Sensors & Transducers, Vol. 179, Issue 9, September 2014, pp. 212-215

Sensors & Transducers

Adaptive Identification for Switched Nonlinear Systems with Linear Parameterization

Xiaoping Zong, Miao Zhang, * Peiguang Wang
College of Electronic and Information Engineering, Hebei University, Baoding 071000, China
Tel.: +86 15933911071
* E-mail: 804258996@qq.com

Received: 11 May 2014 /Accepted: 29 August 2014 /Published: 30 September 2014

Abstract: This paper presents the adaptive control parameter identification of switched nonlinear system that uses model reference adaptive control (MRAC) method to track the variation of the state error to approach the ideal values. MRAC is treated as a class of switched nonlinear systems in which the unknown parameters appear linearly. At the same time, switched systems ensure the whole system stay stability and avoid vibration. The update laws are designed to change the controllers with the arbitrary switching signal so that the systems are closed to the model reference system. The parameters approach the real system parameters when the systems are stable, which achieves the control purpose. Simulation results show that the proposed method is validated. *Copyright © 2014 IFSA Publishing, S. L.*

Keywords: Nonlinear system, Parameter identification, Switched system, Model reference adaptive control.

1. Introduction

Adaptation has been used in the controller design for uncertain systems which are often parameter uncertain and unmodelled dynamics. After some decades development, adaptive control is increasingly applied to the stability of systems and dynamic performance analysis. At the same time, it also used to petrochemical engineering, aerospace and robots, etc. [1-3].

The system can achieve high-performance control since uncertain conditions is the issue which is the automatic control theory field concerning for long time. Until now, the scientists proposed many kinds of control strategies to solve unknown dynamical characteristics. From all of this achievements, when the dynamical characteristic structures are known and parameters are unknown, which satisfies the antecedent parameter conditions, we can use adaptive control to asymptotic track the error [4-6]. Parameter identification has the practical values, because adaptive control systems require online identification which can combine with industrial process control and practical model study. System identification is the method and theory, which establishes the mathematical model of the systems. System modeling and model identification are the basic question of all the problem of control [7, 8].

Switching system is a kind of complicated systems, which is a hybrid system that is composed of a family of continuous-time and discrete-time subsystems and a rule orchestrating the switching between the subsystems [9-11]. Switching and logic have been used to overcome some of the limitations in traditional adaptive control, such as separating the time scales associated with adaptation and the dynamics of the underlying process. Switching systems are always used to select the best controller from all the candidate controllers to enhance the control performance when the control processing.

The adaptive laws of traditional adaptive control for switched systems are composed of the state and state tracking error, because the same state are shared with all the subsystems and the estimation variables will update their values when the corresponding subsystems are inactive. While the method in this paper is that different adaptive laws are devised for both active period and inactive period of each subsystem so that the performance promotes to some extent [12-16].

This paper involves the estimated parameters of adaptive controller asymptotic convergent the true values, i.e. the parameter estimated error becomes zero; the controller is equal to model reference adaptive control feedback controller. Then using Lyapunov method to design adaptive controller and analyze the stability.

2. Controller Design and Stability Analysis

In this part, the error tracking issue for a class of switched nonlinear systems is formulated.

We assume the switched nonlinear system of the form

$$\dot{x} = -f_{\sigma}(x,t,\theta) + g_{\sigma}(x,t) \cdot u + \rho(x,t), \quad (1)$$

where $x \in R^n$, $u \in R^n$ are the state and input of the system respectively. $g_{\sigma}(x,t) \in R$ and $\rho(x,t) \in R^n$ are the known functions. To ensure the controllability of the system, we assume that there is the positive constant $g_0 \in R^+$ and $g_{\sigma}(x,t)$ satisfies that

$$\left| g_{\sigma}(x,t) \right| \geq g_0$$

And the structure of $f_{\sigma}(x,t,\theta) \in R^n$ is known, but the parameter $\theta \in R^m$ is the unknown function, it satisfies the linear condition

$$f_{\sigma}(x,t,\theta) = Y_{\sigma}(x,t)\theta_{\sigma},$$

where $Y_{\sigma}(x,t) \in R^{n \times m}$ symbols the known matrix, when $x(t) \in L_{\infty}$, $Y_{\sigma}(x,t) \in L_{\infty}$.

