Sensors & Transducers, Vol. 160, Issue 12, December 2013, pp. 619-626

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Sensors & Transducers

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Passivity-Based Spacecraft Attitude Control with Disturbance Suppression

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Received: 18 September 2013 /Accepted: 22 November 2013 /Published: 30 December 2013

Abstract: The stability of rigid spacecraft attitude is considered. The mathematical model of spacecraft system is described by quaternion. The robust nonlinear attitude controller without angular velocity measurement is designed based on passivity-based approach. The designed attitude controller has no information related to system parameters. So it has robustness to model error and uncertainty of model parameters. The stability of spacecraft attitude control with considering external disturbance can be proved by applying Lyapunov approach and LaSalle Invariance Principle. Numerical simulation results demonstrate the effectiveness of the designed attitude controller. *Copyright © 2013 IFSA.*

Keywords: Attitude control, Passivity, Quaternion, External disturbance.

1. Introduction

An orbiting spacecraft is subjected to several gravitational influences. These forces have an effect on a spacecraft's trajectory, and tend to its attitude and orbit. Therefore, the attitude control system must counter any rotational torque or movement. With development of technologies, attitude control systems for spacecraft demand better performance. Many scholars have devoted themselves to the problem of spacecraft attitude control over the past several decades. Many methods have been developed to treat this problem, such as robust control approach [1, 2], Lyapunov-based approach [3-5], variable structure control approach [6-10], adaptive control approach [11, 12].

Often, angular velocity and quaternion, are used to deal with the stabilization of feedback control. However, the angular velocity measurement is not necessary in some of the previous works. For example, in [7], a design criterion for a class of PD controllers was firstly proposed by using the Lyapunov-based approach, and then a design criterion of controller without angular velocity measurement was presented based on the passivity approach. The approach proposed in [7] was further extended to the system described by the Rodrigues and modified Rodrigues parameters [8].

The external disturbances, which inevitably affect the motion of spacecraft, are ignored in the above-mentioned literatures. This paper focuses on the attitude stability of spacecrafts in the case of the bounded external disturbances. We use quaternion to represent the spacecraft attitude and design a passivity-based attitude controller without angular velocity measurement. In addition, the designed controller does not contain information related to the system parameters, which makes it robust to the model error and the model parameter uncertainty.

2. Mathematical Model of System

2.1. Spacecraft Attitude Kinematics

We use the unit quaternion to represent spacecraft attitude in order to avoid singularity. Define the unit quaternion as

$$\bar{q} = \begin{pmatrix} q \\ q_0 \end{pmatrix} = \begin{pmatrix} \sin(\theta/2)\hat{n} \\ \cos(\theta/2) \end{pmatrix}, \quad (1)$$

where $\hat{n} \in R^3$ is the rotation axis represented by unit vector, θ is the rotation angular, $q \in R^3$ and $q_0 \in R$ are the components of the unit quaternion, which subject to the following constraint:

$$q^{\mathrm{T}} q + q_0^2 = 1, \quad (2)$$

The kinematic equation represented by the unit quaternion is given by

$$\begin{aligned} \dot{q} &= \frac{1}{2} E(q)\omega = \frac{1}{2}(q_0 I + q^{\times})\omega \\ \dot{q}_0 &= -\frac{1}{2} q^{\mathrm{T}} \omega \end{aligned}, \quad (3)$$

where $\omega = [\omega_1 \ \omega_2 \ \omega_3]^{\mathrm{T}}$ is the spacecraft angular velocity vector with respect to the inertial reference frame, expressed in the spacecraft body-fixed reference frame, I is the 3×3 unit matrix, $q^{\times}$ is the skew symmetric matrix which is defined by Eq. (4).

$$q^{\times} = \begin{pmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{pmatrix}, \quad (4)$$

2.2. Spacecraft Attitude Dynamics

The dynamic model of the spacecraft attitude control system is described by the differential equation in Eq. (5).

$$J\dot{\omega} = -\omega^{\times} J\omega + u + d, \quad (5)$$

where $J = J^{\mathrm{T}} \in R^{3\times3}$ is the inertia matrix which is a symmetric and positive define matrix, $u \in R^3$ is the vector of control torque, $d = [d_1 \ d_2 \ d_3]^{\mathrm{T}}$ is the vector of external disturbance.

Assuming that d is sinusoidal disturbance and satisfies the following condition.

$$\ddot{d} + \gamma^2 d = 0, \quad (6)$$

where $\gamma = \mathrm{diag}\{\gamma_1, \gamma_2, \gamma_3\}$ is the disturbance frequency matrix.

