www.sensorsportal.com

Sensors & Transducers

Published by IFSA Publishing, S. L., 2017
http://www.sensorsportal.com

Trajectory Tracking Optimal Control for Nonholonomic Mobile Robot

Jianwei MA, Jiayu SHI and Haitao ZHANG
Henan University of Science & Technology, Luoyang, 471023, China
Tel.: 15236265350
E-mail: 137034342@qq.com

Received: 19 October 2016 /Accepted: 30 December 2016 /Published: 31 January 2017

Abstract: This paper discussed the problem of trajectory tracking control for the nonholonomic mobile robot's kinematic model with the angular speed of two actuated wheels as its control input. Based on the backstepping control algorithm, a sliding-mode variable structure switching function is designed, in order to improve the quality of motion control, fuzzy logic is used to adjust sliding-mode reaching law parameters. An adaptive control algorithm is proposed to compensate the unknown parameters, on the other hand. The stability of system is easily proven via the Lyapunov function. The simulation results are provided to illustrate the effectiveness of the controller.

Keywords: Mobile robot, Backstepping, Adaptive sliding mode control, Trajectory tracking.

1. Introduction

Motion control of mobile robots has been researchers' concern over the past few years. Many studies have proposed various trajectory tracking control methods considering the nonholonomic constrains of mobile robots. Kanayama, *et al.* [1] design a trajectory tracking control law only for linear systems. Walsh, *et al.* [2] designed a controller through linearizing nearby desired trajectory. Walsh, *et al.* [2] and Annesh [3] use the linearization idea of small disturbance of the model error to design controller, but the controllers do not achieve global stability control. Aneesh [3] designs a global tracking controller based on back-stepping nonlinear state feedback controller. Dong, *et al.* [4] use the backstepping method to determine nonlinear control law for the robot kinematics model and made sure the global asymptotic stability. Cheng [5] designs an asymptotic tracking controller based on global feedback control law. These methods regard the linear velocity of mass center and the steering angular speed as control input, and then design global tracking controllers. These approaches may have some errors, because the angular velocity of the two rear wheels is used as a control input in most cases. Hoang, *et al.* [6] and Wang, *et al.* [7] design trajectory tracking global asymptotically stable controllers and regard the two rear wheels as the control input for the kinematics model. But these controllers did not consider the influence of unknown parameters for trajectory tracking, they have limitations. Huang, *et al.* [8] consider the influence of unknown parameters, and adaptive control algorithm is used to achieve the robust controlling in that environment.

But in practice, the sliding mode control rate parameters will directly influence the control effect. The process of selecting parameters is a troublesome business, and it is not easy to find the best parameters. In this paper we adjust sliding mode reaching law parameters through fuzzy logic to realize the aim of improving the sliding mode motion quality.

Accordingly, this paper concentrates on the tracking control problem of mobile robots with unknown parameters. Based on the backstepping control algorithm, a switch function for variable structure is designed and an adaptive control method is presented to compensate the parameter uncertainties. Moreover, in order to improve the quality of motion control, fuzzy logic is used to adjust sliding mode reaching law parameters. And based on Lyapunov direct method, it's proved that the proposed controller can make globally asymptotically stable.

The paper is organized as follows: Kinematics model of mobile robot is given in Section 2. The trajectory tracking controller is proposed in Section 3, at the same time, the robustness and global stability of the proposed controller is proven by Lyapunov stability theory. Simulation results are shown in Section 4. Section 5 is devoted to concluding remakes.

2. Mobile Robot Kinematics Model

The mobile robot configuration is shown in Fig. 1. It has two differentially driven wheels and a front free wheel.

