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An Improved 2-D DOA Estimation with L-shaped Arrays Based on PM

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Abstract: In this paper, an improved two-dimensional (2-D) direction of arrival (DOA) estimation method is proposed for narrow signals impinging on an L-shaped arrays. Based on the propagator method (PM), the computational loads of the proposed method can be significantly smaller since the PM does not require any eigenvalue decomposition of the received data. With a propagator matrix, the proposed method constructs a new extended matrix to estimate the elevation angle, which improves the DOA estimation performance in low SNR. By exploiting the covariance matrix of the received data, another propagator matrix is achieved, then pair matching and peak searching are used to achieve the corresponding 2-D azimuth angles, which reduces the occurrence of estimation failure and errors. In the case of DOA estimation for two signals, at RMSE = 0.2, the proposed method results in a gain improvement of about 5dB over the joint singular value decomposition (SVD) method and 9.5 dB over the PM method. *Copyright © 2014 IFSA Publishing, S. L.*

Keywords: 2-D DOA, L-shaped, PM, low SNR, SVD.

1. Introduction

Recently, two-dimensional (2-D) direction of arrival (DOA) estimation has received a significant amount of attention [1, 2]. It plays an important role in array processing for improving the quality of the wireless communication [3]. Many effective methods have been proposed for DOA estimation based on the uniform rectangular array (URA) [4] and uniform circular array (UCA) [5]. However, these methods need a large number of sensors to achieve high resolution and give accurate estimates. In reference [6], a 2-D DOA estimation method with two parallel uniform linear arrays is proposed to resolve the uncorrelated signals. However, this method can not resolve signals that have a common direction β, and since there are only two sensors in the y-axis direction, the accuracy of angles β may not be high. In reference [7], it has been proven that the L-shaped array has better estimation than many other simple structured arrays. There has been growing interest in developing 2-D DOA estimators by exploiting L-shaped arrays. Tayem et al. proposed a propagator method (PM) based on L-shaped arrays [8]; however, the independent eigenvalue decompositions cause arbitrary ordering of the eigenvalues. Kikuchi et al. [9] proposed a cross correlation matrix method based on ESPRIT, which has problems at low SNR and encounters estimation failure for small angular separation azimuth angles. The joint singular value decomposition (SVD) method [10] can achieve automatic pairing for 2-D angle estimation, but it performs worse in the estimation of azimuth angle when the number of snapshots is small, furthermore they are computationally intensive because of using SVD.

This paper proposes an effective method for 2-D DOA estimation of signals with a L-shaped array. The proposed method constructs a new propagator matrix to estimate the elevation angle in the same way as the improved PM method [11] with two parallel uniform linear arrays, which improves the performance in low SNR. And the proposed algorithm can achieve automatically paired two-dimensional angle estimation. Finally, numerical simulations show that the proposed method has a higher resolution than the PM method and the joint SVD method of the L-shaped array.

This paper is organized as follows. In section 2, the signal model is described. The proposed method is presented in section 3. Then section 4 gives the computer simulation results. Finally, conclusion is presented in Section 5.

2. Signal Model

Fig. 1 shows the L-shape array configuration which uses x-z plane. Each linear array consists of N omnidirectional element sensors with adjacent spacing d. Suppose that there are K far-field narrow band signal $s_k(t)$ (k = 1,...,K) impinging on the arrays, where λ is the carrier wavelength. The k^{th} signal $s_k(t)$ has an elevation angle θ_k and an azimuth angle ϕ_k. Let the $N \times 1$ signal vectors received at the X and Z subarray be $\mathbf{X}_1(t) = [x_1(t), x_2(t),\ldots, x_N(t)]^T$, and $\mathbf{Z}_1(t) = [z_1(t), z_2(t),\ldots, z_N(t)]^T$, respectively, where the superscript T denotes the transpose. Also let the $(N-1) \times 1$ signal vectors received at the X and Z subarrays be $\mathbf{X}_2(t) = [x_1(t),x_2(t),\ldots,x_{N-1}(t)]^T$ and $\mathbf{Z}_2(t)=[z_1(t), z_2(t),\ldots, z_{N-1}(t)]^T$, respectively.