3. Attitude Controller Design

3.1. Attitude Control without Considering External Disturbance and Angular Measurement

We construct a controller based on passivity approach. It can be described as follows.

$$\begin{cases} \dot{x} = Ax + BCq \\ y = B^{\mathrm{T}} P(Ax + BCq) = B^{\mathrm{T}} P\dot{x} \\ u = -k[\mathrm{sgn}(q_0) - q_0]q - E^{\mathrm{T}}(q)Cy - E^{\mathrm{T}} Fq \end{cases}, \quad (7)$$

where k is the positive constant, B is the full rank matrix, the gain matrices $C = \mathrm{diag}\{c_1, c_2, c_3\}$ and $F = \mathrm{diag}\{f_1, f_2, f_3\}$ are the symmetric positive definite, $E(q)$ is the Jacobian, which satisfies the following property.

$$E(q) = q_0 I + q^{\times}, \quad (8)$$

There exist positive definite matrices P and Q such that

$$A^{\mathrm{T}} P + PA = -Q, \quad (9)$$

The symbolic function $\mathrm{sgn}(q_0)$ is defined by

$$\mathrm{sgn}(q_0) = \begin{cases} 1 & q_0 > 0 \\ -1 & q_0 < 0 \end{cases}, \quad (10)$$

For attitude stabilization and control, $\bar{q} = [\pm 1, 0, 0, 0]^{\mathrm{T}}$ is the stable equilibrium of system when ω is zero, since q and $-q$ represent the same attitude. In fact, the equilibrium tended to is related to the initial symbol of q_0. The different the initial symbol of q_0, the different the control law. Therefore, consider the following Lyapunov function candidate

$$\begin{aligned} V = &\frac{1}{2}\omega^{\mathrm{T}} J\omega + q^{\mathrm{T}} Fq + k(\mathrm{sgn}(q_0) - q_0)^2 + \\ &(Ax + BCq)^{\mathrm{T}} P(Ax + BCq) \end{aligned}, \quad (11)$$

Using Eqs. (3), (5), (7) and (9), the time derivative of V can be computed to be

$$\begin{aligned}\dot{V} &= \omega^{\mathrm{T}} J\dot{\omega} + 2q^{\mathrm{T}} F\dot{q} - 2k\left[(\mathrm{sgn}(q_0) - q_0)\dot{q}_0\right] \\ &\quad + (A\dot{x} + BC\dot{q})^{\mathrm{T}} P\dot{x} + \dot{x}^{\mathrm{T}} P(A\dot{x} + BC\dot{q}) \\ &= -\omega^{\mathrm{T}} E^{\mathrm{T}}(q) Cy - \dot{x}^{\mathrm{T}} Q\dot{x} + 2\dot{q}^{\mathrm{T}} Cy \\ &= -2\dot{q}^{\mathrm{T}} Cy - \dot{x}^{\mathrm{T}} Q\dot{x} + 2\dot{q}^{\mathrm{T}} Cy \\ &= -\dot{x}^{\mathrm{T}} Q\dot{x} \le 0\end{aligned}, \quad (12)$$

The Lyapunov function candidate V is positive definite and radially unbounded from Eq. (12). And $\dot{V}$ is less than or equal to zero in the whole state space. By the LaSalle Invariance Principle [13], all trajectories converge to the largest invariant set $\psi = \{(\omega, q, x): \dot{V} = 0\} = \{(\omega, q, x): \dot{x} = 0\}$. On the invariant set we have that $y = B^{\mathrm{T}} P\dot{x} = 0$, $Ax + BCq = \dot{x} = 0$ from Eq.(5). Since $\dot{x} = 0$, then $A\dot{x} + BC\dot{q} = \ddot{x} = 0$, namely $BC\dot{q} = 0$. The matrices B and C are reversible and Eq.(2), therefore $\dot{q} + \dot{q}_0 = 0$. It is clear that $\dot{q}_0 = 0$. From Eq.(3), $\omega = 0$ in the invariant set implies $\dot{\omega} = 0$. And it can be obtained that $J\dot{\omega} = -\omega^{\times} J\omega + u$ (d=0) or $u = J\dot{\omega} + \omega^{\times} J\omega = 0$ from Eq.(5). From Eq.(7), we have that $u = -k[\mathrm{sgn}(q_0) - q_0]q - E^{\mathrm{T}}(q)Cy - E^{\mathrm{T}} Fq$. That is

$$-k[\mathrm{sgn}(q_0) - q_0]q - E^{\mathrm{T}} Fq = 0, \quad (13)$$

Both sides of Eq.(13) is multiplied by q^{T} , we have that $-k[\mathrm{sgn}(q_0) - q_0]q^{\mathrm{T}} q - q^{\mathrm{T}}(q_0 I - q^{\times})Fq = 0$. Then the above equation can be simplified as $-k[\mathrm{sgn}(q_0) - q_0]q^{\mathrm{T}} q - q_0 q^{\mathrm{T}} Fq = 0$. That is

$$q^{\mathrm{T}}\left\{k[\mathrm{sgn}(q_0) - q_0]I + q_0 F\right\}q = 0, \quad (14)$$

Therefore, $k[\mathrm{sgn}(q_0) - q_0] + q_0 f_i = 0$ (i=1,2,3), q=0 and x=0. The largest invariant set is $\psi = \{(\omega, q, x): \omega = 0, q = 0, x = 0\}$, which corresponds to the desired equilibrium. The designed control law can guarantee that the closed-loop system described by Eqs. (3), (5) and (7) tends to the globally asymptotically stable equilibrium.