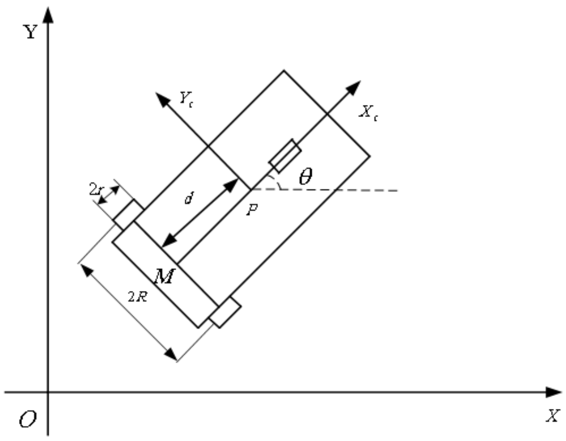


Fig. 1. Mobile robot coordinate system.

Where both wheels have the same radius denoted by r . The two driven wheels are separated by $2R$. M is located in the middle of two driven wheels. d is the distance between P and the axis of the driving wheels. The mobile robot can be described with pose vector: $q=[x \quad y \quad \theta]^{\mathrm{T}}$, where (x, y) is the centre of mobile robot in Cartesian coordinate system, and θ is the angle between the forward direction and the x-axis. The mobile robot satisfies the conditions of pure rolling and no slipping in wheels [9], and we can get:

$$\dot{x}\sin\theta-\dot{y}\cos\theta-d\dot{\theta}=0 \tag{1}$$

The kinematics model of nonholinomic mobile robot is described by the following differential equation.

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix}= \begin{bmatrix}\frac{r}{2}\cos\theta+\frac{r}{2R}d\sin\theta & \frac{r}{2}\cos\theta-\frac{r}{2R}d\sin\theta\\ \frac{r}{2}\sin\theta-\frac{r}{2R}d\cos\theta & \frac{r}{2}\sin\theta+\frac{r}{2R}d\cos\theta\\ \frac{r}{2R} & -\frac{r}{2R}\end{bmatrix}\bullet, \begin{bmatrix}\omega_1\\ \omega_2\end{bmatrix} \tag{2}$$

where ω_1 and ω_2 is the angular velocities of right and left wheels.

The relationship between linear and angular velocities is shown as

$$\begin{bmatrix}\omega_1\\ \omega_2\end{bmatrix}=\begin{bmatrix}1/r & R/r\\ 1/r & -R/r\end{bmatrix}\begin{bmatrix}v\\ \omega\end{bmatrix}, \tag{3}$$

where v is the heading linear velocity, ω is the heading angular velocity.

Substituting (3) into (2)

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix}=\begin{bmatrix}\cos\theta & d\sin\theta\\ \sin\theta & -d\sin\theta\\ 0 & 1\end{bmatrix}\begin{bmatrix}v\\ \omega\end{bmatrix} \tag{4}$$

In the mobile robot coordinate system, the position and orientation error can be defined as $p_e=(x_e \quad y_e \quad \theta_e)^{\mathrm{T}}$

$$p_e=\begin{pmatrix}x_e\\ y_e\\ \theta_e\end{pmatrix}=\begin{pmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}x_r-x\\ y_r-y\\ \theta_r-\theta\end{pmatrix} \tag{5}$$

Take the derivative $\dot{P}_e$ of P_e and mobile robot tracking error differential equation can be obtained as

$$\begin{bmatrix}\dot{x}_e\\ \dot{y}_e\\ \dot{\theta}_e\end{bmatrix}=v\begin{bmatrix}-1\\ 0\\ 0\end{bmatrix}+\omega\begin{bmatrix}y_e\\ -x_e+d\\ -1\end{bmatrix} +\begin{bmatrix}v_r\cos\theta_e+d\,\omega_r\sin\theta_e\\ v_r\sin\theta_e-d\,\omega_r\cos\theta_e\\ \omega_r\end{bmatrix} \tag{6}$$

If ω_1 and ω_2 is used to denote it, mobile robot tracking error differential equation can also be obtained as

$$\begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\theta}_e \end{bmatrix} = \omega_1 \begin{bmatrix} -\frac{r}{2} + \frac{r}{2R} y_e \\ -\frac{r}{2R} x_e + \frac{r}{2R} \\ -\frac{r}{2R} \end{bmatrix} + \omega_2 \begin{bmatrix} -\frac{r}{2} - \frac{r}{2R} y_e \\ \frac{r}{2R} x_e - \frac{r}{2R} \\ \frac{r}{2R} \end{bmatrix} + \begin{bmatrix} v_r \cos\theta + d\,\omega_r \sin\theta_e \\ v_r \sin\theta - d\,\omega_r \cos\theta \\ \omega_r \end{bmatrix}, \quad (7)$$

where v_r and ω_r denote the reference time-varying linear and angular velocity.

