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Dynamics Analysis of the Double Motors Synchronously Exciting Nonlinear Vibration Machine Based on Acceleration Sensor Signal

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Abstract: Using the equivalent linearization method, the compensation and influence of the main vibration spring stiffness to the movement stability of the nonlinear resonance machine which synchronously driven by double exciting motors have been discussed. And using the singular perturbation theory, the influence of the nonlinear force which caused by load fluctuate to the stability of the vibration machine have been researched also. By the experiment and the actual production application, the validity of the mechanical motion stability analysis conclusion for the double motors synchronously exciting nonlinear vibration machine has been verified. The research conclusions that carried out in the paper have important reference value for the dynamic analysis and practical debugging of the double motors synchronously exciting nonlinear vibration machine. *Copyright © 2014 IFSA Publishing, S. L.*

Keywords: Dynamics, Stability, Nonlinear vibration, Synchronization, Acceleration sensor.

1. Introduction

Recent years, nonlinear resonance synchronous vibration machine had been developed in the mining, metallurgy industry [1, 2]. Because of the start, stop process quickly, smoothly and delay time short of the material on the body, which can used to save energy and reduce risk of burn out the exciting motor during the course of the starting stage and other advantages, this kind of vibration machine always praised by the industry [3-7].

In order to analyze the influence of the load fluctuate to the movement stability of the vibration machine during the application process and to discuss the influence of vibration machine's status, the paper make the double motors synchronously exciting nonlinear vibration machine to be the research object, and based on the Lyapunov stability theory to deduce the phase difference value state space model of the double exciting motors.

And the movement stability influence by the load fluctuation had been analyzed in theoretically for this kind of nonlinear machine. Further, the vibration synchronization stability of the double exciting motors which used to excite the nonlinear vibration machine had been researched in the double vibration motors resonance area [8]. And by the actual production application, the validity of the theory analysis conclusions which researched in the paper had been verified.

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2. Dynamics Analysis of the Double Motors Synchronously Exciting Nonlinear Vibration Machine

The nonlinear resonance synchronous vibration machine is composed by the vibrating body's machinery, double exciting motors, main vibration spring, isolation spring and other parts. The structure and working principle of the nonlinear vibration machine is shown as Fig. 1. The vibration body synchronously driven by the double vibration motors does the elliptic vibration which approximate linear under the resonance synchronization conditions. As shown in Fig. 1, to establish the static state coordinate system Oxy, the dynamic state coordinate system $O'x'y'$ and the five generalized coordinates x_1, y_1, x_2, y_2, θ. At the initial static state, the static state coordinate system of the nonlinear vibration machine is coinciding with the dynamic state coordinate system.

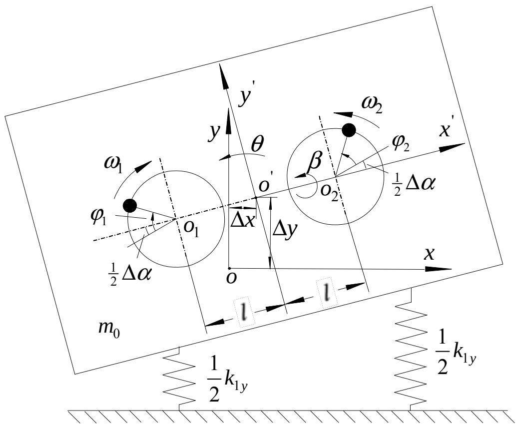


Fig. 1. Mechanical dynamics model of the nonlinear double motors synchronous vibration machine.