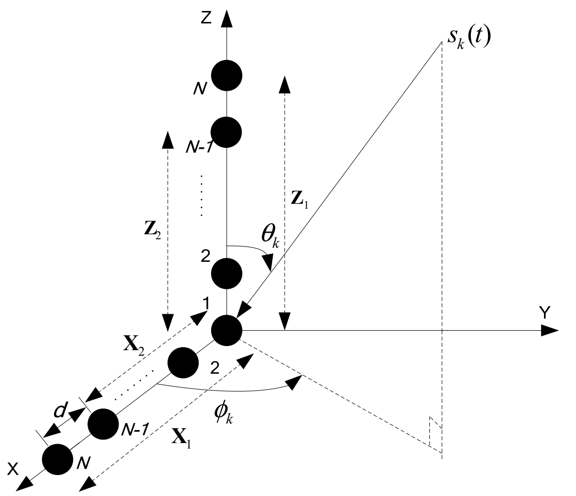

Fig. 1. L-shaped array configuration for 2-D DOA estimation.

These received vectors at the Z and X subarray can be rewritten as

$$\mathbf{Z}_1(t) = \mathbf{A}_z(\theta)\mathbf{S}(t) + \mathbf{n}_{z1}(t), \quad (1)$$

$$\mathbf{Z}_2(t) = \mathbf{A}_z(\theta)\mathbf{\Phi}_1\mathbf{S}(t) + \mathbf{n}_{z2}(t), \quad (2)$$

$$\mathbf{X}_1(t) = \mathbf{A}_x(\theta,\phi)\mathbf{S}(t) + \mathbf{n}_{x1}(t), \quad (3)$$

$$\mathbf{X}_2(t) = \mathbf{A}_x(\theta,\phi)\mathbf{\Phi}_2\mathbf{S}(t) + \mathbf{n}_{x2}(t), \quad (4)$$

$$\mathbf{A}_z(\theta) = [\boldsymbol{\alpha}_z(\theta_1), \boldsymbol{\alpha}_z(\theta_2), \cdots, \boldsymbol{\alpha}_z(\theta_K)]^T, \quad (5)$$

$$\mathbf{A}_x(\theta,\phi) = [\boldsymbol{\alpha}_x(\theta_1,\phi_1), \cdots, \boldsymbol{\alpha}_x(\theta_K,\phi_K)]^T, \quad (6)$$

$$\mathbf{S}(t) = [s_1(t), s_2(t), \cdots, s_K(t)]^T, \quad (7)$$

where $\boldsymbol{\alpha}_z(\theta_k) = [1, e^{-j\frac{2\pi d}{\lambda}\cos\theta_k}, \cdots, e^{-j\frac{2\pi d}{\lambda}(N-1)\cos\theta_k}]^T$, $\mathbf{a}_x(\theta_k,\phi_k) = [1, e^{-j\frac{2\pi d}{\lambda}\sin\theta_k\cos\phi_k}, \cdots, e^{-j\frac{2\pi d}{\lambda}(N-1)\sin\theta_k\cos\phi_k}]^T$, $\mathbf{n}_{z1}(t)$, $\mathbf{n}_{z2}(t)$, $\mathbf{n}_{x1}(t)$ and $\mathbf{n}_{x2}(t)$ are additive white Gaussian noise vectors whose elements have mean zero and variance σ^2. The matrix $\mathbf{\Phi}_1$ and $\mathbf{\Phi}_2$ is a $K\times K$ diagonal matrix containing information bout the elevation angle θ_k and azimuth angle ϕ_k, respectively.