3.2. Attitude Control with Considering External Disturbance and Angular Measurement

The stability of spacecraft is analyzed in the small area of equilibrium (|q_0|=1, q=0) when there exists the external disturbance. To better suppress the effect of the external disturbance, an input part having the dynamic model as same as the external disturbance are introduced into the feedback system using internal model principle. Consequently, we construct a controller as follows.

$$\begin{cases} u = -k[\mathrm{sgn}(q_0) - q_0]q - C\omega \\ \qquad - E^{\mathbf{T}} Fq + E^{\mathbf{T}} G\alpha \\ \ddot{\alpha} + \gamma^2 \alpha = q \end{cases}, \quad (15)$$

where k is the positive constant; the gain matrix $G = diag\{g_1, g_2, g_3\}$ is symmetric positive definite.

A new variable β is introduced before constructing Lyapunov function. That is

$$\beta = \alpha + \mathrm{sgn}(q_0) G^{-1} d, \quad (16)$$

Consider the following Lyapunov function candidate

$$\begin{aligned} V = &\frac{1}{2}\omega^T J\omega + q^T Fq + k(\mathrm{sgn}(q_0) - q_0)^2 \\ &+ \dot{\beta}^T G\dot{\beta} + \beta^T G\gamma^2 \beta - 2\beta^T Gq \end{aligned}, \quad (17)$$

First we prove that Eq. (17) is positive define. Consider $V^* = q^{\mathrm{T}} Fq + \beta^{\mathrm{T}} G\gamma^2 \beta - 2\beta^{\mathrm{T}} Gq$. Obviously, $V - V^*$ is positive defined. V is positive define as long as V^* is positive define. The process is as follows.

Let $F_1 = \mathrm{diag}\{f_{11}, f_{12}, f_{13}\}$, where $f_{11}^2 = f_1$, $f_{12}^2 = f_2$, $f_{13}^2 = f_3$, so $F_1^2 = F$.

$$\begin{aligned} V^* &= q^{\mathrm{T}} Fq + \beta^{\mathrm{T}} G\gamma^2 \beta - 2\beta^{\mathrm{T}} Gq \\ &= (F_1 q - F_1^{-1} G\beta)^{\mathrm{T}} (F_1 q - F_1^{-1} G\beta) \\ &\quad + \beta^{\mathrm{T}} G(\gamma^2 - F_1^{-1} G)\beta \end{aligned}, \quad (18)$$

V^* is positive define when $\gamma_i^2 > \frac{g_i}{f_i}$ (i=1, 2, 3). Consequently, V is positive define.

Then using Eqs. (3), (5), (15) and (16), the first-order time derivative of V can be computed to be

$$\begin{aligned} \dot{V} &= \omega^{\mathrm{T}} J\dot{\omega} + 2q^{\mathrm{T}} F\dot{q} - 2k\left[(\mathrm{sgn}(q_0) - q_0)\dot{q}_0\right] + \\ &\quad 2\dot{\beta}^{\mathrm{T}} G\ddot{\beta} + 2\beta^{\mathrm{T}} G\gamma^2 \dot{\beta} - 2\beta^{\mathrm{T}} G\dot{q} - 2\dot{\beta}^{\mathrm{T}} Gq \\ &= \omega^{\mathrm{T}}\{-\omega^{\times} J\omega + u + d + E^{\mathrm{T}} Fq + \\ &\quad k(\mathrm{sgn}(q_0) - q_0)q - E^{\mathrm{T}} G\alpha - d\} \\ &= -\omega^{\mathrm{T}} Q\omega \le 0 \end{aligned}, \quad (19)$$

It can be seen that the Lyapunov function candidate V is positive definite and radially unbounded. And $\dot{V}$ is less than or equal to zero in the whole state space. By the LaSalle Invariance Principle, all trajectories of the closed-loop system described by Eqs. (3), (5) and (15) converge to the largest invariant set $\psi = \{(\omega, q, \beta): \dot{V} = 0\} = \{(\omega, q, \beta): \omega = 0\}$. So $\dot{\omega} = 0$. From Eqs.(3) and (5), we

know $\dot{q}=0$, $\dot{q}_0=0$. And it can be obtained that $u+d=J\dot{\omega}+\omega^{\times}J\omega=0$ from Eq. (5). From Eq. (15), we have

