

Improvement of Accuracy in Multi-path Ultrasonic Flow Meters

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Abstract: A four-path ultrasonic flow meter was fabricated by mounting of ultrasonic transducers around a pipe. The fluid velocity measurement was carried out by using time difference method. The results were used to estimate the flow rate by Gauss-Legendre (G-L) and Chebyshev-Gauss (Ch-G) quadrature rules. The flow rate was also estimated by the equivalent single-path flow meter. The estimation results were compared with the reference measurements at the velocities from 0.1 m/s to 1.01 m/s. The result indicated that reducing the fluid velocity causes the measurement accuracy to decrease. This phenomenon is attributed to the absolute error of measurement devices. At the velocities below 0.4 m/s the error becomes very significant. At the high velocities, the estimation error of G-L method was about 3 %. On the other hand, the relative error of Ch-G method was 2 % in same flow rate. The relative error of the equivalent single-path meter was 50 % higher than that of four-path one in all flow rates. By extrapolating of the results, the relative error of Ch-G method in velocity beyond 2.0 m/s expected to be about 0.3 %.

Keywords: Ultrasonic flow meter, Multi-path flow meter, Gaussian quadrature, Chebyshev-Gauss, Gauss-Legendre, Relative error.

1. Introduction

For over 50 years, Ultrasonic Flow Meters (UFMs) have been used in various industries. Some of them include: medical [1-2], oil [3], gas [4-5], chemical [6], water management [7] and power plant [8]. Nowadays they are becoming the first choice for measurement of valuable fluid due to their several benefits in terms of precision, uninterrupted measuring, easy installation, low-cost and more [9]. Reliability and negligible depreciation, due to the lack of mechanical parts, are the other prominent features. The operation principle of these meters is based on interaction of ultrasonic wave and fluid. During application of these meters, an ultrasonic wave is injected into the moving fluid and its response is

received. Interaction by the fluid modifies some specifications of the wave and the changes are proportional to the fluid velocity. Thus, by measurement of the changes, the velocity of the fluid can be calculated explicitly. The calculated velocity is then used to estimate the flow rate [10].

UFMs are divided into two general categories, the Doppler meters and transit-time (time-of-travel) meters [3]. The Doppler flow meter (DFM)s use of well-known Doppler Effect [11], in which, a signal of known frequency is injected into the flow stream. Solids, bubbles, or any discontinuity in the fluid, cause the pulse to be reflected to the receiving device. The frequency of the reflected signal changes due to movement of the fluid. The frequency shift is proportional to the fluid's velocity. The main

constraint of Doppler meters is that the flow stream should contain solid particles or gas bubbles, and the pipeline must be acoustically transmissive. So they are not suitable for measurement of clean liquids. On the other hand, their accuracy and repeatability depends on the flow profile and also the particle size and concentrations [3].

The transit-time flow meter (TFM)s differ from Doppler ones in that, they rely on transmission of ultrasonic pulse through the flow stream. So, their performance does not depend on discontinuities or entrained particles in the fluid [3]. The limitation of these meters is that the fluid being measured must be relatively free of entrained gas or solids to minimize signal scattering and absorption. Therefore, the fluid with a high solid or gas bubbles contents cannot be metered by these devices. In the hybrid method the system automatically switches from transit time to Doppler method according to the flow condition. The accuracy of ultrasonic flow meters is in the order of $\pm (1-5) \%$. In general, Doppler flow meters are less accurate than transit-time meters; however, they are less expensive.

Over the years, extensive efforts have been made to improve the accuracy [12], reliability and sustainability of flow meters [13]. Particularly in the oil and gas industry, economic considerations along with the increasing oil production and consumption, justifies the allocation of funds for such researches. For improvement of accuracy several methods are reported [14-15]. Among them are: upgrading of transducers [16], modification of data acquisition hardware [17-19], and development of new data processing algorithms [20-21]. To improve the performance under complex flow profiles, the artificial neural network (ANN) based data integration method is proposed [23-24]. Utilization of new estimation methods is another efficient approach that proposed to improve the accuracy of these systems [16].

In this research, the effect of data analysis method on the accuracy of the TFMs is examined. To accomplish this goal, the flow equation inside the pipe is modified to estimate the flow rate by quadrature rule as numerical integration method. Two different quadratures were applied for estimation of flow rate. The analytical predictions were experimentally verified by fabrication of a four-path ultrasonic flow meter and a suitable test system. Details of fabrication and the test procedure have been already reported [25]. The result of the research indicates that the estimation technique has an effective role in improving the accuracy of transit-time flow meters.