3. Proposed Method

3.1. Estimation of Elevation Angles

Combining the signal vector $\mathbf{Z}_1(t)$ and $\mathbf{Z}_2(t)$, obtain a $(2N-1)\times 1$ vector as

$$\mathbf{Z}(t) = \begin{bmatrix} \mathbf{z}_1(t) \\ \mathbf{z}_2(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A}_z(\theta) \\ \mathbf{A}_z(\theta)\mathbf{\Phi}_1 \end{bmatrix}\mathbf{S}(t) + \mathbf{N}_z(t), \\ = \mathbf{A}\mathbf{S}(t) + \mathbf{N}_z(t) \quad (8)$$

where $\mathbf{N}_z(t) = [\mathbf{n}_{z1}^T(t), \mathbf{n}_{z2}^T(t)]^T$.

Partition the matrix $\mathbf{A}$ as

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \end{bmatrix}, \quad (9)$$

where $\mathbf{A}_1$ and $\mathbf{A}_2$ are the matrices of dimension $K\times K$ and $(2N-1-K)\times K$, respectively.

Assume that $\mathbf{Z}(t)$ are constant for J samples, then the covariance matrix of the received data in Z subarray can be written as

$$\mathbf{R}_z = \frac{1}{J}\sum_{t=1}^{J}\mathbf{Z}(t)\mathbf{Z}^H(t), \quad (10)$$

where the superscript H denotes the conjugate transpose.

As in [11], $\mathbf{R}_z$ is partitioned as $\mathbf{R}_z = [\mathbf{R}_{z1}\ \ \mathbf{R}_{z2}]$, where $\mathbf{R}_{z1}$ and $\mathbf{R}_{z2}$ are matrices of dimension

$(2N\text{-}1)\times K$ and $(2N\text{-}1)\times(2N\text{-}1\text{-}K)$, respectively. And in the noiseless case

$$\mathbf{R}_{z2} = \mathbf{R}_{z1}\mathbf{P}_{z}, \tag{11}$$

where $\mathbf{P}_z$ is the propagator matrix. And the matrix $\mathbf{P}_z$ can be estimated by

$$\hat{\mathbf{P}}_{z} = (\mathbf{R}_{z1}^{H}\mathbf{R}_{z1})^{-1}\mathbf{R}_{z1}^{H}\mathbf{R}_{z2}, \tag{12}$$

With $\hat{\mathbf{P}}_z$, construct a new extended matrix as follow

$$\mathbf{P}_{c1} = \begin{bmatrix} \mathbf{I}_{K\times K} \\ \hat{\mathbf{P}}_{z}^{H} \end{bmatrix}, \tag{13}$$

Then partition $\mathbf{P}_{c1}$ as

$$\mathbf{P}_{c1} = \begin{bmatrix} \mathbf{P}_{x} \\ \mathbf{P}_{y} \end{bmatrix}, \tag{14}$$

where $\mathbf{P}_x$ and $\mathbf{P}_y$ are the matrices of dimension $N\times K$ and $(N\text{-}1)\times K$, respectively.

Let $\mathbf{P}_1$ denote the first N rows of $\mathbf{P}_x$, and we have

$$\mathbf{P}_{1}\mathbf{A}_{1} = \mathbf{A}_{y}, \tag{15}$$

$$\mathbf{P}_{y}\mathbf{A}_{1} = \mathbf{A}_{y}\mathbf{\Phi}_{y}, \tag{16}$$

Define a new matrix as

$$\mathbf{\Psi}_{y} = \mathbf{P}_{1}^{\#}\mathbf{P}_{y} = \mathbf{A}_{1}\mathbf{\Phi}_{y}\mathbf{A}_{1}^{-1}, \tag{17}$$

where the superscript # denotes the pseudoinverse transpose.

Then perform the eigenvalue decomposition of $\mathbf{\Psi}_y$, and the eigenvalues are corresponding to the elevation angle θ_k (k =1, . . . ,K).

3.2. Estimation of Azimuth Angles

Define a $(2N\text{-}1)\times 1$ vector as

$$\begin{aligned}\mathbf{X}(t) &= \begin{bmatrix} \mathbf{X}_{1}(t) \\ \mathbf{X}_{2}(t) \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{A}_{x}(\theta,\phi) \\ \mathbf{A}_{x}(\theta,\phi)\mathbf{\Phi}_{2} \end{bmatrix}\mathbf{S}(t) + \mathbf{N}_{x}(t)\end{aligned}, \tag{18}$$

where $\mathbf{N}_{x}(t) = [\mathbf{n}_{x1}^{T}(t), \mathbf{n}_{x2}^{T}(t)]^{T}$.