The trajectory tracking control problem is how to design proper angular velocities ω_1 and ω_2 respectively to make $p_e = (x_e \;\; y_e \;\; \theta_e)^{\mathrm{T}}$ globally asymptotically stable and satisfy $\lim\limits_{t\to\infty} \left\| (x_e \;\; y_e \;\; \theta_e)^{\mathrm{T}} \right\| = 0$.

Mobile robot kinematics model is a complex multi-input nonlinear system, and the switching functions can be designed based on back-stepping control algorithm.

3. Tracking Controller

3.1. Switching Functions

When $x_e = 0$, given model part Lyapunov function $V_y = \frac{1}{2} y_e^2$.

Suppose $\theta_e = -\arctan(v_r y_e)$, so

$$\dot{V}_y = y_e \dot{y}_e = -y_e x_e \omega - v_r y_e \sin(\arctan(v_r y_e)) \quad (8)$$

if and only if $v_r y_e = 0, \dot{V}_y \le 0$.

When $\theta_e = -\arctan(v_r y_e)$, y_e converges. Thus, if x_e and y_e can converge to zero and θ_e converges $-\arctan(v_r y_e)$ to, the state of the system is convergent.

According to this conclusion, the switching function can be designed as

$$s = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} x_e \\ \theta_e + \arctan(v_r y_e) \end{bmatrix} \quad (9)$$

So if we can make sure $s_1 \to 0$, $s_2 \to 0$, x_e can converge to zero, θ_e converge to zero, and we can achieve $y_e \to 0$ and $\theta_e \to 0$.

3.2. Sliding-Mode Controller

The method combining power reaching law with exponent reaching law can improve approaching quality, sliding mode reaching law can be designed as

$$\dot{s} = -k_1 |s|^{\alpha} \operatorname{sgn}(s) - k_2 s, \quad (10)$$

where $k_1 > 0, k_2 > 0$, $0 < \alpha < 1$.

In order to reduce the chattering, the continuous function can be used to replace sign function

$$\dot{s}_i = -k_{i1} |s_i|^{\alpha_i} \frac{s_i}{|s_i| + \delta_i} - k_{i2} s_i, i = 1, 2, \quad (11)$$

where δ_i is a positive value.

Substituting Equation (6) into (9), we can get

$$\dot{s} = \begin{bmatrix} \dot{s}_1 \\ \dot{s}_2 \end{bmatrix} = \begin{bmatrix} y_e \omega - v + v_r \cos\theta_e + d\,\omega_r \sin\theta_e \\ \omega_r - \omega + \frac{\partial \xi}{\partial v_r} \dot{v}_r + \frac{\partial \xi}{\partial y_e} \\ (-x_e \omega + v_r \sin\theta_e - d\,\omega_r \cos\theta_e + d\,\omega) \end{bmatrix}, \quad (12)$$

where $\xi = \arctan(v_r y_e)$.

Then we can get control law

$$q = \begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} y_e \omega + v_r \cos\theta_e + d\,\omega_r \sin\theta_e \\ + k_{11} |s_1|^{\alpha_1} \frac{s_1}{|s_1| + \delta_1} + k_{12} s_1 \\ [\omega_r + \frac{\partial \xi}{\partial v_r} \dot{v}_r + \frac{\partial \xi}{\partial y_e} (v_r \sin\theta_e - d\,\omega_r \cos\theta) \\ + k_{21} |s_2|^{\alpha_2} \frac{s_2}{|s_2| + \delta_2} + k_{22} s_2]/ \\ (1 + \frac{\partial \xi}{\partial y_e} x_e - \frac{\partial \xi}{\partial y_e} d) \end{bmatrix}, \quad (13)$$

where $\frac{\partial \xi}{\partial v_r} = \frac{y_e}{1 + (v_r y_e)^2}, \frac{\partial \xi}{\partial y_e} = \frac{v_r}{1 + (v_r y_e)^2}$.

