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Study of the Mechanical Behavior of a Hyperelastic Membrane

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Abstract: The benefits in emloying plastics material in microfluidic devices manufactures are extremely attractive that include reduced cost and simplified manufacturing procedures, particularly when compared to silicon. An additional benefit is the wide range of available plastic materials which allow the manufacturer to choose materials' properties suitable for their specific application. The Polydimethylsiloxane is commonly used in a wide range of microfluidic applications due to its flexibility and low cost. In addition the properties of the Polymethyl methacrylate such as the low cost, high transparency, and good chemical properties are needed in microfluidics applications. In this paper, we have used Finit Elements method to simulate the mechanical behavior of Polydimethylsiloxane and Polymethylmethacrylate using hyper elastic and linear elastic model. Sevral parameters have been studied; such as, thickness and number of mesh in order to optimize the dimension of the membrane. Also, we have studied the impact of the mesh form on the membrane's displacement. *Copyright © 2014 IFSA Publishing, S. L.*

Keywords: Finite Element, Microfluidic devices, Hyperelastic membrane.

1. Introduction

This Experimental studies are useful to refine or confirm an object's properties. However, they are expensive and sometimes cumbersome to implement. The prior determination of the behavior of the object is therefore very useful, even essential, if we want to avoid spending a lot of money in long trials. The numerical methods are an inexpensive and flexible way to establish mechanical models. In many cases, it is nevertheless necessary to validate numerical models with physical models.

In the literature much work relating to polymer microfluidic devices has been reported, including microfluidic devices fabricated from a range of polymers. Microfluidic systems fabricated using polydimethylsiloxane (PDMS) have been reported by [1-3]. While Locascio *et al.* [4] and Lee Gwo-Bin *et al.* [5] have reported fabrication of microfluidic devices on poly methylmethacrylate (PMMA).

Polydimethylsiloxane (PDMS) has been the mainstay for rapid prototyping in the academic microfluidics community, because of its low cost, robustness and straightforward fabrication, which are particularly advantageous in the exploratory stages of research. In addition to its simple manipulation, the PDMS has the following properties: it is a hyper elastic polymer; it can undergo very big distortions therefore without deteriorating [6]. It is biocompatible and non porous to the liquids. The

PDMS is the main support for the manufacture of fluidic Microsystems. The PDMS used in our work is silgel 612. In spite of the advantages presented by the use of the PDMS in micro fluidics, a certain number of materials plastic are associated, have been used in answer to the evoked previously difficulties. It is the case of the PMMA. This polymer is spilled enough in MEMS technology, different production methods can be used for the realization of the micro fluidics devices. The composites studied PMMA reinforced to the shock by particles of elastomeric of poly butadiene [7]. The objective of the present paper is to study and simulate the mechanical behavior of polymers using finite element method. The results are presented in two model hyperelastic and linear model. The goal is to examine the effect of material parameters and dimension of membrane at the microfluidic system efficiency.

2. Constitutive Model

In this paper, we compare two materials namely, PMMA and PDMS. Several simulations have been done for evaluating their performances. The mathematical model is separated in two parts: hyper elastic and linear model.

2.1. Hyperelastic Model

A hyperelastic or Green elastic material [8] is a type of constitutive model for ideally elastic material for which the stress-strain relationship derives from a strain energy density function. The hyperelastic material is a special case of a Cauchy elastic material.

A hyperelastic material has a nonlinear behaviour, which means that its deformation is not directly proportional to the load applied. An elastic material is hyperelastic if there is a scalar function, denoted by $W=W(\in)$, such that [8]:

$$S=\frac{\partial W}{\partial \in}=\frac{\partial W}{\partial C}\ , \tag{1}$$

where S is the second Piola Kirchhoff stress tensor, $\in$ is the Green strain tensor C is the Cauchy Green strain tensor. The strain energy function of hyperelastic constitutive models such as Neo-Hookean and Mooney-Revlin are expressed as a function of strain invariants, the Neo-Hookean model is obtained [9-10]:

$$W=\tfrac{1}{2}G(I_1-3)+\tfrac{1}{2}K(J-1)\ , \tag{2}$$

where:

the strain invariant: $I_1=\mathrm{tr}C$ and $J=\det(F)$;

F is the deformation tensor;

$C=F^T F$ and the Green strain tensor: $\in=\tfrac{1}{2}(C-I)$

The Cauchy stress tensor can be expressed as [11]:

$$\sigma=\tfrac{1}{j}FSF^T \tag{3}$$

The shear modulus G and the bulk modulus K of the hyper elastic material are defined by the following relations:

$$G=\frac{E}{2(1+\upsilon)}, \tag{4}$$

$$K=\frac{E}{3(1-2\upsilon)}, \tag{5}$$

where E is the Young's modulus and υ is the Poisson's ratio.

