Bicubic interpolation	1356.083	11.041	16.808
The method based on sparse representation	1330.606	11.124	16.890

4. Conclusion

This paper studies Super-resolution reconstruction method for single frame remote sensing image based on sparse representation. The study includes algorithm flowchart, sample set selection, dictionary training, and reconstruction step. Compare with Bicubic spline surface interpolation method, the method proposed is better in Super-resolution reconstruction.

This paper focus on how to construct the effective dictionary for sparse representation and design sparse decomposition algorithm based on the established dictionary and so on. The method proposed obtains higher accuracy than the traditional method according to Super-resolution reconstruction experiment. Furthermore, it provides theoretical basis for further research on Super-resolution reconstruction method for single frame remote sensing image based on sparse representation and it also has great value in real-world applications.



(a) Low-resolution image.

(b) Reconstruction Result of Bicubic Interpolation.

(c) Super-resolution Reconstruction Result based on Sparse Representation.

(d) The Original High-resolution Image.

Fig. 3. Super-resolution Reconstruction Result of Remotely Sensed Image 2.

References

[1]. B. Waske, S. van der Linden, Classifying multilevel imagery from SAR and optical sensors by decision fusion, *IEEE Transactions on Geoscience and Remote Sensing*, Vol. 46, 2008, pp. 1457-1466.

[2]. T. Komatsu, T. Igarashi, K. Aizawa, T. Saito, Very high resolution imaging scheme with multiple different aperture cameras, *Signal Processing Image Communication*, Vol. 5, 1993, pp. 511-526.

[3]. B. C. Tom, A. K. Katsaggelos, Resolution Enhancement of Monochrome and Color Vidoe using Motion Compensation, *IEEE Transactions on Image Processing*, Vol. 10, No. 2, 2001, pp. 278-287.

[4]. M. K. Ozkan, A. M. Tekalp, M. I. Sezan, POCS-based Restoration of Space varying Blured Images, *IEEE Transaction on Image Processing*, Vol. 3, Issue 4, 1994, pp. 450-454.

[5]. S. Borman, R. L. Stevenson, Simultaneous multi-frame MAP super -resolution video enhancement using spatio-temporal priors, in *Proceedings of IEEE International Conference on Image Processing*, Vol. 3, 1999, pp. 469-473.

[6]. B. C. Tom, A. K. Katsaggelos, Reconstruction of a high-resolution image by simultaneous registration, restoration, and interpolation of low-resolution images, in *Proceedings of IEEE International Conference on Image Processing*, Vol. 2, pp. 539-542.

[7]. R. Hardie, A Fast Image Super-resolution Algorithm Using an Adaptive Wiener Filter, *IEEE Transactions on Image Processing*, Vol. 16, Issue 12, 2007.

[8]. J. Yang, J. Wright, T. Huang, Image Super-Resolution via Sparse Representation, *IEEE Transactions on Image Processing,* Vol. 19, pp. 2861-2873.

[9]. P. Thevenaz, T. Blu, M. Unser, Interpolation revisited, *IEEE Transactions on Medical Imaging*, Vol. 19, Issue 7, 2000, pp. 739-758.

[10]. M. Irani, S. Peleg, Improving resolution by image registration, in *Proceedings of CVGIP: Graphical Models and Image*, Vol. 53, 1991, pp. 231-239.

[11]. H. Stark, P. Oskoui, High-resolution image recovery from image-plane arrays, using convex projection, *Journal of the Optical Society of America A*, Vol. 6, 1989, pp. 1715-1726.

[12]. W. T. Freeman, T. R. Jones, E. C. Pasztor, Example-based super-resolution, *IEEE Computer Graphics and Applications*, Vol. 22, 2002, pp. 56-65.
[13]. H. Chang, D. Y. Yeung, Y. Xiong, Super-resolution through neighbor embedding, in *Proceedings of IEEE Conference CVPR*, Vol. 1, 2004, pp. 275-282.
[14]. J. A. Tropp, Just relax: Convex program, mming methods for subset selection and sparse approximation, *IEEE Transactions on Information Theory,* Vol. 52, 2006, pp. 1030–1051.
[15]. S. Mallat, Z. Zhang, Matching pursuits with time-frequency dictionaries, *IEEE Transactions on Signal Processing*, 1993, Vol. 41, Issue 12, 1993, pp. 3397-3415.
[16]. Y. Pati, R. Rezaiifar, P. Krishnaprasad, Orthogonal matching pursuit: Recursive function approximation with applications to wavelet decomposition, in *Proceedings of the 27th Annual Asilomar Conference on Signals and Systems,* Vol. 1, 1993, pp. 40-44.
[17]. S. S. Chen, D. L. Donoho, M. A. Saunders. Atomic decomposition by basis pursuit, *Technical Report Stanford,* 1995.
[18]. K. Engan, S. O. Aase, J. H. Husey, Method of optimal directions for frame design, in *Proceedings of IEEE International Conference on Acoustics, Speech*, and Signal Processing, Washington, DC USA, Vol. 5, 1999, pp. 2443-2446.
[19]. R. Vidal, Y. Ma, S. Sastry, Generalized principal component analysis (GPCA), *IEEE Transactions on Pattern Analysis & Machine Intelligence*, 2005, Vol. 27, Issue 12, pp. 1945-1959.
[20]. M. Aharon, M. Elad, A. M. Bruckstein, The K-SVD: An algorithm for designing of over -complete dictionaries for sparse representation, *IEEE Transactions on. Signal Processing*, Vol. 54, 2006, pp. 4311–4322.

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