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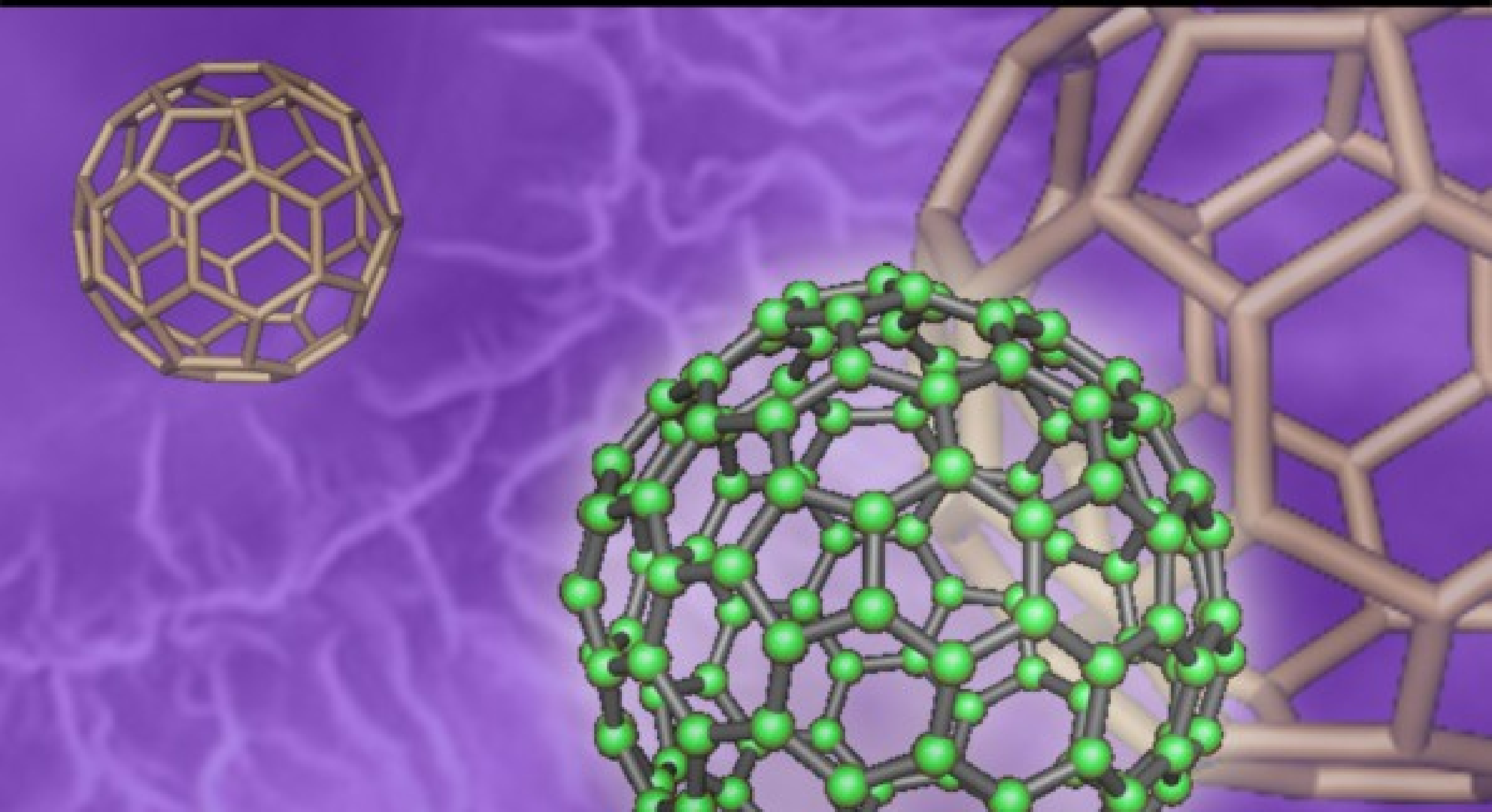

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Pull-in Phenomena and Dynamic Response of a Capacitive Nano-beam Switch