2. Theory

2.1. Operation Principle of Transit Time Ultrasonic Flow Meter

A schematic of a single-path transit-time flow meter is presented in Fig. 1. The system consists of

two transducers A and B mounted on opposite side of a pipe. The velocity measurement is carried out by transmission of an ultrasonic signal between A and B, first in the direction of flow and then in opposite direction. Time of travel of the signal between the transducers is modified by the velocity of flow stream as below [18].

$$t_{B \rightarrow A} = \frac{L}{u \cdot \cos(\theta) - V}, \quad t_{A \rightarrow B} = \frac{L}{u \cdot \cos(\theta) + V}, \quad (1)$$

where V is the velocity of fluid, u is the velocity of sound in the medium and $L = D / \tan(\theta)$, in which D is the pipe bore diameter. $t_{B \rightarrow A}$ and $t_{A \rightarrow B}$ are the propagation times needed to signal travel from transducers B to A and A to B respectively, and the θ is the tilt angle between wave propagation path and the direction of the fluid flow.

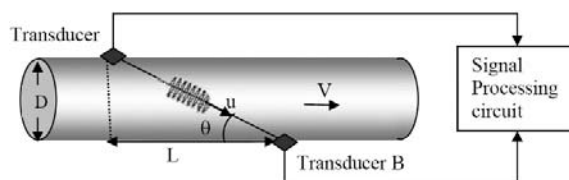


Fig. 1. Operation principle of a single path ultrasonic flow meter.

The travel time difference is

$$\Delta t = t_{B \rightarrow A} - t_{A \rightarrow B} = L \frac{2V}{(u \cdot \cos(\theta))^2 - V^2} \quad (2)$$

It's obvious that the time difference depends on the velocity of flow stream. So knowing θ and u , the measurement of time difference leads to the calculation of velocity of fluid. In this case, the calculated velocity depends on the relevant layer of fluid that encompassing the ultrasonic path. So, the measured value is susceptible to flow profile effects.

Based on fluid dynamics fundamentals, the fluid velocity in a pipe is not uniform and varies from the maximum value (V_{\max}) in the pipe centerline to a minimum (or zero) in its surface. On the other hand, in a pipe with constant cross-section, the flow rate Q is calculated by:

$$Q = V_{\text{avg}} \times A, \quad (3)$$

where V_{avg} is the average velocity of the fluid and A is the pipe cross-sectional area [19]. It is obvious that, the velocity calculated by Eq. (2) can not be considered as average velocity to calculate the flow rate.

There exist three fluid flow regimes in a pipe; these are laminar, transition and turbulent flow. In the laminar flow the velocity profile is parabolic in nature and the velocity at the centerline is twice the average

velocity [4] (i.e. $V_{max}=2 \times V_{avg}$). In fully turbulent flow, there are numerous empirical velocity profiles. Among those, the simplest and the best known is the power-law velocity profile which can be expressed as [26]:

$$V(r) = V_{max} \times \left(1 - \frac{r}{R}\right)^{\frac{1}{n}}, \quad (4)$$

where the exponent n is a constant whose value depends on the flow conditions. Fig. 2 shows the velocity profiles for turbulent flow for different n , and its comparison with the laminar flow. For turbulent flow after a sufficient straight pipe run, the flow profile becomes fully developed in which the velocity at the centerline is only about 1.2 times the mean velocity. The velocity profile for transitional flow is unknown and the V_{avg} can not be calculated easily [7]. However, by proper calibration, an empirical relationship can be found to obtain the average velocity.

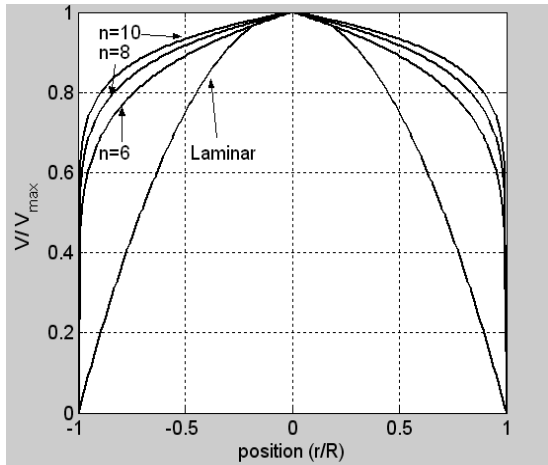


Fig. 2. Velocity profiles for turbulent flow in a pipe for different n , and its comparison with the laminar velocity profile.