The covariance matrix of the received data in X subarray is

$$\mathbf{R}_{x} = \frac{1}{J}\sum_{t=1}^{J}\mathbf{X}(t)\mathbf{X}^{H}(t), \tag{19}$$

which is partitioned as $\mathbf{R}_x$= [$\mathbf{R}_{x1}$ $\mathbf{R}_{x2}$], where $\mathbf{R}_{x1}$ and $\mathbf{R}_{x2}$ are matrices of dimension $(2N\text{-}1)\times K$ and $(2N\text{-}1)\times(2N\text{-}1\text{-}K)$, respectively. And the propagator matrix can be estimated by

$$\hat{\mathbf{P}}_{x} = (\mathbf{R}_{x1}^{H}\mathbf{R}_{x1})^{-1}\mathbf{R}_{x1}^{H}\mathbf{R}_{x2}, \tag{20}$$

then define a new matrix $\mathbf{P}_{c2} = \begin{bmatrix} \mathbf{I}_{K\times K} \\ \hat{\mathbf{P}}_{x}^{H} \end{bmatrix}$, and partition P_{c2} as

$$\mathbf{P}_{c2} = \begin{bmatrix} \mathbf{P}_{x2} \\ \mathbf{P}_{y2} \end{bmatrix}, \tag{21}$$

where $\mathbf{P}_{x2}$ and $\mathbf{P}_{y2}$ are the matrices of dimension $N\times K$ and $(N\text{-}1)\times K$, respectively.

Let $\mathbf{Q}_x$ denote the first N rows of $\mathbf{P}_{x2}$, then the k^{th} azimuth angle ϕ_k can be found from the maximum peaks of the following formula

$$\hat{\phi}_{k} = \arg\max_{\phi}\left\|\frac{1}{\mathbf{a}_{x}^{H}(\theta_{k},\phi)\mathbf{Q}_{x}\mathbf{Q}_{x}^{H}\mathbf{a}_{x}(\theta_{k},\phi)}\right\|, \tag{21}$$

3.3. Summary of the Algorithm

In this paper, the 2-D DOAs of multiple signals are estimated at two different stages. First, with the improved PM method [11], a new propagator matrix $\mathbf{P}_{c1}$ is constructed, which use fully all elements of the array. It can obtain good performance in 2-D DOA estimation of elevation angles, especially under low SNR situation. Second, the matrix $\mathbf{Q}_x$ is achieved to associate the azimuth angles with the corresponding elevation angles in the proposed method, and the peak searching procedure improves the performance of DOA estimation.

4. Numerical Results

Computer simulations are carried out to illustrate the performance of the proposed method. The elements of each ULA with N = 11 sensors were separated by a half-wavelength. 300 Monte Carlo trials were performed for each experiment.

In the first simulation, there are K=2 signals impinging on the array. The DOAs of elevation and azimuth angles are (45°, 45°) and (60°, 30°). The number of the snapshots is 200 with a SNR of 10 dB. Fig. 2 shows that the proposed method can estimate the DOAs of all the signals.

In the second simulation, there are K=2 signals impinging on the array from the DOAs (65°, 70°) and (40°, 55°). Fig. 3 shows the root mean square error (RMSE) of the joint elevation and azimuth angle estimation versus the SNR in dB, using the proposed method, the PM method and the joint SVD method.

The number of the snapshots is 500, and the SNR is varied from -5 dB to 30 dB. The RMSE is defined as

$$\mathrm{RMSE} = \sqrt{\frac{1}{PK}\sum_{p=1}^{P}\sum_{k=1}^{K}\left[(\hat{\theta}_k(p)-\theta_k)^2+(\hat{\phi}_k(p)-\phi_k)^2\right]}, \quad (23)$$

where P is the number of the independent trials. It can be observed that at RMSE = 0.2, the proposed method achieves a gain improvement of about 5 dB over the joint SVD method, and about 9.5 dB over the PM method.