2.2. Hook's Relation (Linear Elastic Model)

The most general way to represent a linear relation between the stress tensor σ_{ij} and the strain tensor $\in_{kl}$ is given by Hooke's law.

$$\sigma_{ijkl}=C_{ijkl}\in_{kl}\ , \tag{6}$$

where σ_{ij} components of the Cauchy stress tensor $\in_{kl}$ are components of the strain tensor and C_{ijkl} is called the elastic constants tensor of fourth order.

The stress-strain relation for an isotropic linear elastic material can be written as

$$\sigma_{ij}=3K\left(\tfrac{1}{3}\in_{kk}\ \delta_{ij}\right)+2G\left(\in_{ij}-\tfrac{1}{3}\in_{kk}\ \delta_{ij}\right), \tag{7}$$

where δ_{ij} is the Kronecker Delta, under small strain the deformation of PMMA and PDMS membrane are very little. We can use the linear model for the first order approximation.

3. Numerical Simulation

A three dimensional constitutive model of rectangular elastomeric membrane has been used for simulation in our study, the FE model of the active structure and boundary conditions are shown in Fig. 1.

The modeling by finite element is commonly used to calculate the deformation of structure in Microsystems, because of its capacity to simulate the system in two and three dimensions. We took care of using a meshing enough refined to minimize the error in the modeling, and non-uniform mesh arrangement is implemented in the present model. Let us note that the relative error is less than 8 % in it we choose the small number of element to optimize the calculation time.

We have studied the influence of the variation of the force's angle inclination θ via the tree axis (Fig. 1), the obtained results.

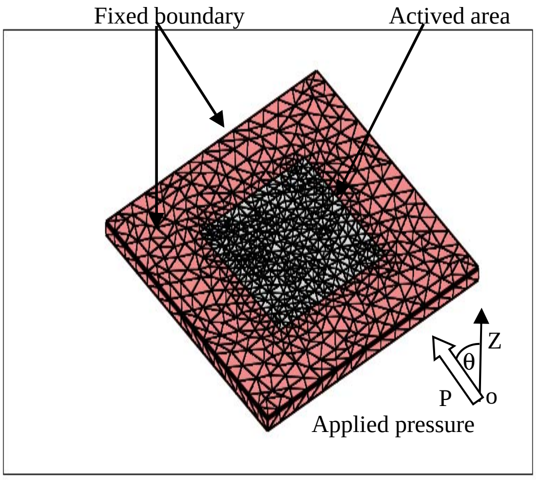

Fig. 1. Model of elastomeric membrane and boundary conditions.

4. Results and Discussion

The Fig. 2 present the deformation in three dimensions of PDMS membrane obtained by simulation. We observe an important change in the center of the membrane under a pressure of 30 kPa.

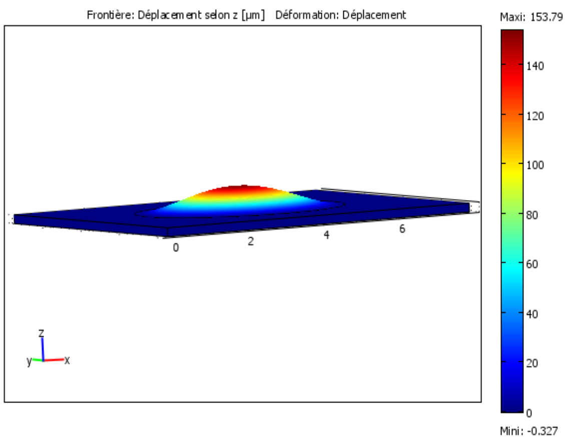

Fig. 2. Deformation of PDMS membrane induced by applied pressure with θ=0°.

To describe the mechanical behavior of the membrane we used two models: the hyperelastic model and the linear model. The obtained results are represented in the Fig. 3, we notice that the model of NH and the linear model give similar results for low values of pressure. What allowed us to replace the model complicated with NH by a simple linear model to solve such a problem.