On a pipe with constant cross-sectional area the V_{avg} is supposed to remain constant. However, the velocity in some applications may change somewhat because of changes in density with temperature. Therefore, the estimated flow rate is also environmental conditions dependent. This degrades the accuracy of the single-path flow meter. On the other hand, the single-path flow meter measures velocity across the center of the pipe, so its accuracy is susceptible to flow profile effect. To overcome this problem and improve the accuracy, the multi-beam approach is proposed, in which, the velocity is measured at the several chordal paths. This approach needs many pairs of ultrasonic transducers. They should be located at the specific positions around the pipe. The resultant device is known as multi-path ultrasonic flow meter. Schematics and operational principle of a typical four-path meter is presented in

Fig. 3. In this system, the measurement is performed over four predefined paths which are shown in Fig. 3 (c). Four pairs of ultrasonic transducers are needed for measurements.

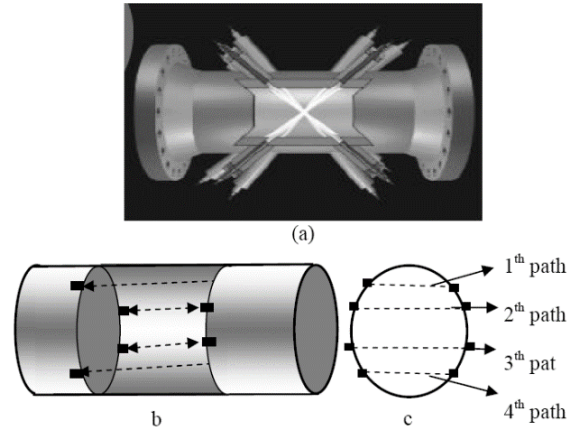


Fig. 3. (a) Operation principle of a four-path ultrasonic flow meter; (b) Schematic model, (c) Side view of paths.

Let consider the side view of the pipe at Fig. 4 with a thin horizontal chord passing the point $P(0, y)$ as fluid filament (gray ribbon parallel to X axis). The volume of fluid passing through the filament per second will be given by [10]:

$$q(y) = V(y)D(y)dy, \quad (5)$$

where $D(y)$ is the length dy is the thickness of the filament. R is the pipe bore radius and y is filament spacing from the pipe centerline. $V(y)$ is the average velocity of the fluid in the mentioned filament. The total amount of flow passing through the pipe can be calculated by:

$$Q = \int_{-R}^R q(r)dr = \int_{-R}^R D(r)v(r)dr \quad (6)$$

Putting $D(y) = 2\sqrt{R^2 - y^2}$ in 6 we have:

$$Q = \int_{-R}^R 2\sqrt{R^2 - r^2}v(r)dr \quad (7)$$

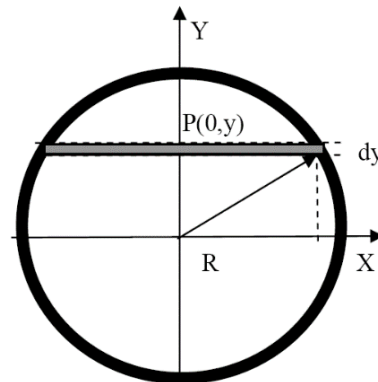


Fig. 4. The side view of fluid flow in the pipe at $P(0,y)$.

The calculation of the flow rate by Eq. (7) requires that the velocity of the fluid be measured at various intervals of different y . This needs infinite transducers to be used in all parallel chords of the pipe. In practice, the realization of this condition is not possible. This difficulty leads us to use quadrature rules as numerical integration algorithms.

2.2. Quadrature Rules

Quadrature is the process for approximating definite integrals of a given function. Typically this approximation takes the form of a weighted sum of function evaluations. An N -point quadrature rule is given by [27].

$$\int_{-1}^1 f(x)dx = \sum_{i=1}^N w_i f(x_i) \quad (8)$$

for some set of nodes $\{x_k\}$ and weights $\{w_k\}$.