$$-k[\mathrm{sgn}(q_0)-q_0]q-C\omega-E^TFq+E^TG\alpha+d=0$$

That is

$$-k[\mathrm{sgn}(q_0)-q_0]q-E^TFq+E^TG[\alpha+\mathrm{sgn}(q_0)G^{-1}d]=0$$

Then the above equation can be simplified as

$$-k[\mathrm{sgn}(q_0)-q_0]q-E^TFq+E^TG\beta=0, \quad (20)$$

Differentiate Eq.(20), then $\dot{\beta}=0$, which is in the invariant set, so $\ddot{\beta}=0$. According to Eqs.(6), (15) and (16), we know $\dddot{\beta}+\gamma^2\dot{\beta}=q$. Consequently,

$$\gamma^2\dot{\beta}=q, \quad (21)$$

Substituting Eq.(21) into Eq.(20), we have that

$$-k[\mathrm{sgn}(q_0)-q_0]q-E^TFq+E^TG(\gamma^2)^{-1}q=0, \quad (22)$$

Both sides of Eq. (22) is multiplied by q^{T}, we have that

$$-k[\mathrm{sgn}(q_0)-q_0]q^Tq-q^T(q_0I-q^{\times})Fq+q^T(q_0I-q^{\times})G(\gamma^2)^{-1}q=0$$

Then the above equation can be simplified as $-k[\mathrm{sgn}(q_0)-q_0]q^Tq-q_0q^TFq+q_0q^TG(\gamma^2)^{-1}q=0$

That is

$$q^T\{-k[\mathrm{sgn}(q_0)-q_0]I-q_0F+q_0G(\gamma^2)^{-1}\}q=0$$

Therefore,

$$\{k[\mathrm{sgn}(q_0)-q_0]+q_0(f_i-\frac{g_i}{\gamma_i^2})\}q_i^2=0 \ (i=1,2,3)$$

Because $\mathrm{sgn}(q_0)-q_0$ and q_0 have the same symbol and are not equal to zero at the same time. And k and $f_i-\frac{g_i}{\gamma_i^2}$ are positive constant. We have

$k[\mathrm{sgn}(q_0)-q_0]+q_0(f_i-\frac{g_i}{\gamma_i^2})\neq 0$ (i=1,2,3), thus $q=0$. According to Eq.(21), we have $\beta=0$. Consequently, the largest invariant set is $\psi=\{(\omega,q,\beta):\omega=0,q=0,\omega=0\}$, which corresponds to the desired equilibrium. The designed control law can guarantee that the closed-loop system described by Eqs. (3), (5) and (15) tends to the globally asymptotically stable equilibrium.

3.3. Attitude Control with Considering External Disturbance and without Angular Measurement

According to Subsection 3.1 analysis, we know that the designed control law can enable the spacecraft system to achieve global asymptotic stable. The stability of spacecraft is analyzed in the small area of equilibrium ($|q_0|$=1, q=0) when there exists the external disturbance. The controller is constructed as follows.

$$\begin{cases}\dot{x}=Ax+BCq\\ y=B^{\mathrm{T}}P(Ax+BCq)=B^{\mathrm{T}}P\dot{x}\\ u=-k[\mathrm{sgn}(q_0)-q_0]q-E^{\mathrm{T}}(q)Cy\\ \quad -E^{\mathrm{T}}Fq+E^{\mathrm{T}}G\alpha\\ \ddot{\alpha}+\gamma^2\alpha=q\\ \beta=\alpha+\mathrm{sgn}(q_0)G^{-1}d\end{cases}, \quad (23)$$

where $G=\mathrm{diag}\{g_1,g_2,g_3\}$ is the symmetric and positive define matrix, β satisfies the following condition.

$$\ddot{\beta}+\gamma^2\beta=q, \quad (24)$$

Consider the following Lyapunov function candidate

$$V=\frac{1}{2}\omega^{\mathrm{T}}J\omega+q^{\mathrm{T}}Fq+k\left(\mathrm{sgn}(q_0)-q_0\right)^2+(Ax+BCq)^{\mathrm{T}}P(Ax+BCq)+\dot{\beta}^{\mathrm{T}}G\dot{\beta}+\beta^{\mathrm{T}}G\gamma^2\beta-2\beta^{\mathrm{T}}Gq, \quad (25)$$