The SPR (strictly real positive) reference model is that

$$\dot{x}_M(t) = A_M x_M(t) + B_M r(t)$$

The purpose of control system is tracking the expect track $x_m \in R^n$, we define the tracking error

$$e = x_m - x, \quad (2)$$

The derivation of (2) and combine with (1), we can obtain that

$$\begin{aligned} \dot{e} = \dot{x}_m - \dot{x} = \dot{x}_m + f_{\sigma}(x,t,\theta) \\ - g_{\sigma}(x,t) \cdot u - \rho_{\sigma}(x,t) \end{aligned}$$

The problem switches to design a controller u_{σ} to guarantee the error e is asymptotic convergence. So we define that

$$u_{\sigma} = Y_{\sigma}(x,t)\hat{\theta}_{\sigma}(t) + ke(t), \quad (3)$$

where k is the positive gain, $\hat{\theta}_{\sigma}(t)$ is the estimation of the parameter of θ_{σ}.

So we can receive that

$$\dot{e} = Y_{\sigma}(x,t)\tilde{\theta}_{\sigma}(t) - ke(t), \quad (4)$$

where

$$\tilde{\theta}_{\sigma}(t) = \theta_{\sigma} - \hat{\theta}_{\sigma}(t), \quad (1)$$

Therefore, the adaptive laws are designed as

$$\dot{\hat{\theta}} = \Gamma Y^T(x,t)Pe(t)$$

where Γ and P are the symmetric positive matrixes.

Consider the Lyapunov function

$$V = \frac{1}{2}e^T Pe + \frac{1}{2}\Phi^T \Gamma^{-1}\Phi$$

where $\Phi = \begin{bmatrix} \tilde{\theta}_1 \\ \vdots \\ \tilde{\theta}_N \end{bmatrix}$ and $\Gamma = \begin{bmatrix} \Gamma_1 & & \\ & \ddots & \\ & & \Gamma_2 \end{bmatrix}$.

Taking the time derivative of V and using (4), we obtain

$$\begin{aligned} \dot{V} &= e^T P\dot{e} + \Phi^T \Gamma^{-1}\dot{\Phi} \\ &= e^T P(Y_{\sigma}(x,t)\tilde{\theta}_{\sigma}(t) - ke) - \tilde{\theta}_{\sigma}^T \Gamma^{-1}\dot{\hat{\theta}}_{\sigma} \\ &= \tilde{\theta}_{\sigma}^T \left[Y_{\sigma}^T(x,t)Pe - \Gamma^{-1}\dot{\hat{\theta}}_{\sigma} \right] - ke^T e \\ &= -ke^T Pe \leq 0 \end{aligned}$$

This implies that under arbitrary switching signal, $e \in L_2 \cap L_{\infty}$, $\Phi \in L_{\infty}$ and $\dot{\Phi} \in L_2 \cap L_{\infty}$.

According to Barbalat lemma, we can prove that $\lim_{t\to\infty} e(t) = 0$, which means that the tracking error asymptotic convergent zero.

3. Prove

For (4), according to the closed-loop characteristic of error, we can get the derivation of it and combining with (5)

$$\ddot{e}(t) = \dot{Y}_\sigma(x,t)\tilde{\theta}_\sigma(t) + Y_\sigma(x,t)\dot{\tilde{\theta}}(t) - k\dot{e}(t)$$
$$= \dot{Y}(x,t)\tilde{\theta}_\sigma(t0 - Y(x,t)\Gamma Y^T(x,t)e(t) - k\dot{e}(t)$$

So $\ddot{e}(t) \in L_\infty$, therefore $\dot{e}(t)$ uniform continuity. We also know that $\lim_{t\to\infty} e(t) = 0$, based on this and used Barbalat lemma we can get

$$\lim_{t\to\infty} \dot{e}(t) = 0$$

Combine with (5)

$$\lim_{t\to\infty} Y_\sigma(x,t)\tilde{\theta}_\sigma(t) = 0, \tag{6}$$

Define 1 (persistent excitation) for the signal $w(t) \in R^n$ can be used to arbitrary $t_0 \in R^+$ when there are positive constants α_1, α_2, $\delta \in R^+$ and $\alpha_1 I_n \le \int_{t_0}^{t_0+\delta} w(\tau)w^T(\tau)d\tau \le \alpha_2 I_n$, where I_n is the n degree unit matrix, so this called that $w(t)$ is persistent excitation.

Theorem 1 (parameter identification of adaptive control theorem) the defined adaptive control algorithm for (3) and (5) when the signal $Y(x,t)$ satisfies the persistent excitation, this algorithm is not only can achieve error asymptotic convergence, but also can get the parameter asymptotic identify the target, i.e. $\lim_{t\to\infty} \hat{\theta}_\sigma(t) = \theta_\sigma$.