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Abstract: In this paper static and dynamic responses of a nano-beam subjected to the electrostatic force have been studied. For this purpose, the governing nonlinear equations for static and dynamic behavior of a nano-beam have been obtained. Due to the nonlinearity of electrostatic, van der Waals (vdW) and Casimir forces, the governing differential equation for static analysis has been linearized using step by step linearization method (SSLM) and the developed linearized equation has been discretized using Galerkin weighted residual method. Dynamic responses have also been studied using linearized form of the Galerkin based reduced order model. In this model, nonlinear force terms have been taken into account using an iteration procedure. Using this model, dynamic response of a nano-switch to a stepwise DC voltage excitation at the presence of Casimir and vdW forces has also been studied. The results show that the Casimir force for some orders of geometric properties has more effect on the static pull-in voltage than the vdW force; and therefore the effect of the vdW force, in some cases can be ignored. The results also show that un-damped dynamic pull-in voltage at the presence of Casimir and vdW forces is 89 % of the static pull-in voltage, whereas, this ratio rises to 90.8 % when the effect of these forces has been ignored. By considering damping effects, pull-in voltage increases up to a definite value of DC voltage. At this level, the nano beam tends to be critically damped and the dynamic pull-in voltage approaches the static pull-in voltage limit. *Copyright © 2009 IFSA.*

Keywords: NEMS, Nanobeam, Static pull-in, Dynamic pull-in, Casimir force, Van der Waals force

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1. Introduction

Nowadays, fast development of micro/nano scale technologies made it possible and also prominent to fabricate very useful and supper precise devises. MEM/NEM devices such as, micro switches [1, 2], micro accelerometers [3], micro resonators [4],micro mirrors [5], nano tweezers [6], super sensitive sensor [7, 8] and random access memories [9] are widely designed, analyzed, fabricated and used.

Most components of MEM/NEM devices act based on electrostatic actuation of a conductive flexible beam/plate suspended over a ground plate. In these devices, the Pull-in phenomenon can be faced as a side affect due to the overload. The nonlinear nature of electrostatic actuation also adds to the complexity of this problem.

In studying the static equilibrium of these devises, it has been observed that when electrostatic forces/torques is less than a critical value, after any disturbance, flexible beam/part of the system reaches a stable state. With increasing the applied voltage, the induces electrostatic forces/torques overcomes the existing forces/torques so that the flexible part of the system deforms and collapse on the fixed electrode. This phenomenon is known as “Pull-in phenomenon” and the associated minimum voltage which cause pull-in, is called “Pull-in voltage”.

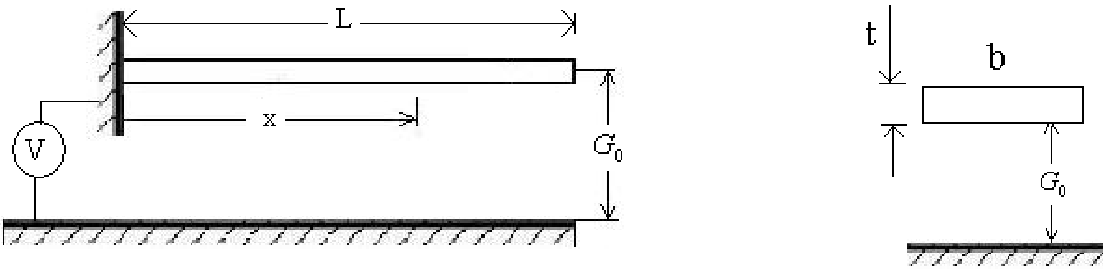
In NEM systems, by decreasing the geometric dimensions, Casimir and van der Waals (vdW) effects play major role especially in terms of the mechanical behavior of these systems. The Casimir force represents the attraction of two uncharged material bodies due to the modification of the zero-point energy associated with the electromagnetic modes in the space between them [10]. The well known example is two conductive parallel plates that are pulled towards each other with a force proportional to d^{-4}, where d is the distance between two plates. The vdW force is related to the electrostatic interaction between dipoles at the atomic scale [11]. One of the most important differences between these two forces is that the value of Casimir force between two plates depends only on the geometry of the objects whereas the vdW force depends on both the material properties and the distance between two plates. Hendrik Casimir predicted Casimir effects in 1948 for perfectly conductive parallel plates [12] and Kenneth *et al.* have extended these calculations to real-world materials [13]. Although these forces have been explored for decades, but they are rarely considered in analyzing the mechanical behavior and in calculating the Pull-in voltage of the NEM systems especially the nano tube structures. For instance Dequesnes, et al. [14] have studied the Pull-in phenomena and pull-in voltage of a carbon-based nano-electromechanical tube-switch without considering the effects of Casimir force. Also W.H. Lin, et al. [15] have studied the pull-in phenomena and calculated the pull-in voltage for a nano-electromechanical switch with the assumption of one degree of freedom for a nano switch without taking into account the effect of vdW force.