There are several quadrature rules and choice of the best rule depends on the integrand function, $f(x)$ [28-29]. An N point Gaussian quadrature is a rule constructed to yield an exact result for polynomials of degree $2N - 1$ or less by a suitable choice of the points x_i and weights w_i , $i = 1, \dots, N$ [29]. This rule will only produce good results if the function $f(x)$ is well approximated by a polynomial within the interval $[-1, 1]$. The method is not suitable, for example, for functions with singularities. However, if the integrand is written as $f(x) = \omega(x).g(x)$, where $g(x)$ is a polynomial and $\omega(x)$ is known, then there are alternative weights w 's and points x 's that depend on the weighting function $\omega(x)$:

$$\int_{-1}^1 f(x)dx = \int_{-1}^1 \omega(x)g(x)dx \approx \sum_{i=1}^N w'_i g(x'_i) \quad (9)$$

To applying the Gaussian quadrature rule to (7), it is necessary to change the interval. The interval $[-R, R]$ can be replaced by $[-1, 1]$ using linear transformation of $x=r/R$ in Eq. (7). The result yield to:

$$\begin{aligned} Q &= \int_{-R}^R 2R^2 \sqrt{1 - \left(\frac{r}{R}\right)^2} v(r) d\left(\frac{r}{R}\right) \\ &= 2R^2 \int_{-1}^1 \sqrt{1 - x^2} v(Rx) dx \end{aligned} \quad (10)$$

If also assumed $f(x) = \sqrt{1 - x^2} v(Rx)$, then we have:

$$\begin{aligned} Q &= 2R^2 \int_{-1}^1 f(x)dx = 2R^2 \sum_{i=1}^n w_i f(x_i) \\ &= 2R^2 \sum_{i=1}^n w_i \sqrt{1 - x_i^2} v(Rx_i) \end{aligned} \quad (11)$$

This is the simplest integration problem with $\omega(x) = 1$, which is usually known as Gauss-Legendre (G-L) quadrature [30]. In this case the i^{th} evaluation node x_i , is the i^{th} roots of Legendre orthogonal polynomials, $P_n(x)$; and the weight coefficients are given by $w_i = 2/\left[(1 - x_i^2)(P'_n(x_i))^2\right]$. Although calculation of weight coefficients w_i from the mathematical relations are possible, but they are extensively tabulated in relevant references [30-31] for different n . Table 1 presents these values along with the roots of Legendre polynomials for $n=4$. By applying the G-L method to the four path system, the flow arte is approximated as follows:

$$\begin{aligned} Q &= R^2(0.3536808V(r_1) + 1.2265968V(r_2) \\ &\quad + 1.2265968V(r_3) + 0.3536808V(r_4)) \end{aligned} \quad (12)$$

where $r_i = R \times x_i$.

Now, consider another alternative in which, the weighting functions is $\omega(x) = \sqrt{1 - x^2}$ and $f(x) = 2R^2 v(Rx)$. In this case Eq. (10) can be expressed as:

$$Q = \int_{-1}^1 \sqrt{1 - x^2} f(x)dx \quad (13)$$

This is common format for Chebyshev-Gauss (Ch-G) quadrature (second kind) [28]. In this case the integral is approximated by:

$$\begin{aligned} Q &= \int_{-1}^1 \sqrt{1 - x^2} f(x)dx = \sum_{i=1}^4 w'_i f(x'_i) \\ &= \sum_{i=1}^4 w'_i 2R^2 v(Rx'_i), \end{aligned} \quad (14)$$

where

$$x'_i = \cos\left(\frac{i}{n+1}\pi\right) \quad i = 1, 2, \dots, n \quad (15)$$

and the weights coefficients are given by.

$$w'_i = \frac{\pi}{n+1} \sin^2\left(\frac{i}{n+1}\pi\right) \quad (16)$$

For $n=4$, nodes $\{x'_k\}$ and weights $\{w'_k\}$ are also presented in the third and fourth columns of Table 1. By applying the new constants, the flow arte can be approximated by:

$$\begin{aligned} Q &= R^2(0.434157 \times v(r'_1) + 1.1366388 \times v(r'_2) \\ &\quad + 1.1366388 \times v(r'_3) + 0.434157 \times v(r'_4)) \end{aligned} \quad (17)$$

where $r'_i = R \times x'_i$.