Based on the Lagrange equations and the Hamilton principle, the dynamics equations of the nonlinear vibration machine which shown as Fig. 1 can be expressed as

$$\begin{aligned}
& M\ddot{y}_1 - c_{2y}\dot{y}_1 + c_{2y}\dot{y}_1 - a_y\Delta y + b_y\Delta y^3 \\
& \quad = m_0 r\omega^2 \sin(\omega t + \alpha + \beta) \\
& M\ddot{x}_1 - c_{2x}\dot{x}_1 + c_{2x}\ddot{x}_1 - a_x\Delta x + b_x\Delta x^3 \\
& \quad = m_0 r\omega^2 \cos(\omega t + \alpha + \beta) \\
& (J_1 + m_1 r_1^2 + J_2 + m_2 r_2^2)\ddot{\theta} + c_\theta\dot{\theta} \\
& \quad + [k_{1\theta}(l_1\cos\alpha + l_2\sin\alpha - r_1\cos\beta)^2 \\
& \quad - k_{2\theta}(l_2\cos\alpha - l_2\sin\alpha + r_1\cos\beta)^2] \\
& \quad = 2m_0 r\omega^2 r_2 \cos\omega t
\end{aligned} , \quad (1)$$

where $M = m_1 + m_2 + 2m_0$ is the vibrating mass of the nonlinear vibration systems.

According to the literature [9-11] to know that, the design conditions of this kinds of vibration machine is

$$\begin{aligned}
& k_{1y}(y_1 + l_1\theta) + k_{2y}(y_1 - l_2\theta) << k_{ay}\Delta y - k_{by}\Delta y^3 \\
& k_{1x}(x_1 + l_1\theta) + k_{2x}(x_1 - l_2\theta) << k_{ax}\Delta x - k_{bx}\Delta x^3
\end{aligned} \quad (2)$$

And because the system's torsional pendulum response θ is small, when the nonlinear vibration machine which shown as Fig. 1 works in practice [1, 12], and the vibrating isolation spring stiffness is far less than the main vibration spring stiffness, so to deal with them as minor term ε.

Set

$$\begin{aligned}
\Delta y &= A\sin^2(\alpha + \beta) \\
\Delta x &= A\cos^2(\alpha + \beta)
\end{aligned} , \quad (3)$$

Then lead expression (3) into expression (1), the follow equation will be obtained

$$m\ddot{H} + c_2\dot{H} + aH - bH^3 = m_0 r\omega^2 \sin(\omega t), \quad (4)$$

where $m = \dfrac{m_1 m_2}{m_1 + m_2}$ is the nominal mass.

To set the solution of expression (4) is

$$H = A\sin\varphi, \quad (5)$$

To equivalent linearization the nonlinear vibration equation expression (4), and lead expression (5) into it, then expression (6) will be obtained as

$$m\ddot{H} + c_2\ddot{H} + K_e H = m_0 r\omega^2 \sin(\omega t), \quad (6)$$

where $K_e = \dfrac{1}{A\pi}\displaystyle\int_0^{2\pi}(aH - bH^3)\sin\varphi d\varphi$.

Lead expression (5) into expression (6), then

$$K_e = a - \frac{3}{4}bA^2, \quad (7)$$

Then the equivalent natural frequency and the relative amplitude of the nonlinear vibration machine which shown as Fig. 1 can be obtained as expression (8) and expression (9).

$$\omega_e = [(a - \frac{3}{4}bA^2)/m]^{\frac{1}{2}}, \quad (8)$$

$$A = \frac{m_0 r\omega^2}{[(K_e - m\omega^2)^2 + c_2^2\omega^2]^{\frac{1}{2}}}, \quad (9)$$

Analyze expression (9) to know that, when the exciting frequency ω falling, the amplitude A of the nonlinear vibration machine will be increased. But from expression (8) to know that, right now the equivalent natural frequency ω_e will be reduced, which make the frequency ratio z increase, leading the amplitude A to do shrink fluctuations. All will make the amplitude A of the nonlinear vibration machine to stable at a certain fixed value, despite the ω changes make the amplitude values A is not equal to the value of the changes before, but during the actual application the difference is slight. So the conclusion can be gotten that the amplitude A has the automatic compensation function when the working frequency ω fluctuate of the exciting motor.