As it is illustrated in the Fig. 4, the maximum of vertical displacement increases with the applied pressure, and the membrane in PDMS undergoes a wide deformation for an applied maximal pressure.

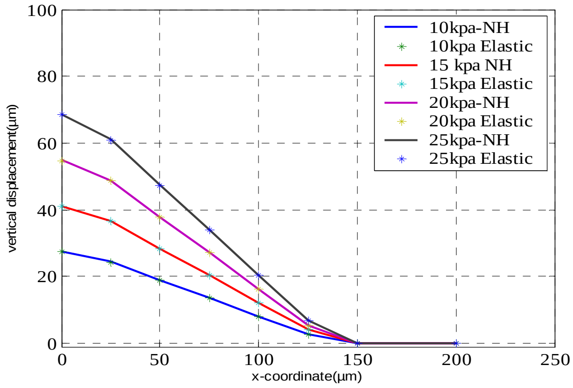

Fig. 3. Vertical displacement on surface of PDMS membrane at different pressure: comparison between N- Hookeen and Linear model.

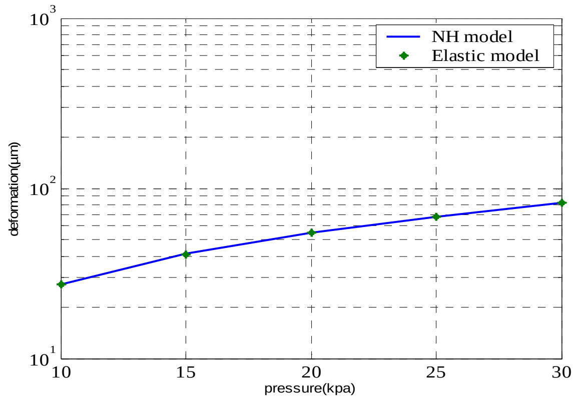

Fig. 4. Maximum of vertical displacement on surface of PDMS membrane at different pressure.

To see the influence of the dimensions membrane on the rigidity of the system, we simulate the relation between deformation and exercised pressure by including the membrane thickness variation (Fig. 5).

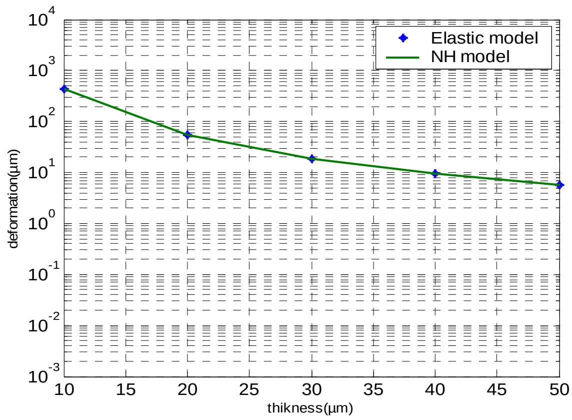

Fig. 5. Maximum of vertical displacement onsSurface of PDMS membrane at different thickness.

The analysis of this curve show that the membrane deformation decreases. With increasing thickness. So to obtain a larger deformation at smaller pressure, adds us interested to decrease the membrane thickness.

4.1. Comparison Between PDMS and PMMA

The flowing step consists in comparing between the PDMS and the PMMA, these two materials which are known for some time, of part their characteristics and their common use in micro-technology. Furthermore it is easy to integrate these materials into a technological enchainment of microsystem type.

The Fig. 6 represents the deformation in three dimensions in of the membrane of PMMA at 30 kPa, the Fig. 7 illustrates the variation of the vertical displacement of the PMMA according to the pressure. Curves represent a good coincidence between the linear model and the hyper elastic model. To compare the PDMS and PMMA materials in term of maximum deformation, we drew the curves of the maximal vertical displacement according to the pressure and thickness.