Table 1. Evaluation node x_i and weight coefficients for 4 path ultrasonic flow meter.

node	Gauss-Legendre rule		Chebyshev-Gauss rule	
	$x_i=r_i/R$	$2w_i\sqrt{1-x_i^2}\times R^2$	$x'_i=r_i/R$	$2w'_i\times R^2$
1	0.8611363	0.3536808R ²	0.8090170	0.4341574R ²
2	0.3399810	1.2265968R ²	0.3090170	1.1366388R ²
3	-0.3399810	1.2265968R ²	-0.3090170	1.1366388R ²
4	-0.8611363	0.3536808R ²	-0.8090170	0.4341574R ²

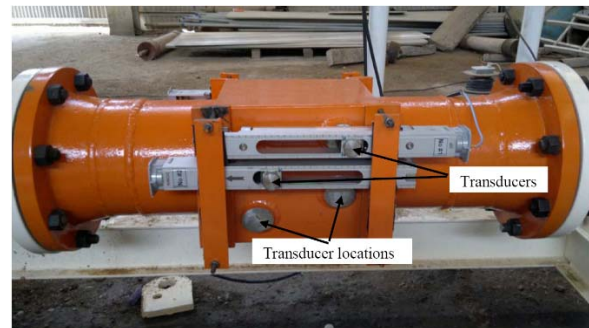
3. Experimental

3.1. Device Fabrication

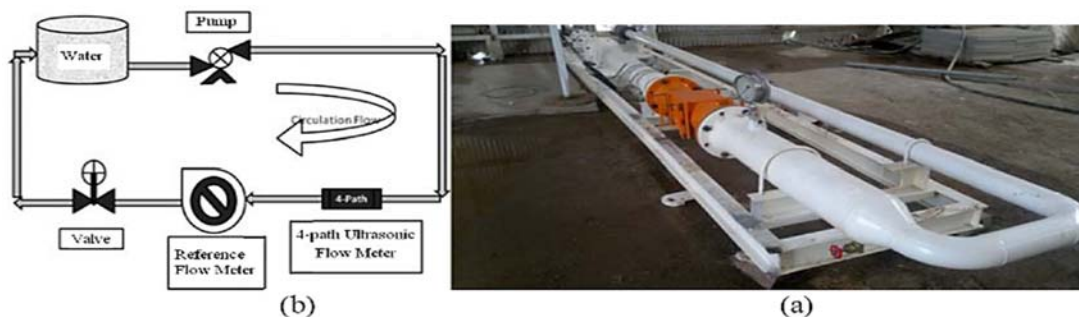
The analytical calculations were verified experimentally by design and fabrication of prototype four-path flow meters and suitable test system. The photograph of the fabricated prototype is presented in Fig. 5. It consists of a metal pipe with the bore diameter of 8 inch (20.32 cm). Appropriate locations are embedded on the outer wall of the pipe to be equipped with four ultrasonic transducers in each side. The positions of transducers are defined in accordance to G-L rule. So, putting $R= 10.16$ cm in Table 1, the distance of the sensors from the tube center, r_i , is determined as Table 2. Four pair of transducers are needed on each side, but in Fig. 5 just two transducers have been installed and two other places are left empty to present the sensor locations. The transducers used in this system are of OPTISONIC 6300 type. They are commercially available and manufactured by KROHNE Messtechnik GmbH & Co. KG.

These transducers are combination of OPTISONIC 6000 Clamp-on sensor(s) and a UFC300 ultrasonic flow converter. More details about fabrication of the system and the test procedure were already reported in reference [25].

In order to test fabricated meter, a test set up was installed as Fig. 6(a). The schematics presentation of the test system is shown in Fig. 6(b).

**Fig. 5.** The photograph of fabricated prototype flow meter.**Table 2.** Location of sensors and weight coefficients for 8 inch pipe.

node	Gauss-Legendre rule		Chebyshev-Gauss rule	
	r_i (cm)	w_i	r'_i (cm)	$2w'_i\times R^2$
1	8.749144808	0.003650891275	8.21961272	0.0044816158
2	3.45420696	0.012661619075	3.13961272	0.011733022
3	-3.45420696	0.012661619075	-3.13961272	0.011733022
4	-8.749144808	0.003650891275	-8.21961272	0.0044816158

**Fig. 6.** (a) Photograph of test facility, (b) Schematic presentation of the system.

The system acts as a closed water circulation loop. It consists of a water pump, a water storage tank, a control valve, and a precise mechanical flow meter.

All the system parts are connected in series with the prototype flow meter. To ensure the stability of the velocity distribution (profile) inside the pipe, the

appropriate distance between the control valve and meters is considered. The reference meter was (F4 PD Meter) type. It was manufactured by Smith Meter™ Inc. Based on factory test certificate, the accuracy of reference meter reported to be less than 0.1 %. It was just used as reference for comparison purposes and has no other roll on system operation. The pump circulates the water inside the system and the flow rate is measured and recorded simultaneously by the both flow meters. The control valve keeps the flow rate at the desired level.