In this paper, a nano scaled capacitive switch is studied and the transverse bending of the nano-beam for this switch is investigated under the existing electrostatic force. Secondly a stepwise DC voltage is assumed to be applied. In both assumptions, the effects of the Casimir and vdW forces are considered. Continuum domain method, which is reliable enough in these issues, is considered and by implementing the well known Euler-Bernoulli beam theory, some nonlinear differential equations are obtained. The equations of the static deflection have been linearized using the step by step linearization method (SSLM). Then, Galerkin weighted residual method is used to discretize obtained linearized equation. The dynamic equations including the effect of the stepwise DC voltage excitation is solved using Galerkin based reduced order model and nonlinear forcing terms are accounted using an iteration procedure for different damping ratios.

2. Model Description

In Fig. 1 a schematic view of a NEM switch is illustrated which consists of a nano beam suspended over a stationary conductor plate, with length *L*, and initial gap of G_0. Fig. 1.b shows a cross section of the nano-beam with thickness of *t* and width of *b*. Casimir, vdW and attractive electrostatic forces due to the applied voltage pull nano-switch down towards the substrate. Minimum length of the nano-beam in which only Casimir and vdW forces lead nano-beam to collapse on the substrate (in lack of the electrostatic force) is known as detachment length.

The nano-beam is considered isotropic with Young modulus *E*, density ρ and cross section inertia moment of *I*.



(a) A beam-based NEM switch.

(b) Cross section of the nano beam.

Fig. 1. Schematic view of a beam-based NEM switch.

3. Mathematical Modeling

Based on the continuum beam theorem, governing equation for dynamic response of the nano beam can be obtained using well-known Euler-Bernoulli equation [16]:

$$EI\frac{\partial^4 w}{\partial x^4} + \rho S\frac{\partial^2 w}{\partial t^2} + f\frac{\partial w}{\partial t} = q_{ext} , \tag{1}$$

where *w*(*x*, *t*) is the flexural deflection of the beam, *f* is an equivalent damping ratio, *S* is the cross section area of the nanostructure and q_{ext} is the summation of the electrostatic, vdW and Casimir forces so that:

$$q_{ext} = q_{elect} + q_{vdW} + q_{casimir} \tag{2}$$

When the actuating voltage is applied between the nano beam and substrate, the electrostatic force per unit length are computed using a standard capacitance model [17] and is equal with:

$$q_{elect} = \frac{\varepsilon_0 b V^2}{2(G_0 - w)^2} \tag{3}$$

The Casimir force per unit length between nano beam and substrate is given by [12]:

$$q_{caimir} = \frac{b\pi^2\hbar c}{240(G_0 - w)^4} \tag{4}$$

and the vdW force per unit length of nano beam is given by [18]:

$$q_{vdW} = \frac{Ab}{6\pi(G_0 - w)^3} \tag{5}$$

For equations 3-5 $\varepsilon_0 = 8.854 \times 10^{-12} C^2 N^{-1} m^{-2}$ is the permittivity of vacuum within the gap, *b* is the width of nano beam, *V* is the electrical potential difference within the beam and ground, G_0 is the initial gap between the beam and substrate, $\hbar = 1.055 \times 10^{-34} Js$ is the Planck's constant , $c = 2.998 \times 10^8 ms^{-1}$ is the speed of light, and $A = \pi^2 C \rho_1^2$ is Hamaker constant which lies in the range of $(0.4 - 4) \times 10^{-19} J$ [19].

The governing equation for static deflection of the nano-beam from Euler-Bernoulli beam model is given as:

$$EI\frac{d^4 w}{dx^4} = q_{ext} \ , \tag{6}$$

where *w* is the displacement of the neutral axis.

4. Numerical Approach

Due to the nonlinearity of the governing equations, the analytical solution methods can not be used to obtain the results, so in order to solve these equations, the SSLM [20, 21] method has been used to linearize the equations. Using this method, the smooth and continuous behavior of the beam can be approximated in each step and the amount of nonlinear forces in each step will be obtained from foregoing iterations. Different techniques to apply this method to calculate the static and dynamic response of the nano-beam will be discussed below.