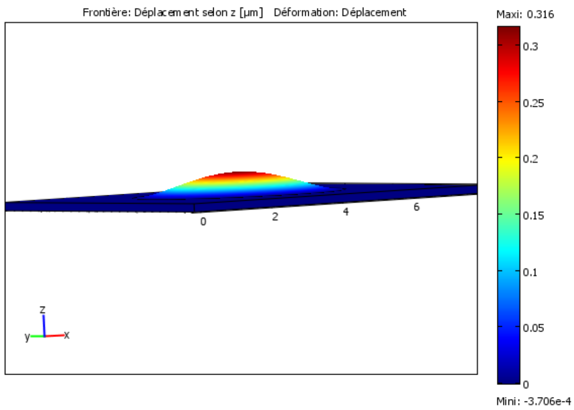

Fig. 6. Deformation of PMMA membrane induced by applied pressure with θ =0°.

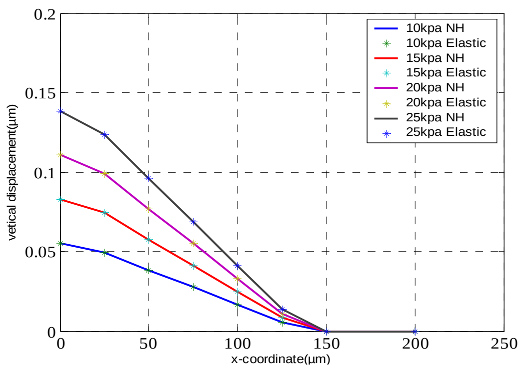

Fig. 7. Vertical Displacement on Surface of PMMA Membrane at Different Pressure: Comparison between N-Hookeen and Linear Model.

The obtained results are represented in Fig. 8 and Fig. 9. Particularly, the results show that the PDMS membrane has a large deformation compared to PMMA membrane at same conditions.

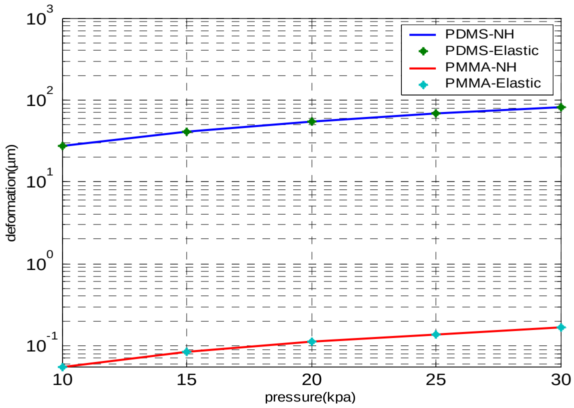

Fig. 8. Maximum of Vertical Displacement on Surface of PDMS and PMMA Membrane at Different Pressure.

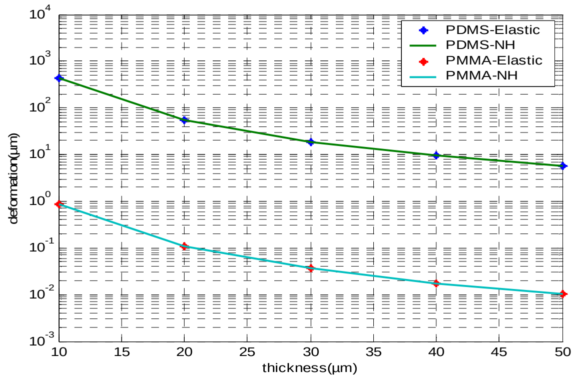

Fig. 9. Maximum of Vertical Displacement on Surface of PDMS and PMMA Membrane at Different Thickness.

To study the effect of inclination of the pressure force, we trace the variation of maximum displacement with the angle of inclination in Y-Z plan for different values of membrane thickness. The obtained result is presented in Fig. 10 and Fig. 11.

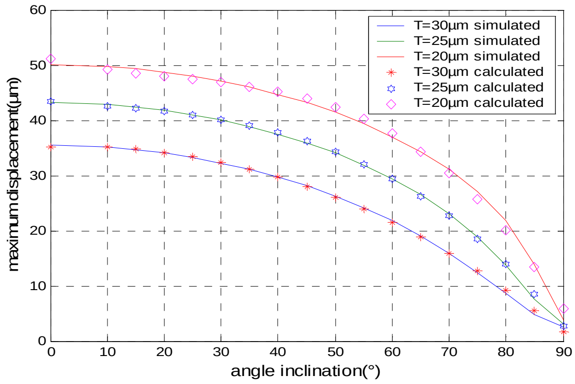

Fig. 10. Maximum of Displacement on Surface of PMMA to Different Values of Inclination Angle.