About expression (8), when the vibrating mass M of the nonlinear vibration machine gets large, the equivalent natural frequency of the nonlinear vibration system will decrease, and then the frequency ratio z increase, which lead the amplitude A that expressed as expression (9) to decrease. But now know from expression (8) that the decrease of amplitude A will make the equivalent natural frequency ω_e increases, and then the frequency ratio z will decrease again. All lead the amplitude value A to gravel to compensate the decrease ahead. Then the other conclusion can be gotten that the amplitude A have automatic compensation function also, when the vibrating mass M fluctuation.

When the nonlinear vibration machine working under the resonance conditions, the load such as ore has small and weak nonlinear damping characteristics. So to add the small parameter ε on the damping term and the load nonlinear force term of expression (1), and then set $y_1 = \psi_1$, $\dot{y}_1 = \psi_2$, $y_2 = \psi_3$, $\dot{y}_2 = \psi_4$, $x_1 = \psi_5$, $\dot{x}_1 = \psi_6$, $x_2 = \psi_7$, $\dot{x}_2 = \psi_8$.

Now expression (1) will become as

$$\frac{d\psi_i}{dt} = \sum_{j=1}^{8} a_{ij}\psi_j + L_i(\omega t) + \varepsilon F_i(\omega t, \psi, \varepsilon), \quad (10)$$

where $i = 1,2,\cdots,8$, and

$$\begin{cases} a_{ij} = 1 & (i = j) \\ a_{ij} = -\dfrac{1}{m_1 + m_2} - \left[k_{iy} + K_e \cos^2(\alpha + \omega t)\right] & (i \neq j) \end{cases}, \quad (11)$$

$$L_i = \frac{2m_0 r\omega}{m_1 + m_2}\cos(\omega t + i\alpha), \quad (12)$$

$$F_i(\omega t, \psi, \varepsilon) = F_{i0}(\psi, \varepsilon) + \sum_{n=1}^{\infty}\left[F_{in}(\psi, \varepsilon)\cos(n\omega t) + F_{in}'(\psi, \varepsilon)\sin(n\omega t)\right], \quad (13)$$

where

$$\begin{cases} F_{i0}(\psi, \varepsilon) = 2m_0 r\omega \\ F_{in}(\psi, \varepsilon) = -\dfrac{\varepsilon(c_{1y} + c_{2y})\psi_i}{m_1 + m_2} + \left(a_{yi}\cos(n\psi_i) + b_{yi}\sin(n\psi_i)\right) \\ F_{in}'(\psi, \varepsilon) = -\dfrac{\varepsilon(c_{1x} + c_{2x})\psi_i}{m_1 + m_2} + \left(a_{xi}\cos(n\psi_i) + b_{xi}\sin(n\psi_i)\right) \end{cases}, \quad (14)$$

Using nonlinear vibration system average method [13] to solve expression (10), remove ε^2 and above high order item, then the one time periodic approximate solution can be obtained as

$$\psi_i = A_i \sin(\omega t + \alpha - \vartheta_i), \quad (15)$$

where

$$A_i = \frac{2m_0 r\omega^2\cos(\vartheta_i + i\alpha) - \left(K_e - m\omega_e^2\right)}{M}, \quad (16)$$

$$\vartheta_i \approx \tan^{-1}\frac{\sqrt{a_{yi}^2 + b_{yi}^2}\cos(\omega t + i\alpha) + \sqrt{a_{xi}^2 + b_{xi}^2}\sin(\omega t + i\alpha)}{K_e - m\omega_e^2} \quad (17)$$

To know that, when nonlinear vibration system which shown as Fig. 1 works under the resonant conditions, the following conclusions can be gotten

$$\begin{cases} \vartheta_i \approx 90^\circ \\ A_i = \dfrac{2m_0 r\omega^2\cos(i\alpha)}{M} \end{cases}, \quad (18)$$

2. Simulation Experiments and the Analysis of the Practical Application Results

Based on the Harmonic vibration synchronously test bench that driven by two exciting motors shown as Fig. 2, the experiments can be carried out. On the test bench, there are three acceleration sensors set around the two exciting motors and the beam of the test bench. And the acceleration signal can be obtained by the sensor, the acceleration signal of the two exciting motors are shown as Fig. 3.