4.1. Static Deflection

The use of static SSLM demands smooth force application. In the case of electrostatic force, the voltage can be increased from zero to the final value gradually so that it satisfies the quasi-equilibrium condition but the vdW and Casimir forces depended on the gap size and their value can be determined based on the gap-size. In order to solve this problem, it has been assumed that these forces have been applied gradually in a virtual manner. For this purpose, a virtual variable "λ" has been introduced which changes from zero to one [21]. Multiplying this variable to each of these forces, the assumption of the step by step change of the loads can be satisfied.

It is supposed that w^i is the displacement of nanostructure when subjected to V^i under the virtual nano scaled force of q_{ext}^i. With increasing the voltage and consequent virtual force variable (λ) the deflection of (i+1)th can be obtained as:

$$V^{i+1} = V^i + dV \ \& \ \lambda^{i+1} = \lambda^i + \delta\lambda \ \Rightarrow \ w^{i+1} = w^i + \delta w = w^i + \varphi^i \ , \tag{7}$$

where dV and $\delta\lambda$ are the voltage and virtual force variable changes between two successive steps respectively. By considering a small value of dV and $\delta\lambda$ the φ^i will be small enough that we can use the first two term of the Taylor expansion in each step instead of exact main excitation function. The equation for ith step is:

$$EI\frac{d^4w^i}{dx^4} = q_{elect}(V^i, w^i) + \lambda^i\left(q_{vdW}(w^i) + q_{casimir}(w^i)\right) \tag{8}$$

and for (i+1)th step:

$$EI\frac{d^4w^{i+1}}{dx^4} = q_{elect}(V^{i+1}, w^{i+1}) + \lambda^{i+1}\left(q_{vdW}(w^{i+1}) + q_{casimir}(w^{i+1})\right) \tag{9}$$

Substituting w^{i+1}, V^{i+1} and λ^{i+1} in equation 9 and using Taylor expansion, the equation changes to the form of:

$$\begin{gathered}EI\frac{d^4w^i}{dx^4} + EI\frac{d^4\varphi^i}{d^4x} = q_{elec}(V^i, w^i) + \lambda^i\left(q_{vdW}(w^i) + q_{casimir}(w^i)\right) + \\ \left(\left.\frac{\partial q_{elect}}{\partial w}\right|_{w^i} + \lambda^i\left.\frac{\partial q_{vdW}}{\partial w}\right|_{w^i} + \lambda^i\left.\frac{\partial q_{casimir}}{\partial w}\right|_{w^i}\right)\varphi^i + \frac{\partial q_{elect}}{\partial V}dV + \left(q_{vdW}(w^i) + q_{casimir}(w^i)\right)\delta\lambda\end{gathered} \tag{10}$$

With subtracting equation 8 in 10:

$$\begin{gathered}EI\frac{d^4\varphi^i}{d^4x} - \left(\left.\frac{\partial q_{elect}}{\partial w}\right|_{w^i} + \lambda^i\left.\frac{\partial q_{vdW}}{\partial w}\right|_{w^i} + \lambda^i\left.\frac{\partial q_{casimir}}{\partial w}\right|_{w^i}\right)\varphi^i = \\ \frac{\partial q_{elect}}{\partial V}dV + \left(q_{vdW}(w^i) + q_{casimir}(w^i)\right)\delta\lambda\end{gathered} \tag{11}$$

Equation 11 is a linear ordinary differential equation that represents the variation of deflection along the nano-beam. This equation can be solved using Galerkin Method in which $\phi(x)$ can be expressed as:

$$\phi(x) = \sum_{i=1}^{\infty} q_i\psi_i(x) \tag{12}$$

where $\psi_i(x)$ is the ith shape function satisfying the boundary conditions of the nano-beam. The unknown $\phi(x)$ is approximated by truncating the summation series to a finite number, N:

$$\phi_N(x) \cong \sum_{i=1}^{N} q_i\psi_i(x) \tag{13}$$

By substituting the equation 13 into equation 11 and multiplying by $\psi_j(x)$ as a weight function in Galerkin method and integrating the outcome, a set of algebraic equations will be obtained. Solving the set of algebraic equations, deflection of the nano-beam at any given applied voltage can be determined.