k_{i1} and k_{i2} can be given directly, but this method does not take into account the approaching time and chattering. The process of selecting parameters is a troublesome business, and it is not easy to find the best parameters. In order to improve the quality of the motion, we introduce fuzzy logic to solve this problem [10].

3.3. Fuzzy Controller

Considering approaching law $\dot{s} = -k_1 |s|^{\alpha} \operatorname{sgn}(s) - k_2 s$ (k_1 and k_2 represent the approaching velocity), through real-time adjustment of two parameters, we can achieve fast and stable tracking control system.

We can select $|s_1|$ as the input variable of fuzzy controller, and k_{11} and k_{12} as the fuzzy controller's output variables. Fuzzy sets are defined as: VS (Very small), S (small), M (Medium), B (Big), VB (Very big). The size of input ($|s_1|$) and output (k_{11} and k_{12}) are selected as [0,4] and [0,100]. According to the relevant control experience, fuzzy control rules are designed as: when $|s_1|$ is VS, in fuzzy role, k_{11} and k_{12} should be VS. The fuzzy logic control rule is designed as Table 1:

Table 1. Fuzzy logical control rules.

$\lvert s_1 \rvert$	**VS**	**S**	**M**	**B**	**VB**
k_{11}	VS	S	M	B	VB
k_{12}	VS	S	M	B	VB

Another fuzzy logic control rule can be designed for another input variable $|s_2|$, and output variables k_{21} and k_{22} can be selected in the same way.

But the angular velocities ω_1 and ω_2 can't still be obtained due to the unknown parameters r and b .

3.4. Adaptive Controller

The adaptive algorithm can be used to estimate two unknown parameters. The control input ω_1 and ω_2 are designed as

$$\begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix} = \begin{bmatrix} \hat{\beta}_1 & \hat{\beta}_2 \\ \hat{\beta}_1 & -\hat{\beta}_2 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} \beta_1 + \hat{\beta}_1 & \beta_2 + \tilde{\beta}_2 \\ \beta_1 + \hat{\beta}_1 & -\beta_2 - \tilde{\beta}_2 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}, \quad (14)$$

where $\beta_1 = 1/r$, $\beta_2 = R/r$, $\hat{\beta}_1$ and $\hat{\beta}_2$ are their estimated value, $\beta_1 = \hat{\beta}_1 - \tilde{\beta}_1$, $\beta_2 = \hat{\beta}_2 - \tilde{\beta}_2$.

Substituting (14) into (7), we can get

$$\begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\theta}_e \end{bmatrix} = \left(1 + \frac{\tilde{\beta}_1}{\beta_1}\right) v \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} + \left(1 + \frac{\tilde{\beta}_2}{\beta_2}\right) \omega \begin{bmatrix} y_e \\ -x_e + d \\ -1 \end{bmatrix} + \begin{bmatrix} v_r \cos\theta_e + d\,\omega_r \sin\theta_e \\ v_r \sin\theta_e - d\,\omega_r \cos\theta_e \\ \omega_r \end{bmatrix} \quad (15)$$

The Lyapunov functional candidate is defined as

$$V = \frac{1}{2} s_1^2 + \frac{1}{2} s_2^2 + \frac{1}{2\gamma\beta_1} \tilde{\beta}_1^2 + \frac{1}{2\gamma\beta_2} \tilde{\beta}_2^2 \quad (16)$$