Using Eqs. (3), (5), (16) and (23), the time derivative of V can be computed to be

$$\begin{aligned}\dot{V}&=\omega^{\mathrm{T}}J\dot{\omega}+2q^{\mathrm{T}}F\dot{q}-2k\left[(\mathrm{sgn}(q_0)-q_0)\dot{q}_0\right]+(A\dot{x}+BC\dot{q})^{\mathrm{T}}P\dot{x}+\dot{x}^{\mathrm{T}}P(A\dot{x}+BC\dot{q})+2\dot{\beta}^{\mathrm{T}}G\ddot{\beta}+2\beta^{\mathrm{T}}G\gamma^2\dot{\beta}-2\beta^{\mathrm{T}}G\dot{q}-2\dot{\beta}^{\mathrm{T}}Gq\\ &=\omega^{\mathrm{T}}(-\omega^{\times}J\omega+u+d)+q^{\mathrm{T}}FE\omega+k(\mathrm{sgn}(q_0)-q_0)q^{\mathrm{T}}\omega+\dot{x}^{\mathrm{T}}(A^{\mathrm{T}}P+PA)\dot{x}+2\dot{q}^{\mathrm{T}}Cy+2\dot{\beta}^{\mathrm{T}}G(\ddot{\beta}+\gamma^2\beta-q)-\dot{\beta}^{\mathrm{T}}GE^{\mathrm{T}}\omega\\ &=-\omega^{\mathrm{T}}E^{\mathrm{T}}(q)Cy-\dot{x}^{\mathrm{T}}Q\dot{x}+2\dot{q}^{\mathrm{T}}Cy\\ &=-2\dot{q}^{\mathrm{T}}Cy-\dot{x}^{\mathrm{T}}Q\dot{x}+2\dot{q}^{\mathrm{T}}Cy\\ &=-\dot{x}^{\mathrm{T}}Q\dot{x}\le 0\end{aligned}, \quad (26)$$

According to Subsection 3.2 analysis, we know that the Lyapunov function candidate V is positive definite and radially unbounded. And $\dot{V}$ is less than or equal to zero in the whole state space. By the LaSalle Invariance Principle, all trajectories converge to the largest invariant set $\psi = \{(\omega, q, x, \beta): \dot{V} = 0\} = \{(\omega, q, x, \beta): \dot{x} = 0\}$. On the invariant set we have that $y = B^{\mathrm{T}} P\dot{x} = 0$, $Ax + BCq = \dot{x} = 0$ from Eq. (23). So $A\dot{x} + BC\dot{q} = \ddot{x} = 0$, namely $BC\dot{q} = 0$. The matrices B and C are reversible and Eq. (2), therefore $\dot{q} + \dot{q}_0 = 0$. It is clear that $\dot{q}_0 = 0$. From Eq. (3), $\omega = 0$ in the invariant set implies $\dot{\omega} = 0$. And it can be obtained that $J\dot{\omega} = -\omega^{\times} J\omega + u + d$ or $u + d = J\dot{\omega} + \omega^{\times} J\omega = 0$ from Eq. (5). From Eq.(23), we have

$$-k[\mathrm{sgn}(q_0) - q_0]q - E^{\mathrm{T}}(q)Cy - E^{\mathrm{T}}Fq - E^{\mathrm{T}}G\alpha + d = 0$$

That is

$$-k[\mathrm{sgn}(q_0) - q_0]q - E^{\mathrm{T}}Fq - E^{\mathrm{T}}G[\alpha + \mathrm{sgn}(q_0)G^{-1}d] = 0$$

Then the above equation can be simplified as

$$-k[\mathrm{sgn}(q_0) - q_0]q - E^{\mathrm{T}}Fq - E^{\mathrm{T}}G\beta = 0, \quad (27)$$

Eq. (27) is same as Eq. (20). Therefore, $k[\mathrm{sgn}(q_0) - q_0] + q_0\left(f_i - \frac{g_i}{\gamma_i^2}\right) \neq 0$ (i=1, 2, 3), q=0 and x=0. The largest invariant set is $\psi = \{(\omega, q, x, \beta): \omega = 0, q = 0, x = 0, \beta = 0\}$, which corresponds to the desired equilibrium. The designed control law can guarantee that the closed-loop system described by Eqs. (3), (5) and (23) tends to the globally asymptotically stable equilibrium and suppress the effect of external disturbance. Therefore the spacecraft attitude achieves stabilization.

Remark: 1) The designed attitude controller of spacecraft is only related to quaternion. So it needn't angular measurement. 2) The matrix gain in the control law u can balance the relationship between the scalar gain and control torque. 3) $\mathrm{sgn}(q_0)$ is introduced into the control law u, so that the approachable equilibrium point is related to the symbol of q_0 initial value. 4) The control law u includes two parts. One is the passive control part, $-k[\mathrm{sgn}(q_0) - q_0]q - E^{\mathrm{T}}(q)Cy - E^{\mathrm{T}}Fq$, which can guarantee that the system achieves global asymptotic stable to the equilibrium point. The other is the suppress vector, $E^{\mathrm{T}}G\alpha$, which can suppress the external disturbance. 5) The control law u has no information related to system parameters, thereby having robustness to model error and uncertainty of model parameters.

4. Numerical Simulations and Results

In order to demonstrate the effectiveness of the designed attitude controller, several numerical simulations are presented in this section.