4.2. Dynamic Analysis

The dynamic response of the nano beam has also been studied by calculating the dynamic pull-in voltage for a nano-beam under step-wise DC voltage actuation.

Assuming $w(x,t)=\sum_{i=1}^{N} q_i(t)\psi_i(x)$ for equation 1 and using Galerkin approximation method, the equation of dynamic response has been obtained as:

$$[M][\ddot{q}]+[f][\dot{q}]+[K][q]=[F]\ , \tag{14}$$

where:

$$\begin{aligned} &[M]_{ij}=\rho S\int_{0}^{L}\psi_i(x)\psi_j(x)dx\ ,\ [f]_{ij}=f\int_{o}^{L}\psi_i(x)\psi_j(x)dx \\ &[K]_{ij}=EI\int_{o}^{L}\frac{d^4\psi_i}{dx^4}\psi_j dx\ ,\ [F]_i=\int_{0}^{L}q_{ext}\psi_i(x)dx \end{aligned} \tag{15}$$

are the effective mass, damping, spring and actuating force matrixes respectively. $q(t)$ can be obtained from above set of ordinary differential equations using an integration scheme.

5. Results and Discussions

5.1. Validation and Convergence of Numerical Method

In order to validate the numerical solution, the static and dynamic response of a fixed-fixed nano-beam of a micro switch without residual stress has been calculated and the results have been compared with those obtained by Rezazadeh, et al. [20]. The dimensions and physical properties of the micro-switch nono-beam is given in Table 1. The results of this comparison are presented in Table 2. As shown in this table, the results have good agreement.

Table 1. Geometrical and material properties of micro switch.

Properties	Value
Length	350 μm
Width	50 μm
Height	3 μm
Young's modulus	169 GPa
G_o	1 μm

Table 2. The values of calculated pull-in voltage for the fixed-fixed micro switch.

dV	2	1	0.5	0.1	0.05
$V_{pull-in}$ obtained by Rezazadeh, et al.[20]	22	21	20.5	20.2	20.2
$V_{pull-in}$ - calculated	22	21	21	20.3	20.25

5.2. Convergence of the Results

The convergence of the solution has been controlled using different number of grid points. For this purpose the pull-in voltage and deflection of a fixed-fixed end nano-beam has been obtained for different $\delta\lambda$'s and at the presence of the Casimir and vdW forces but without electrostatic excitation. The physical properties of the fixed-fixed end nano-beam are: Young' Modulus *E* is 169 GPa, density of the material $\rho = 2300 {kg}/{m^3}$. The dimensions of the nano-beam are: *length L =70 nm,* G_0= 10 nm, width *b* =6 nm and height *t* = 6 nm. The results are presented in Table 3.

Table 3. The values of $\delta\lambda$ for calculated initial beam deflection without electrostatic excitation.

$\delta\lambda$	0.5	0.25	0.2	0.1	0.02	0.01	0.005
Dimensionless center deflection ($\frac{w(L/2)}{G_0}$)	0.007	0.1	0.011	0.012	0.013	0.013	0.013

The convergence of pull-in voltage under different values for *dV* and number of grid points, using $\delta\lambda = 0.01$ are presented in Table 4 and 5 respectively.

Table 4. The obtained pull-in voltage using 100 grid points and $\delta\lambda = 0.01$ for different dV.

dV	1	0.5	0.2	0.1	0.05	0.02	0.01	0.005
$V_{pull-in}$	7	6.5	6.4	6.4	6.3	6.28	6.28	6.28

Table 5. The obtained pull-in voltage using $dV = 0.01$ and $\delta\lambda = 0.01$ for different number grid points.

Number of gird Points	2	4	10	20	50	100
$V_{pull-in}$	7.3	6.3	6.29	6.29	6.28	6.28

These results show that with the change of $\delta\lambda$, *dV* and number of grid points, the results have been calculated with acceptable convergence and therefore the numerical solution provides good stability.

5.3. Static Response of the Nano-Beam

The pull-in voltage and deflection of the both end fixed nano-beam subjected to electrostatic, Casimir and vdW forces have been obtained using SSLM and are presented in this section. The effects of Casimir and vdW forces are illustrated in figures 2-6. Fixed-fixed nano-beams material properties are: Young' Modulus *E* is 169 GPa., density of nano-beam $\rho = 2300 {kg}/{m^3}$. The effect of beam dimenstion has also been studied using different values for the beam width *b*, thickness *t*, length *L* and primary gap G_0.