Clearly $V \geq 0$, differentiating (16), we can obtain that

$$\dot{V} = s_1 \dot{s}_1 + s_2 \dot{s}_2 + \frac{\tilde{\beta}_1}{2\gamma_1 \beta_1} \dot{\hat{\beta}}_1 + \frac{\tilde{\beta}_2}{2\gamma_2 \beta_2} \dot{\hat{\beta}}_2$$
$$= -k_{11} |s_1|^{\alpha_1} \frac{s_1^2}{|s_1| + \delta_1} - k_{12} s_1^2 - k_{21} |s_2|^{\alpha_2} \frac{s_2^2}{|s_2| + \delta_2} - k_{22} s_2^2 + \frac{\tilde{\beta}_1}{\gamma_1 \alpha_1} \left(\dot{\hat{\beta}}_1 - \gamma_1 s_1 v\right) + \frac{\dot{\hat{\beta}}}{\gamma_2 \beta} \left[\dot{\hat{\beta}}_2 + \gamma_2 s_2 y_e \omega - \gamma_2 \omega s_2 \left(1 + \frac{\partial \xi}{\partial y_e} (x_e - d)\right)\right. \quad (17)$$

In order to make sure $\dot{v} \leq 0$, the parameter adaptive law can be designed as

$$\dot{\hat{\beta}}_1 = \gamma_1 s_1 v$$
$$\dot{\hat{\beta}}_2 = -\gamma_2 s_2 y_e \omega + \gamma_2 \omega s_2 \left(1 + \frac{\partial \xi}{\partial y_e} (x_e - d)\right) + f\left(\hat{\beta}_2\right), \quad (18)$$

where $f\left(\hat{\beta}_2\right) = \begin{cases} 0 & \hat{\beta}_2 > \lambda \\ \left(\lambda - \hat{\beta}_2\right)\left(f_0 - \lambda\right)^2 & \hat{\beta}_2 \leq \lambda \end{cases}$,

$f_0 = -\gamma_2 s_2 y_e \omega + \gamma_2 \omega s_2 \left(1 + \frac{\partial \xi}{\partial y_e} (x_e - d)\right)$,

$0 < \lambda \leq \beta_2$.

Substituting (18) into (17), we can get

$$\begin{aligned}\dot{V} &= -k_{11}|s_1|^{\alpha_1}\frac{s_1^2}{|s_1|+\delta_1} - k_{12}s_1^2 \\ &\quad -k_{21}|s_2|^{\alpha_2}\frac{s_2^2}{|s_2|+\delta_2} - k_{22}s_2^2 + \frac{\tilde{\beta}_2-\beta_2}{\gamma_2\beta_2}f\left(\hat{\beta}_2\right) \\ &\le -k_{12}s_1^2 - k_{22}s_2^2 + \frac{\tilde{\beta}_2-\beta_2}{\gamma_2\beta_2}f\left(\hat{\beta}_2\right)\end{aligned} \tag{19}$$

When $\hat{\beta}_2 > \lambda, \dot{V} \le 0$, when $\hat{\beta}_2 \le \lambda$, $f\left(\hat{\beta}_2\right) \ge 0$ and $\hat{\beta}_2 - \beta_2 \le \lambda - \beta_2 \le 0$, thus $\dot{V} \le 0$.

Therefore, when s_1 and s_2 can converge to zero, $\lim\limits_{t\to\infty} y_e = 0$. When $v_r \ne 0, \lim\limits_{t\to\infty}\theta_e = 0$ and it satisfies $\lim\limits_{t\to\infty}\left\|\begin{pmatrix}x_e & y_e & \theta_e\end{pmatrix}^T\right\| = 0$. The adaptive trajectory tracking controller is established, and its conclusion is given as follows:

Theorem 1. Assume $\forall t \in [0,+\infty)$, $v_r, \dot{v}_r, \omega_r$ and $\dot{\omega}_r$ are bounded. The nonholonomic robot tracking error system (7) is globally asymptotically stable under the control law (14) and parameter adaptive law (18), which satisfies $\lim\limits_{t\to\infty}\left\|\begin{pmatrix}x_e & y_e & \theta_e\end{pmatrix}^T\right\| = 0$.