3.2. Measurement

The test process starts by turning on the pump. The flow rate is then adjusted at the desired level by the control valve. The flow rate applied to the system was increased from 11 m³/h to 117 m³/h. Assuming the pipe diameter of 8 inches (20.32 cm), the corresponding average velocity of the fluid varies from 0.1 m/s to 1.01 m/s. Due to the fact that, the accuracy of the measurement depends on the pressure and temperature, all the experiments were carried out in the environment with constant pressure and temperature and a few possible deviations were ignored.

With any change of control valve, a time is allowed for the flow velocity to be stabilized. Then, the ultrasonic signal was injected into the fluid and its response was received. The transit time measurement was achieved by analyzing of the pulses. An example of the sent and received signals recorded by the Chauvin digital oscilloscope (Scopix III (OX-7104)) is shown in Fig. 7.

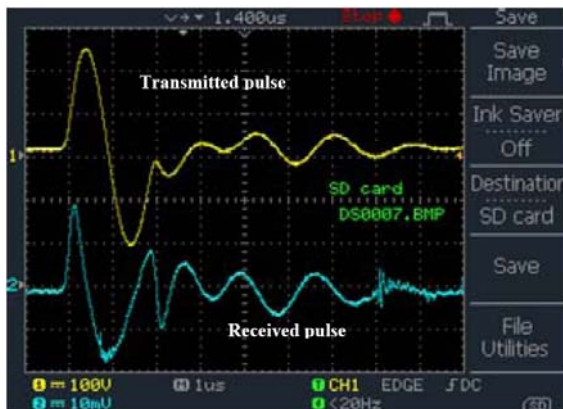


Fig. 7. The actual shape of sent and received ultrasound pulses.

Due to the significant time interval between sent and received signals, it is not possible to display them simultaneously on a same screen. So, the Fig. 7 is offline presentation the signals and their time dependencies are not included in time axes; (i.e. they are recorded and stored individually and then displayed in the same screen).

Based on the figure, the amplitude of the sent signal is in order of 100 V and the amplitude of the received signal is in order of 10 mV. Presentation of the received and sent pulses helps us to design and develop the relevant electronic systems for data acquisition, data conditioning and data processing. In our system, the data recording and calculation of transit time was performed simultaneously by commercial fast A/D and processors. More detail on the design considerations of the electronic system for data acquisition and processing are available in references [18-19, 32].

4. Results and Discussion

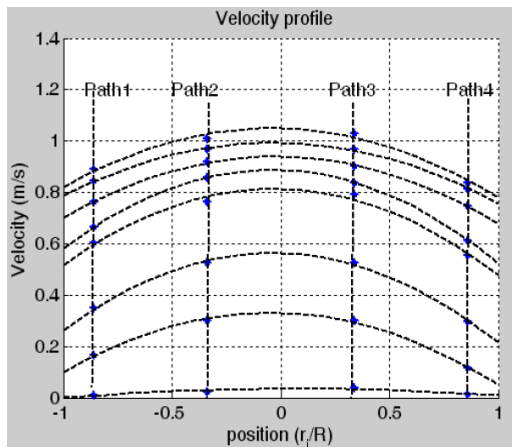
The measured velocities across the four paths are presented in Fig. 8(a). The measurement was carried out at different flow rates from 11 m³/h to 117 m³/h. Each point is obtained by performing up to four measurements and calculating their average. The unrealistic values have been removed from the results. To examine the similarity of the results with the laminar flow, the measured values were fitted by parabolic curves with different parameters. The fitting results are also presented as dashed curves. It is clear that, there is high degree of parabolic symmetry in low velocities. At high velocities, however, the velocity distribution deviates from parabolic. Although, these deviations do not have any effect in numerical integration.

The measured velocities were applied to Eq. (11) to estimate the flow rate through the pipe by G-L method as:

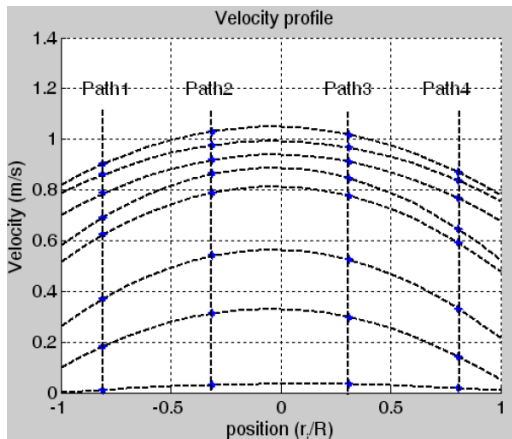
$$Q = 2R^2 \sum_{i=1}^4 w_i f(x_i), \quad (18)$$

where the w_i 's are the weight coefficients, and $f(x_i) = \sqrt{1-x_i^2} v(Rx_i)$, where x_i 's are as Table 2 and $v(Rx_i)$ is the measure velocity at i^{th} layer. Putting $R=10.16$ cm in Table 1, the weight coefficients corresponding to G-L rule will be obtained as second row of Table 2. The flow rates calculated by G-L method, along with the flow rate measured by the reference meter are shown in Fig. 9 as "-+-" curve.

In order to apply Ch-G quadrature rule, it is necessary to fabricate another prototype in which the distance of the transducers was defined according to the Ch-G rule (i.e. ± 0.8090170 and ± 0.3090170). In practice, this did not happen, instead, another solution was suggested in which the velocities at the positions corresponding to Ch-G rule was estimated by interpolation from the fitted curves of Fig. 8(a). The estimated results are presented in Fig. 8(b). In this case the weight coefficients w_i 's, are obtained from Eq. (16) by putting $R=10.16$. The calculated weight coefficients are presented in fourth column of Table 2. It is evident that the result obtained by this approach will be slightly different from the results of actual measurement.



(a)



(b)

Fig. 8. a) Measured velocities at positions of G-L rule; b) Estimated velocities at positions of Ch-G rule. The dashed lines presents the curve fitted to experimental data.

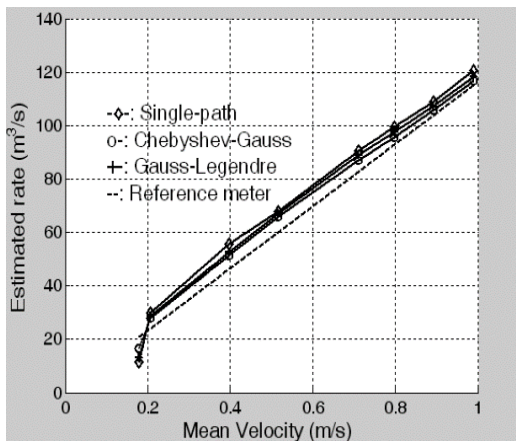


Fig. 9. Flow rate calculated by three systems and their comparison with values measured reference meter.

The estimated velocities in new points are applied to Eq. (14) to calculate the flow rate by Ch-G rule. The calculated flow rate by the Ch-G method is shown in Fig. 9 as "-o-" curve.

Finally, in order to compare the performance of the four-path systems with a single-path one, the flow rate was also estimated by equivalent single-path meter. To achieve this, the information of Fig. 8(a) were used to estimate the maximum velocity in the pipe centerline, V_{max} . Assuming that the fluid flow is turbulent, the mean velocity was calculated by $V_{avg} = V_{max}/1.2$. The V_{avg} were applied to Eq. (3) to estimate the flow rate of equivalent single-path meter. The results of the calculations, is also presented in Fig. 9 as "-.-" curve.

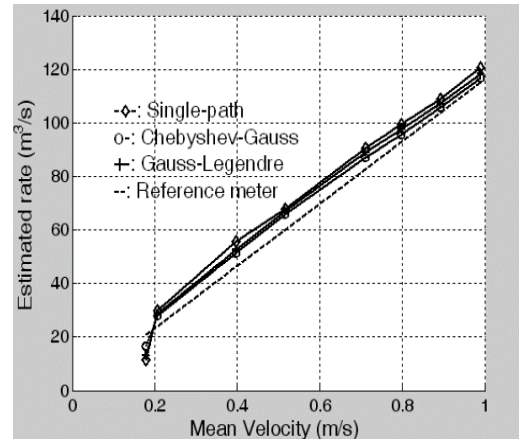


Fig. 9. Flow rate calculated by three systems and their comparison with values measured reference meter.