A spacecraft with the following inertia matrix is considered: $J = \begin{pmatrix} 15 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 10 \end{pmatrix} (\mathrm{kg \cdot m^2})$. The other main parameters are k=8, $-A$=P=B=I, $C = \begin{pmatrix} 6 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 8 \end{pmatrix}$, $F = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 5 \end{pmatrix}$. Assuming that the sinusoidal disturbance torque is $d = \begin{pmatrix} 0.1\sin(0.2t) \\ -0.06\sin(0.3t) \\ 0.14\sin(0.4t) \end{pmatrix} (\mathrm{N \cdot m})$. Then $\gamma = \mathrm{diag}(0.2, 0.3, 0.4)$.

First we consider the attitude stability around the equilibrium point. The initial states are $\hat{n} = \begin{pmatrix} 0.5345 \\ 0.2673 \\ 0.8018 \end{pmatrix}$, $\theta = \frac{11\pi}{6}$, $\omega(0) = 0$ rad/s. Then the corresponding quaternion representations are $q(0) = \begin{pmatrix} 0.1383 \\ 0.0692 \\ 0.2075 \end{pmatrix}$ and $q_0(0) = -0.9659$. At this time, yaw angle, roll angle and pitch angle are 23.56º, 17.21º and 4.98º, respectively.

Case 1: Consider the attitude controller with angular measurement. Given $G = \begin{pmatrix} 0.1 & 0 & 0 \\ 0 & 0.35 & 0 \\ 0 & 0 & 0.7 \end{pmatrix}$. The simulation result is shown in Fig. 1.

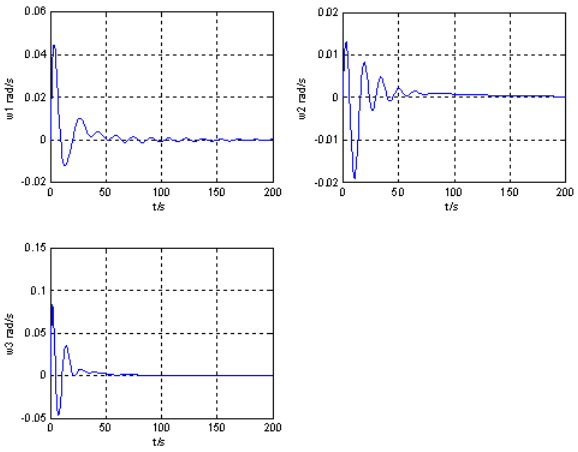

Fig. 1. The angular velocity curve with angular velocity measurement.

Case 2: Consider the attitude controller without angular measurement. Given $G = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ and $G = \begin{pmatrix} 0.1 & 0 & 0 \\ 0 & 0.35 & 0 \\ 0 & 0 & 0.7 \end{pmatrix}$. The simulation results are shown from Fig. 2 to Fig. 5.

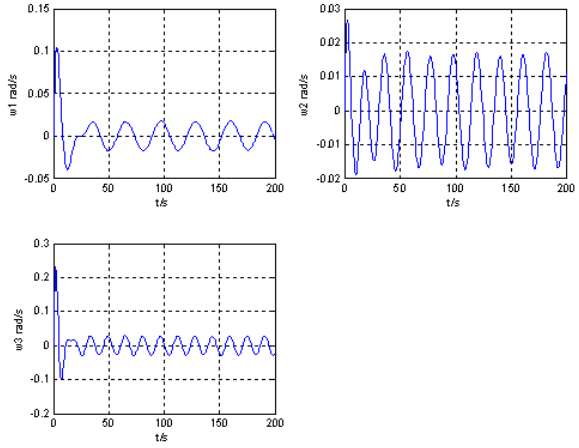

Fig. 2. The angular velocity curve without angular velocity measurement and suppress of external disturbance.

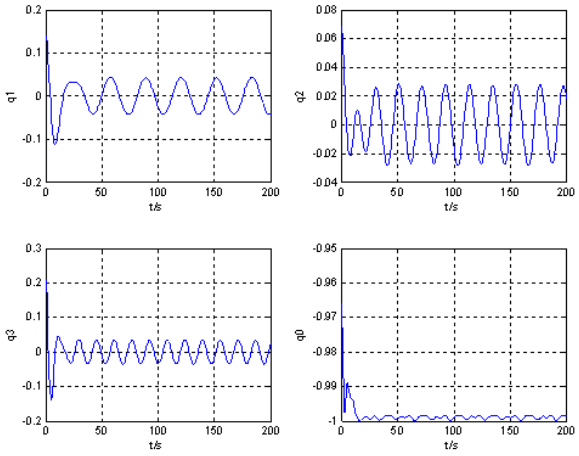

Fig. 3. The quaternion curve without angular velocity measurement and suppress of external disturbance.

Case 3: Consider the attitude controller without angular measurement. Given $G = \begin{pmatrix} 0.1 & 0 & 0 \\ 0 & 0.35 & 0 \\ 0 & 0 & 0.7 \end{pmatrix}$ and $\theta = \frac{\pi}{6}$. The value of $q_0(0)$ is changed to 0.9695 and other parameters remain unchanged. And the simulation result is shown in Fig. 6.