From (6), we know

$$\lim_{t\to\infty} \left[Y_\sigma(x,t)\tilde{\theta}_\sigma(t)\right]^T Y_\sigma(x,t)\tilde{\theta}_\sigma(t) = 0$$
$$\lim_{t\to\infty} \tilde{\theta}_\sigma^T(t)\left[Y_\sigma^T(x,t)Y_\sigma(x,t)\right]\tilde{\theta}_\sigma(t) = 0$$

So we obtain that

$$\lim_{t\to\infty} \int_{t_0}^{t_0+\delta} \tilde{\theta}_\sigma^T(\tau)\left[Y_\sigma^T(x,\tau)Y_\sigma(x,\tau)\right]\tilde{\theta}_\sigma(t)d\tau, \quad = 0 \tag{7}$$

According to the persistent excitation of signal $Y_\sigma(x,t)$, obtain that

$$\lim_{t\to\infty} \int_{t_0}^{t_0+\delta} \tilde{\theta}_\sigma^T(\tau)\left[Y_\sigma^T(x,\tau)Y_\sigma(x,\tau)\right]\tilde{\theta}_\sigma(t)d\tau, \quad \ge \alpha_1 \int_{t_0}^{t_0+\delta} \tilde{\theta}_\sigma^T(\tau)\tilde{\theta}_\sigma(\tau)d\tau \ge 0 \tag{8}$$

According to (7) and (8) we know

$$\lim_{t\to\infty} \tilde{\theta}_\sigma(t) = 0$$

In other words, the system achieves parameter asymptotic identification $\lim_{t\to\infty} \hat{\theta}_\sigma(t) = 0$.

4. Example

Let $N = 2$ and the subsystems of the switched system are

$$\dot{x} = -0.45x^3 - 0.6x^2 + u(t)$$
$$\dot{x} = -0.42x^4 - 0.9x + u(t)$$

The reference model system is

$$\dot{x}_m = -x_M + r(t)$$

The reference input is

$$r(k) = \begin{cases} 1 & k = 1,2,\cdots,N \\ 0 & other \end{cases}$$

The initial conditions are chosen as

$$x_{M0}(k) = \begin{pmatrix} 0 & 0 \end{pmatrix} \quad x_0(k) = \begin{pmatrix} 1 & 0 \end{pmatrix}$$
$$\Gamma = \begin{pmatrix} 10 & 0 \\ 0 & 10 \end{pmatrix}$$
$$P = \begin{pmatrix} 20 & 0 \\ 0 & 20 \end{pmatrix}$$

Simulation results are described in Fig. 1-Fig. 4.

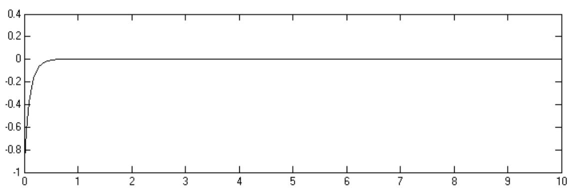

Fig. 1. Output error of the switched system.

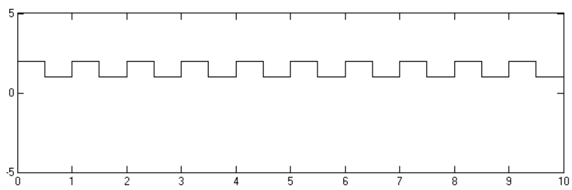

Fig. 2. Switching signal.

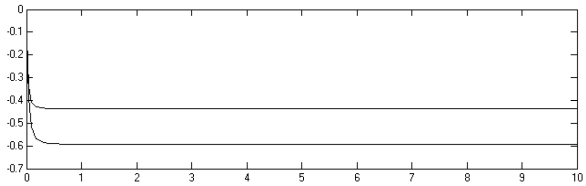

Fig. 3. Parameter identification of system 1.

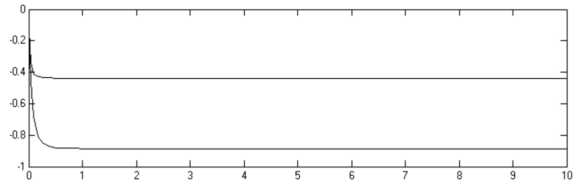

Fig. 4. Parameter identification of system 2.