Comparing the results of flow rate calculation with the three methods mentioned above, in Fig. 9, shows that with increasing the velocity, the estimated results slowly approach the reference value presented as solid curve. On the other hand, these results clearly show that, the flow rate approximation by Ch-G method has the highest accuracy; the accuracy of approximation by G-J method is less than those approximated by Ch-G method. And the lowest accuracy is belongs to the equivalent single-path method. In order to have better comparison, the relative error is calculated as follow:

$$e = \frac{|f - f_r|}{f_r} \times 100, \quad (19)$$

where f is the estimated flow rate and f_r is the flow rate measured by reference meter. Variation of relative error verses flow rate is presented in Fig. 10. Based on this figure the relative error decreases with increasing of the velocity in all three methods. This is a common feature of flow meters, whose accuracy increases with increasing fluid velocity. The relative error of G-L method changes form 40 % at low rates to about 2 % in high rates. On the other hand, the relative error of Ch-G method changes form 20 % at low rates to about 1 % in high rates. This represents a significant improvement in measurement performance of Ch-G method compared to G-L method. The accuracy of single-path device is far from the four-path devices. In

this case the relative error is more than 45 % in low rates and 4 % in low rates. This is 30 % higher than G-L method and 50 % higher than Ch-G method.

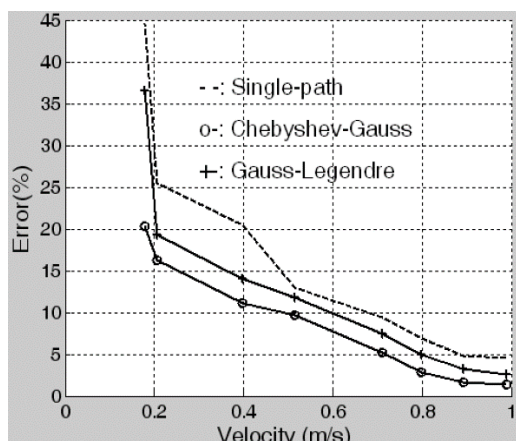


Fig. 10. Comparison of relative error for single-path and four-path flow meters estimated by Ch-G and G-L methods.

The trend of error reduction in Fig. 10 predicts that the measurement error of Ch-G method will reach to less than 0.3 % at higher velocity. This value is in the range of accuracy of commercial flow meters [19]. On the other hand, at very low velocities the relative error significantly increases. Therefore, for the low velocities the other measurement systems should be considered. The relative error reduction in low velocities can be attributed to the error of sensing devices. In fact, the absolute measurement error of ultrasonic transducers plays an important role in reducing relative error. As can be seen in Fig. 9, the absolute error (the distance estimated points from the reference line) is almost constant and decreases very slowly by velocity. Therefore, at low rates, when the fluid velocity is comparable to the error of the measurement device, the relative error suddenly begins to increase. This fact can be clearly deduced from Eq. (19); because the numerator of the equation, which is representative of the absolute error, is almost constant. Therefore, by decreasing the flow rate in the denominator, the relative error will increase. The absolute measurement error is inherent of the sensing elements and processing hardware. It is constant and independent from fluid velocity and estimation method. Due to this fact, the use of the system is not recommended at velocities below 0.6 m/s. We need more precise transducers to reduce the error at lower velocities.

5. Conclusion

By a mathematical model, the amount of fluid passing through the pipe was derived as a definite integral. According to the format of integrand equation, the Gauss-Legendre(G-L) and Chebyshev-

Gauss(Ch-G) quadrature rules were employed to approximate the flow rate. The analytical results were verified experimentally. A four-path ultrasonic flow meter was fabricated and the average velocities along the paths were measured by transit-time method. The measured results were applied to mathematical model to estimate the flow rate. The measurements were performed at different fluid velocity from 0.1 m/s to 1.01 m/s. The flow rate was also estimated by an equivalent single-path meter. The comparison of analytical results and empirical measurements performed by a reference meter showed that the measurement error is almost constant and changes very slowly by fluid velocity; on the other hand, the relative error increases at low fluid velocity in all methods. This phenomenon is attributed to the absolute error of measurement devices. At the velocities below 0.6 m/s, this increase becomes very fast.

The relative error of Ch-G method at velocity of 1.0 m/s was less than 2 %. On the other hand, the relative error of G-L method was about 3 %. This is 50 % higher than that of Ch-G method. The relative error of equivalent single-path meter in same velocity was about 4 % which is twice the error of the Ch-G method. The trend of error reduction with velocity predicts that the relative error of Ch-G method at higher velocities will be less than 0.3 %. This is comparable with the accuracy of commercially available flow meters. On the other hand, at the velocities lower than 0.6 m/s the relative error of Ch-G method will be higher than 7 % and the relative error of G-L method is about 10 %. Due to this fact the use of the system is not recommended at fluid velocity below 0.6 m/s. More precise transducers needed for lower velocities.

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