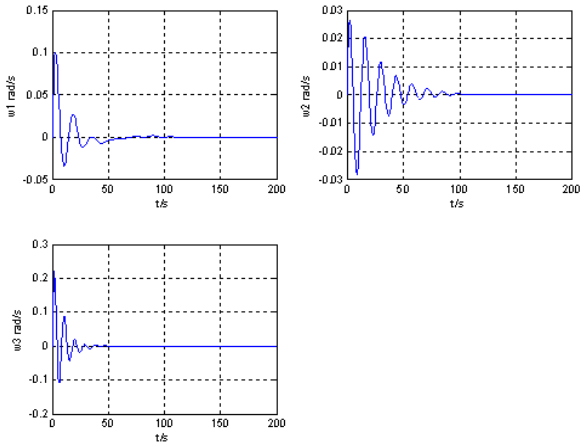

Fig. 4. The angular velocity curve without angular velocity measurement and with suppress of external disturbance.

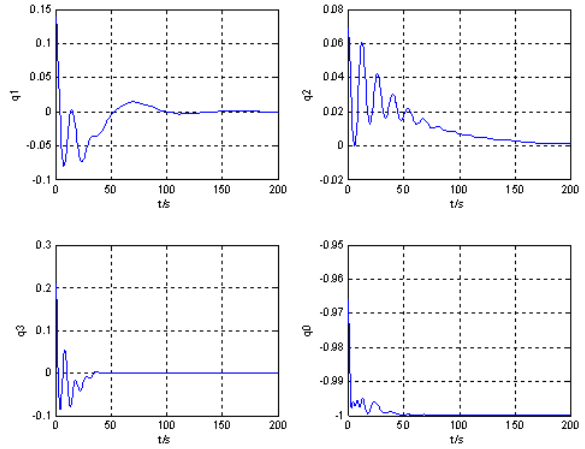

Fig. 5. The quaternion curve without angular velocity measurement and with suppress of external disturbance.

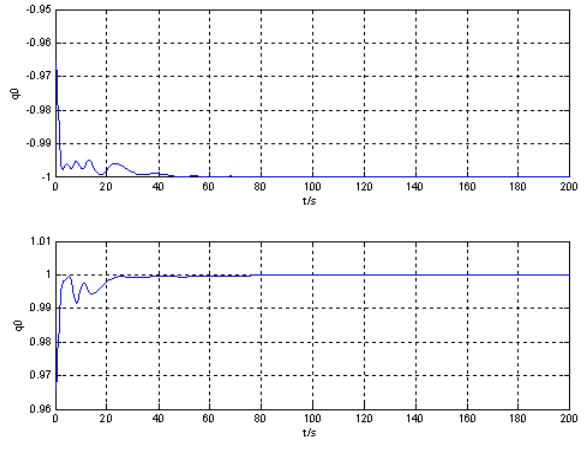

Fig. 6. The q_0 curve with different initial value of q_0.

Case 4: Consider the attitude controller without angular measurement. Given

$G = \begin{pmatrix} 0.1 & 0 & 0 \\ 0 & 0.35 & 0 \\ 0 & 0 & 0.7 \end{pmatrix}$. And assume that there exist the model error and model parameter uncertainty, that is $J = \begin{pmatrix} 15+0.5 & 0 & 0 \\ 0 & 20+0.4 & 0 \\ 0 & 0 & 10+0.6 \end{pmatrix} (\text{kg}\cdot\text{m}^2)$. The simulation result is shown in Fig. 7.

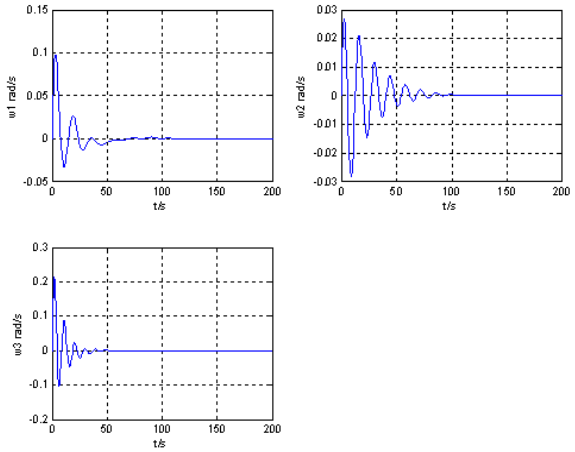

Fig. 7. The angular velocity curve with the existence of model error and model parameter uncertainty.