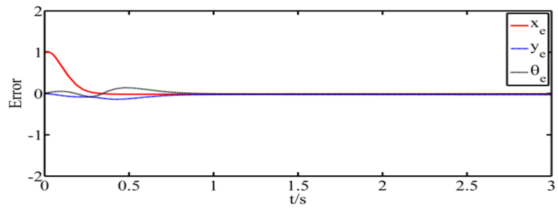
4. Simulation Research

In this section, a simulation is used to prove the effectiveness of the proposed controller which is designed in the previous section. The parameters of mobile robot are chosen as: $R = 0.75m$, $r = 0.15m$, $d = 0.3m$, where R and r are unknown. The desired trajectory are selected as $\omega_r = 1.0 rad/s$, $v_r = 2.0 m/s$, $x_r = r\cos(\omega_r t) = 2.0\cos t$, $y_r = r\sin(\omega_r t) = 2.0\sin t$, and $\theta_r = \omega_r t = t$. The parameters of controller are designed as: $\delta_1 = \delta_2 = 0.05$, $\alpha_1 = \alpha_2 = 0.5$, $\gamma_1 = \gamma_2 = 6$, the initial position and orientation error $P_e(0) = [1 \quad 0 \quad 0]$. The initial conditions are $\hat{\beta}_1(0) = \hat{\beta}_2(0) = 0$, when conventional controller is used, $k_{11} = k_{12} = k_{21} = k_{22} = 5$.

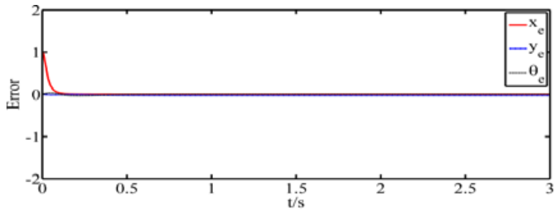
The simulation is carried out in MATLAB in 5 seconds. The comparison effect of the conventional sliding mode control and fuzzy sliding mode control after optimization are shown in Fig. 2 - Fig. 5.

Fig. 2 show that system position tracking error can converge to zero within 0.2 seconds under optimize control law, convergence speed is faster.

Fig. 3 show that actual velocities are almost confused with the desired velocities. The control law after optimization is better.

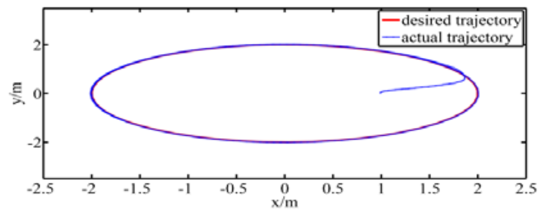


(a) Before optimization

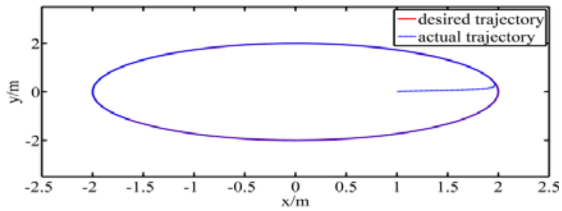


(b) After optimization

Fig. 2. The comparison of error.

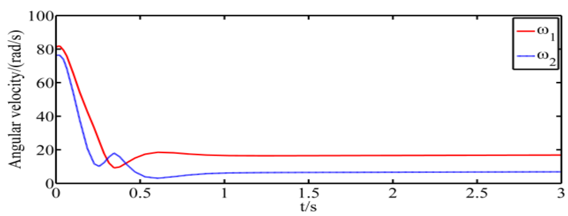


(a) Before optimization

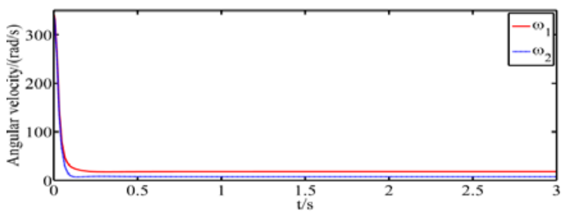


(b) After optimization

Fig. 3. The comparison of circular track position tracking.