5. Conclusions

The purpose of this paper is to design a controller with the adaptive laws for nonlinear system to achieve asymptotic identify the parameters, at the same time, switch the system which uses the switching function when switching signal is coming. From the simulation results, we can find that different switched systems have different identified processes. The problem is that there are slight errors between the estimated parameters and the real parameters, which is for the systems are not static error control systems. The stability of system which proves with Lyapunov function must find out a suitable adaptive law, so it needs to link relation between the controller and the Lyapunov function. When we find the suitable parameters, the system can approach the model reference system. There is still one problem which is the state error that we cannot observe in practical control. So the next work is to design a controller observer to track the state error.

Acknowledgements

The work presented in this paper is supported by projects of the National Natural Science Foundation of China (11271106).

References

[1]. M. L. Chiang, L. C. Fu. Variable structure model reference adaptive control of unknown switching linear systems with relative degree greater than one, in *Proceedings of the Joint 48th IEEE Conference on Decision and Control and 28th Chinese Control Conference*, Shanghai, P. R. China, 2009, pp. 4240-4245.

[2]. J. P. Hespanha, D. Liberzon, A. S. Morse, Overcoming the limitations of adaptive control by means of logic-based switching, *Systems & Control Letters*, Vol. 49, Issue 1, 2003, 49-65.

[3]. B. D. O. Anderson, A. Dehghani, Challenges of adaptive control: past, permanent and future, *Annual Reviews in Control*, Vol. 32, Issue 1, 2008, 123-135.

[4]. M. D. Bernardo, U. Montanaro, S. Santini, Novel switching model reference adaptive control for continuous piecewise affine systems, in *Proceedings of the 47th IEEE Conference on Decision and Control*, Cancun, Mexico, 2008, pp. 1925-1930.

[5]. Y. Q. Zheng, Y. J. Liu, S. C. Tong, Tie-Shan Li, Combined adaptive fuzzy control for uncertain MIMO nonlinear systems, in *Proceedings of the American Control Conference (AACC)*, St. Louis, MO, USA, 2009, pp. 4266-4271.

[6]. Wang Hao-Yu, Zhang Yun-Sheng, Zhang Guo, Implementation on system identification and adaptive control system simulation algorithm, *Control Engineering of China*, Vol. 15, S1, September 2008, pp. 77-80.

[7]. Z. Qu, Adaptive and robust controls of uncertain systems with nonlinear parameterization, *IEEE Transactions on Automatic Control*, Vol. 48, Issue 10, 2003, pp. 1817-1823.

[8]. M. L. Chiang, L. C. Fu, Variable structure adaptive backstepping control for a class of unknown switching linear systems, in *Proceedings of the American Control Conference (AACC)*, Baltimore, MD, USA, 2010, pp. 2476-2481.

[9]. Zhan Zhengtao, Yu Lei, Huang Jun, Multi-model switching control for SISO discrete systems, in *Proceedings of the Chinese Control Conference*, Xi'an, China, July 26-28, 2013, pp. 160-163.

[10]. Cheng Dai-Zhan, Guo Yu-Qian, Advances on switched systems, *Control Theory & Applications*, Vol. 32, No. 6, 2005, pp. 954-960.

[11]. Liu Xiangbin, Hou Zhongsheng, Jin Shangtai, Switching adaptive control of a class of non-affine nonlinear systems, in *Proceedings of the Chinese Control Conference*, Xi'an, China, July 26-28, 2013, pp. 2986-2991.

[12]. Sun Ming-Xuan, Yu Lin-Jiang, Adaptive iterative learning control of discrete time-varying systems, *Journal of Zhejiang University of Technology*, Vol. 41, No. 1, February 2013, pp. 84-90.

[13]. B. Ma, Y. Fang, X. Xiao, Switching logic based adaptive robust control of nonlinearly parameterized uncertain systems, *Journal of Automation*, Vol. 33, Issue 6, 2007, pp. 668-672.

[14]. M. L. Chiang, L. C. Fu, Variable structure adaptive backstepping control for a class of unknown switching linear systems, in *Proceedings of the American Control Conference (AACC)*, Baltimore, MD, USA, 2010, pp. 2476-2481.

[15]. Sun Ming-Xuan, Yan Qiu-Zhen, Error tracking of iterative learning control systems, *Acta Automatica Sinica*, Vol. 39, Issue 3, March 2013, pp. 251-262.

[16]. Lu Shuai, Su Hong-Ye, Liu Xiang-Bin, Liu Zhi-Tao, Adaptive robust control for a class of nonlinear parameterized systems, *Control Theory & Applications*, Vol. 29, No. 10, October 2012, (http://en.cnki.com.cn/Article_en/CJFDTOTAL-KZLY201210015.htm).