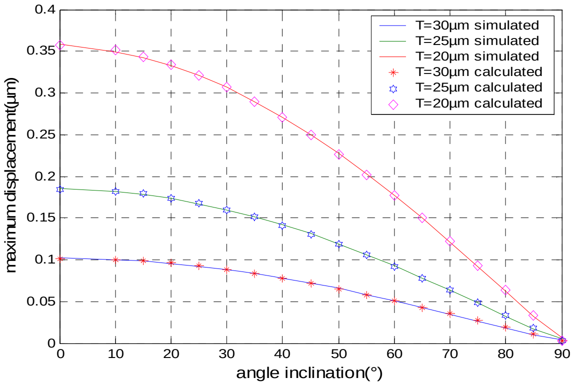

Fig. 11. Maximum of Displacement on Surface of PMMA to Different Values of Inclination Angle.

According to the presented figures, we can notice that even at θ=90 °C, the displacement is done, in other word, the materials present a small displacement when the force is parallel to the membrane's surface. We can explain this result by the rigidity of materials.

5. Conclusions

In this paper we presented two mechanical models (NH and Linear elastic model) to describe the behavior of PDMS and PMMA membrane. We have showed that the similar results are obtained for the two models under small loading. In this case we can replace the complicate hyperelastic model by the simple elastic model for small loading.

The comparison of membrane's deformation between the PDMS and PMMA are studied at different values of pressure and thickness. The results obtained by FE modelling show that the deformation increase with pressure and reversely decrease with increasing thickness.

The miniaturization in the MEMS is a requirement, thus we always try to decrease the thickness of the membrane, this stage is positive but it will be limited by the rigidity of the system.

The PDMS is better than PMMA in term of maximum deformation, this special characteristic it can be used in microfluidic us active membrane especially us active microvalves with mechanical moving parts, despite the advantages of PDMS rapid prototyping for microfluidics technology, this material suffers from a serious drawback in that it swells in most organic solvents [12]. The swelling of PDMS-based devices makes it impossible for organic solvents to flow inside the micro channels.

The PMMA is an elastomer with little deformation in comparison with the PDMS, this property can be used for the construction of canalization of micro valve when the rigidity is required.

References

[1]. Martin P., Matson D., Fabrication of plastic microfluidic component's, *Proc. SPIE Microfluidic Devices*, 3515, 1998, pp. 172-176.

[2]. Kampar K., Dopper J., A self filing low cost membrane micropump, in *Proceedings of the MEMES'98*, pp. 432-437.

[3]. Schartz M. M., Joining of composite material's, *American Society for Metals*, 1994, pp. 35-88.

[4]. Dopper J., Clemens M., Development of low cost injection moulded micropumps, in *Proceedings of the Actuators'96*, pp. 37-40.

[5]. Lee Gwo-Bin, Chen S., Microfabricated plastic chips by hot embossing methods and their applications, *Sensors and Actuators B*, 75, 2001, pp. 142-148.

[6]. M. Unger, H. Chou, T. Thorsen, A. Sherer, S. Quake, Monolithic microfabricated valves and pumps by multilayer soft lithography, *Science*, 5463, 2000, pp. 113-116.

[7]. N. Belayachi, N. Benseddiq, M. Abdelaziz, Numeric and experimental survey of the behavior of polymeric shocks by an approach of homogenizations, in *Proceedings of the 18th French Congress of Mechanics*, Grenoble, France, 2007.

[8]. J. Bonet, R. D. Wood, No linear continuum mechanics for finite element analysis, *Cambridge University Press*, 1997, pp. 248.

[9]. E. Mardsen, J. R. Hughest, Mathematical foundations of elasticity, *Prentice-hall, Englewood Clifs*, 1983, pp. 556.

[10]. L. R. G. Treloar, Stress-strain data for vulcanized rubber under various types of deformation, *Transaction Faraday Society*, 40, 1944, pp. 59-70.

[11]. H. Daanouni, Y. Tillier, F. Bay, Numeric Survey of Hyperelastic Membrane Subjected to Muliaxial Loadings, in *Proceedings of the 19th French Congress of Mechanics*, Marseille, France, 2009.

[12]. Lee J. N., Park C., *et al.*, G M 2003 Solvent compatibility of polydimethylsiloxane - based microfluidic devices, *Anal. Chem.*, 75, 2003, pp. 6544–6554.