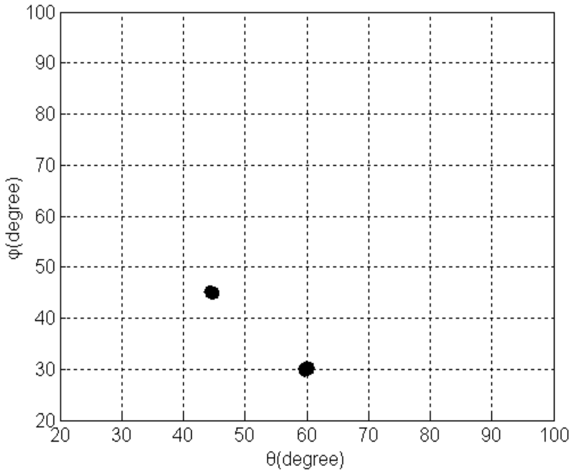

Fig. 2. Scatter plots of joint elevation and azimuth angle estimation for K=2 signals at (45°, 45°) and (60°, 30°) with 200 snapshots, SNR=10 dB.

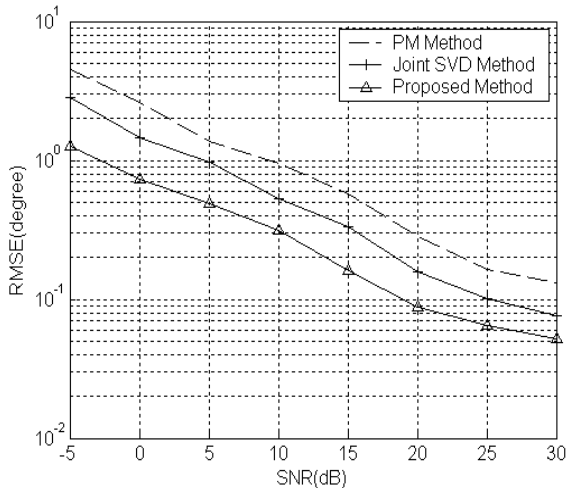

Fig. 3. RMSE of joint elevation and azimuth angle estimation versus SNR for K=2 signals with 500 snapshots.

In the third simulation, Fig. 4 describes the detection probability versus the number of signals. The snapshot is 100 and SNR is 3 dB, respectively. The result illustrates that the success rate of the proposed method is better than that of the joint SVD method and the PM method, when the number of snapshots is small and the SNR is low.

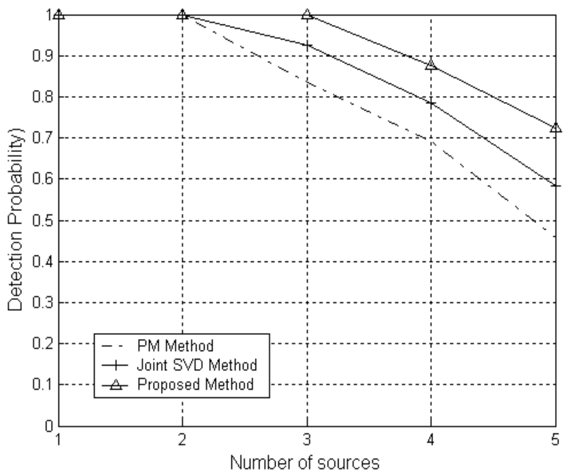

Fig. 4. Detection probability versus number of the coherent signals with 100 snapshots, SNR=3 d B.

5. Conclusion

A new method for estimating 2-D angles of wave arrival is proposed using L-shaped ULAs. The proposed method constructs a new propagator matrix to estimate the elevation angle, which improves the performance in low SNR. Then the automatic pair matching method is used to achieve the corresponding 2-D angles. Simulation results show that the proposed method has better 2-D DOA estimation accuracy.

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