Then we consider the attitude stability around the unequilibrium point when the external disturbance exists. The initial states are $\hat{n} = \begin{pmatrix} 0.4618 \\ 0.2032 \\ 0.8408 \end{pmatrix}$, $\theta = \frac{3\pi}{2}$. The corresponding quaternion representations are $q(0) = \begin{pmatrix} 0.3462 \\ 0.1437 \\ 0.5996 \end{pmatrix}$ and $q_0(0) = -0.7071$. At this time, yaw angle, roll angle and pitch angle are −87.51º, −18.50º and −86.18º, respectively. The initial angular velocity is $\omega(0) = 0$ rad/s.

Consider the attitude controller without angular measurement. Given $G = \begin{pmatrix} 0.1 & 0 & 0 \\ 0 & 0.35 & 0 \\ 0 & 0 & 0.7 \end{pmatrix}$. And the simulation results are shown in Fig. 8 and Fig. 9.

By comparing the Fig. 1 and Fig. 4, we can see that the designed spacecraft attitude controller without angular velocity measurement has the better performance than the attitude controller with angular velocity measurement. By comparing Fig. 2 with Fig. 4 and comparing Fig. 3 with Fig. 5, it can be seen that the closed-loop system without the suppression vector of external disturbance is greatly affected by the external disturbance and can not converge to the equilibrium point and be not any more stable.

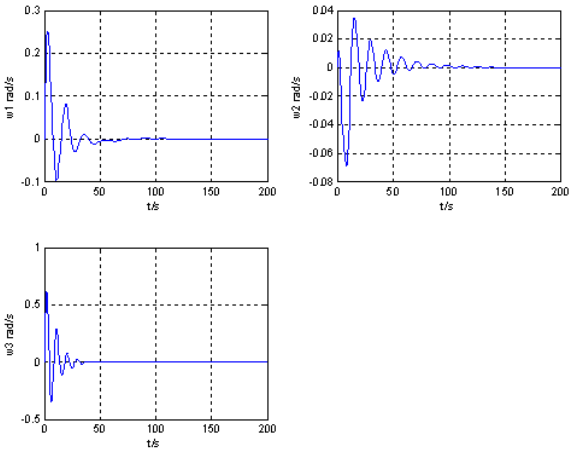

Fig. 8. The angular velocity curve without angular velocity measurement (The initial location of spacecraft is around the unequilibrium point).

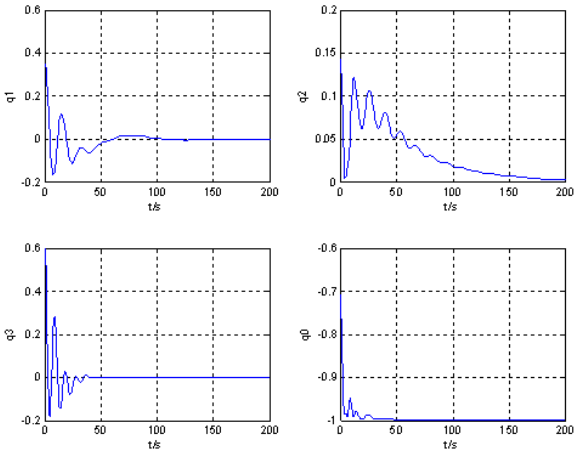

Fig. 9. The quaternion curve without angular velocity measurement (The initial location of spacecraft is around the unequilibrium point).

It fluctuates around the equilibrium point. In addition, simulation results have shown that when the condition $\gamma_i^2 > \frac{g_i}{f_i}$ is satisfied, the amplitude of fluctuations decreases gradually with the gradual increase of g_i after the closed-loop system approaches the equilibrium point. When g_i increases to a certain extent, the closed-loop system can be stable at the equilibrium point. Morover, the closed-loop system with the suppression vector of external disturbance can converge fastly to the equilibrium point. The observation from Fig. 2 and Fig. 3 is that the designed passivity-based controller without angular velocity measurement can make the closed-loop system described by Eqs. (3), (5) and (23) stable around the equilibrium point and suppress the effect

of external disturbance. Fig. 6 shows that the equilibrium tended to is related to the initial symbol of q_0. When there exist model error and model parameter uncertainty, the performance of the closed-loop system under the control torque is given in Fig. 7. Obviously, the system can be still stable at equilibrium point. It shows that the designed controller is robust to model error and model parameter uncertainty. Fig. 8 and Fig. 9 show the stability of the attitude control system acted by control torque around the equilibrium point. The attitude control system has better stability and is asymptotically stable at equilibrium point finally.

5. Conclusions

We considered the attitude stability of spacecrafts with the existence of external disturbances and uncertain inertia in this paper. A passivity-based attitude controller is designed, which introduces the suppression vector of external disturbance into the control law. The designed controller does not need the angular velocity measurement and can suppress the effect of external disturbance to a certain extent. In addition, the control law doesn't contain information related to the system parameters, which makes the attitude control system robust to model error and model parameter uncertainty.

Acknowledgements

This work was supported by the Fundamental Research Funds for the Central Universities (FRF-TP-12-088A) and Scientific Research Funds.

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