Lead the mechanical characteristic parameters and motor parameters into expression (16)-(17), to get the value of A_i and ϑ_i. Then based on expression (15), the numerical simulation of the nonlinear system can be carried out. The Fig. 4 which used to express the limit cycles trajectory of systemic solution will be obtained. Know from the Fig. 4 that in spite of the position of initial point is set in the interior region of systemic limit cycles, the trajectory of systemic solution will infinitely approach to the limit cycles with helix form, when the time t is increasing. The final result is that the trajectory arrives at the limit cycles and the motion state of system comes into stable state. So the assertion can be obtained that nonlinear machine which shown as Fig. 1 has the stable periodic solution [14-16].

Fig. 2. Harmonic vibration synchronously test bench that driven by two exciting motors.

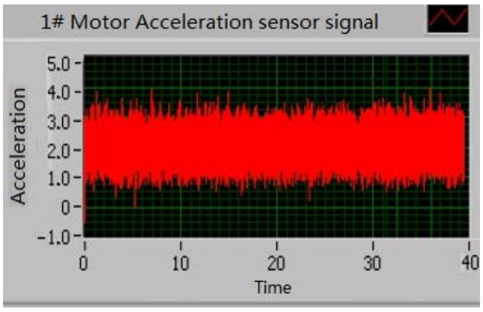


Fig. 3 (a). Time domain acceleration signal of two exciting motors that come from the acceleration sensors: Acceleration signal of 1# exciting motor.

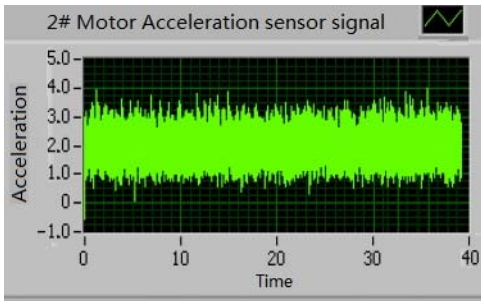


Fig. 3 (b). Time domain acceleration signal of two exciting motors that come from the acceleration sensors: Acceleration signal of 2# exciting motor.

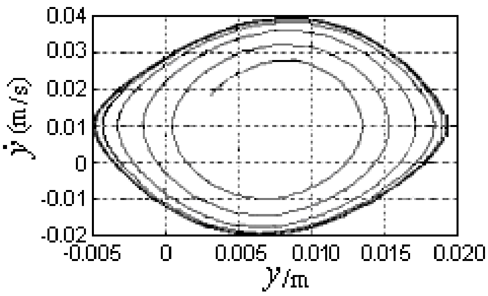


Fig. 4. Phase trajectory simulation of nonlinear system solution of expression (1) under the initial conditions $(0.003\ \text{m}, 0.018\ \text{m/s})$.

In order to validate the synchronous movement stability of the nonlinear vibration system shown as Fig. 1, set vibrating mass M small-scope fluctuation case, to discuss the changes of the amplitude the rotate speed ω_i of the double exciting motors. The experimental results are shown as Table 1.

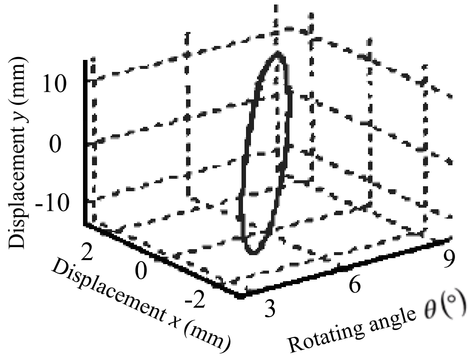
Table 1. Amplitude and rotate speed experiment when vibrating mass fluctuate.