By comparing the results shown in Fig. 2 and Fig. 3, it can be seen that by increasing the length of the beam from *180 nm* to *230 nm*, the pull-in voltage reduces from 6.3 to 3.4 volts when both vdW and Casimir are considered. The results presented in Fig. 4 show that by decreasing the thickness of the nano-beam from *t=6 nm* to *4 nm*, the pull-in voltage at the presence of both forces reduces from 6.3 to 2.75 volts. Fig. 5 shows that by decreasing the G_o*=10 nm* to *8 nm*, the pull-in voltage at the presence of both forces reduces from 6.3 to 3.75 volts. By comparing the results shown in Fig. 2 and Fig. 6, it can be seen that the width of the beam, has no effect on the pull-in voltage of the nano-beam. Also the results shown in these figures prove that by decreasing the thickness and primary gap, and increasing the length of the nano-switch, the role of the vdW and Casimir forces increase and the Casimir force has more significant effect on the pull-in voltage than the vdW force; and therefore, the effect of the vdW force, in some cases can be ignored.

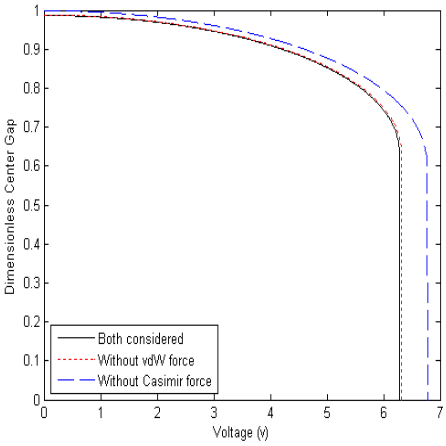


Fig. 2. Dimensionless center gap deflection versus voltage for fixed-fixed nano-beam with L =180 nm, G_0 =10 nm, b =9 nm and t = 6 nm.

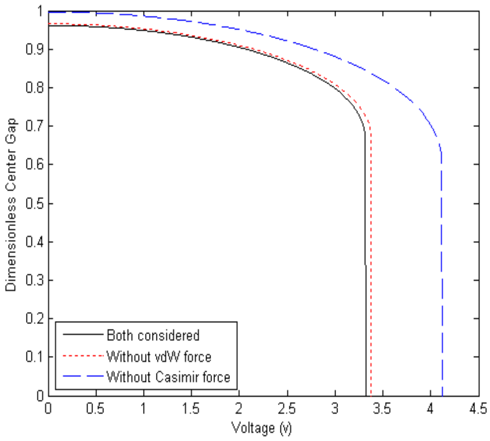


Fig. 3. Dimensionless center gap deflection versus voltage for fixed-fixed nano-beam with L =230 nm, G_0 =10 nm, b =9 nm and t = 6 nm.

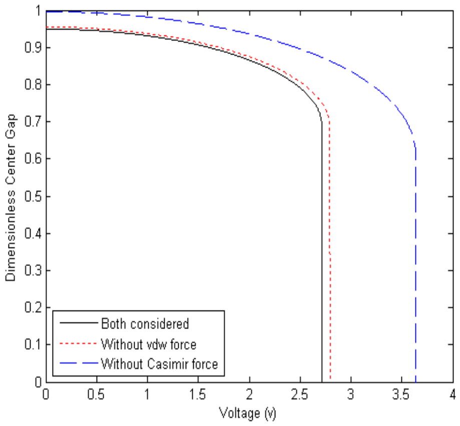


Fig. 4. Dimensionless center gap deflection versus voltage for fixed-fixed nano-beam with L =180 nm, G_0 =10 nm, b =9 nm and t = 4 nm.