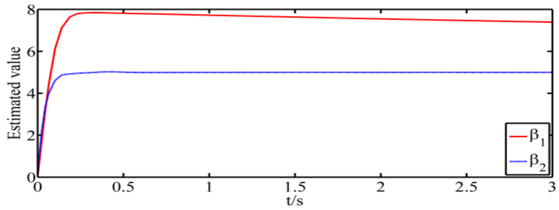


(a) Before optimization

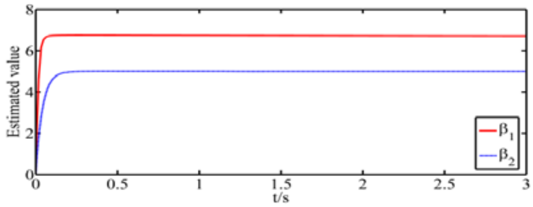


(b) After optimization

Fig. 4. The comparison of angular velocities control.



(a) Before optimization



(b) After optimization

Fig. 5. The comparison of parameter estimation.

Fig. 4 show that control input torque can converge to a global bound state faster under optimize control law.

Fig. 5 show that the estimated parameters can converge to a global bound state faster under optimize control law.

5. Conclusions

The mobile robot trajectory tracking control problem is considered in this paper. The sliding mode control method is first designed based on back-stepping. Then fuzzy logic is proposed so as to improve the quality of the motion. Furthermore, an adaptive sliding mode control strategy is proposed to compensate the parametric uncertainties, and the globally asymptotic stability of the proposed controller is proved by Lyapunov theory. Finally the simulation results demonstrate that the optimal controller has a better control effect, this scheme is effective.

References

[1]. Kanayama Y., Kimura Y., *et al.*, A stable tracking control method for an autonomous mobile robot, in *Proceeding of the IEEE Conference on Robotics and Automation*, 1990, pp. 184-189.

[2]. Walsh G., Tilbury D., Sastry S., *et al*., Stabilization of trajectories for systems with nonholonomic constrains, *IEEE Trans on Automatic Control*, Vol. 39, No. 1, 1994, pp. 216-222.

[3]. Divya Aneesh, Tracking controller of mobile robot, in *Proceeding of IEEE Conference on Computing, Electronics and Electronics Technologics (ICCEET)*, 2012, pp. 343-349.

[4]. F. Dong, W. Hinemann, R. Kasper, Nonlinear control design for row guidance system of an automated asparagus harvesting robot, in *Proceeding of the IEEE International Conference on Advanced Intelligent Macaronis*, 2011, pp. 1087-1092.

[5]. L. Cheng, Trajectory tracking control of nonholonomic mobile robots by back-stepping, in *Proceeding of the International Conference on Modeling Identification and Control*, 2011, pp. 134-139.

[6]. T. T. Hoang, D. T. Hiep, P. M. Duong, *et al.*, Proposal of algorithms for navigation and obstacles avoidance of autonomous mobile robot, in *Proceeding of the IEEE 8th Conference on Industrial Electronics and Applications*, 2013, pp. 1308-1313.

[7]. J. H. Wang, Z. G. Lu, W. H. Chen, *et al.*, An adaptive trajectory tracking control of wheeled mobile robots, in *Proceeding of the 6th IEEE Conference on Industrial Electronics and Applications*, 2011, pp. 1156-1160.

[8]. Dawei Huang, Junyong Zhai, Trajectory tracking control of wheeled mobile robots based on disturbance observer, in *Proceeding of the Chinese Automation Congress (CAC)*, 2015, pp. 1761-1765.

[9]. J. Barraquand, J. C. Latombe, Nonholonomic multibody mobile robots controllability and motion planning in the presence of obstacle, in *Proceeding of the IEEE International Conference on Robotics and Automation*, 1991, pp. 2328-2335.

[10]. R. Fierro, F. L. Lewis, Control of a nonholonomic mobile robot: backsteping kinematics into dynamic, *Journal of Robotic Systems*, Vol. 14, No. 3, 1997, pp. 149-163.