	Vibrating mass M (kg)										
	99.0	99.2	99.4	99.6	99.8	100.0	100.2	100.4	100.6	100.8	101.0
A_i (mm)	13.63	13.70	13.66	13.65	13.61	13.59	13.60	13.56	13.53	13.51	13.57
ω_1 (rad/s)	47.49	47.49	47.44	47.40	47.38	47.39	47.37	47.35	47.36	47.37	47.31
ω_2 (rad/s)	47.48	47.46	47.41	47.44	47.40	47.39	47.38	47.33	47.33	47.34	47.33

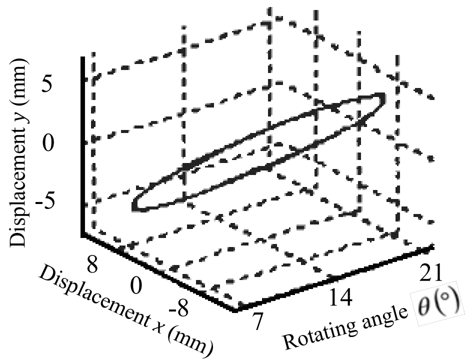
From the experimental data that shown as Table 1 to know that, when the vibrating mass of the nonlinear vibration machine small-scope fluctuating (fluctuation ratio around 2 %), despite the amplitude value of the nonlinear system occur certain change, but the change range limited in about 1.32 % scope, which can be considered that the amplitude approximate remain stability unchanged when vibrating mass fluctuation cases. The practical application results of this case also prove that the stable amplitude can completely satisfy the industrial application. In order to ascertain the synchronization of the double exciting motors which used to drive the nonlinear vibration machine when the vibrating mass fluctuation situation, in Table 1, the rotor frequency of the double exciting motors are given at the same time under different vibrating mass. Know from Table 1 data, along with the increase of the vibrating mass, the rotor frequency of the double exciting motors occur decrease trend, but the synchronization of still remain good status, the maximum rotor frequency difference ratio is about 0.81 %, which make the synchronous vibration machine perfectly meet the industrial application.

In the experiments, to set a series of phase difference angle $\Delta\alpha$ of the double exciting motors, to discuss the dynamics motion trajectory of the nonlinear vibration machine which shown as Fig. 1.

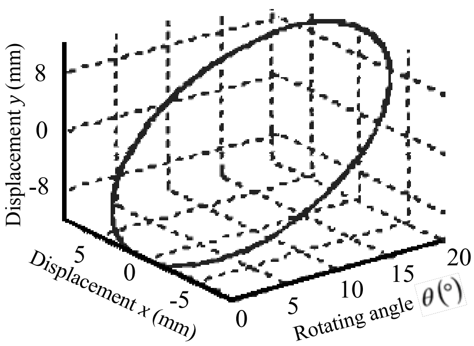
Based on the measured center of mass motion track data of the nonlinear vibration machine, systemic vibration response trajectory can be expressed as Fig. 5. Know from Fig. 5, when $\Delta\alpha = 0°$, the motion in y direction is given priority to linear motion, accompany with the elliptical motion in x direction and the twist vibration in θ direction. And when $\Delta\alpha = 180°$, the motion in x direction is given priority to elliptical motion, accompany with the elliptical motion in y direction and the twist vibration in θ direction. When $\Delta\alpha \approx 0° \sim 180°$, the motion trajectory of the nonlinear vibration machine which shown as Fig. 1 is composed by elliptical motion and the twist vibration.



(a) $\Delta\alpha = 0°$



(b) $\Delta\alpha = 180°$



(c) $\Delta\alpha = 0 \sim 180°$

Fig. 5. Motion trajectory of the double exciting motors nonlinear vibration machine under different value of Δα.

3. Conclusions

The research results which carried out in the paper showed that, about the nonlinear vibration system which supported by the soft nonlinear characteristics spring, the amplitude value of the nonlinear system can be automatically compensated, when the vibrating mass of the vibrating system fluctuating in small-scope, which make the amplitude approximate remaining constant. At the same time, despite the rotor frequency of the double exciting motors changed, but the synchronization motion status of the double exciting motors can remain better. All these can make the vibration trajectory of the nonlinear vibration machine meet the industrial application requirements perfectly. The dynamic analysis method and the research conclusions that shown in the paper have important refer significance for the nonlinear vibration mechanical design and application debugging.

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