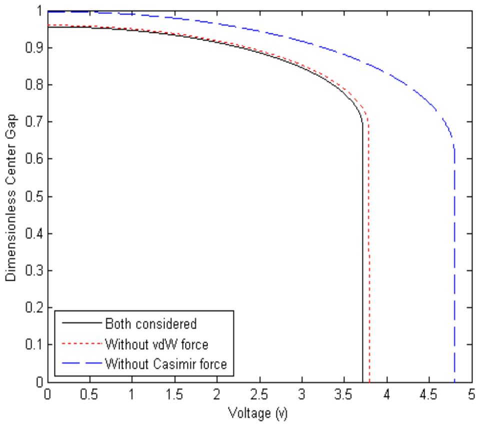


Fig. 5. Dimensionless center gap deflection versus voltage for fixed-fixed nano-beam with L =180 nm, G_0 =8

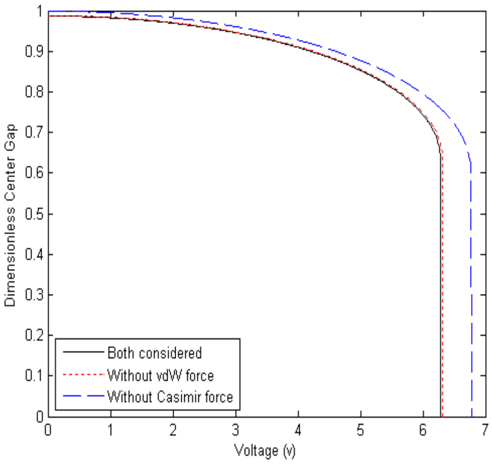
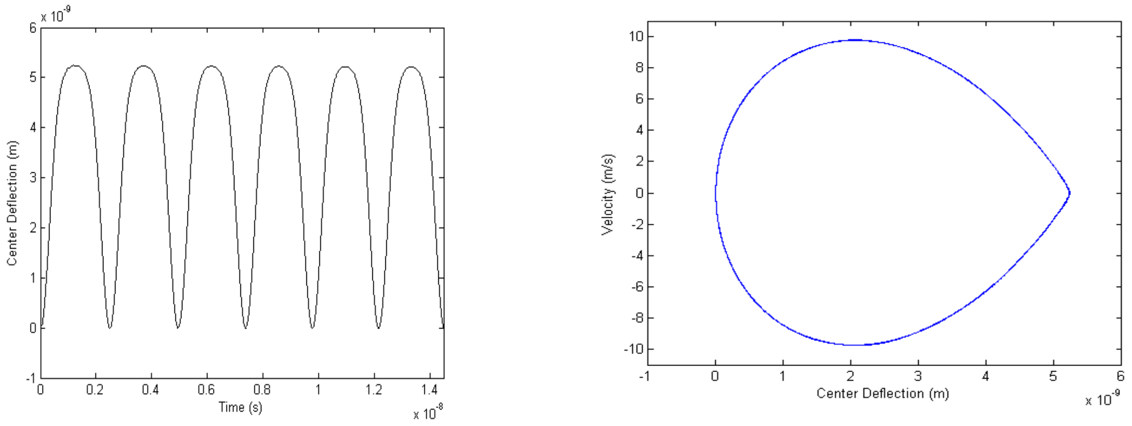


Fig. 6. Dimensionless center gap deflection versus voltage for fixed-fixed nano-beam with L=180 nm, G_0 =10 nm, b =6 nm and t = 6 nm.

5.4. Dynamic Response

The dynamic response of the nano-beam to the step-wise change of DC voltage is studied using Galerkin based reduced order model. For this study, a nano-beam with L =*180 nm*, G_0 =*10 nm*, b =*9 nm* and t = *6 nm* has been used. The pull-in voltage under electrostatic actuation (without considering Casimir and vdW forces) is 6.84v. The dynamic pull-in voltage considering no damping effects is 6.21v about 90.8 % of the static pull-in voltage, which shows that the results have good agreement with those reported by Ananthasuresh, et al. [22].

As shown in Fig. 2 the pull-in voltage of the nano-beam under electrostatic actuation and at the presence of both Casimir and vdW forces is 6.28 v. Fig. 7a and Fig. 7b show the time history and phase diagram of the center deflection of the nano-beam with no damping effect and under the actuating step voltage load of 5.6 v. It can be seen that the nano-beam under this value of step-wise DC voltage (5.6 v) dose not collapse and oscillates with 5nm amplitude. However, the results depicted in Fig. 8 show that the nano-beam collapses under the step-wise DC voltage of 5.61 v (about 89 % of the static pull-in voltage), and therefore the dynamic pull-in phenomena occurs.



(a) Time history of center deflection. (b) Amplitude of center deflection versus velocity.

Fig. 7. Dynamic response of a fixed-fixed nano-switch subjected to the actuating step-wise voltage of V_{DC} = 5.6 v.

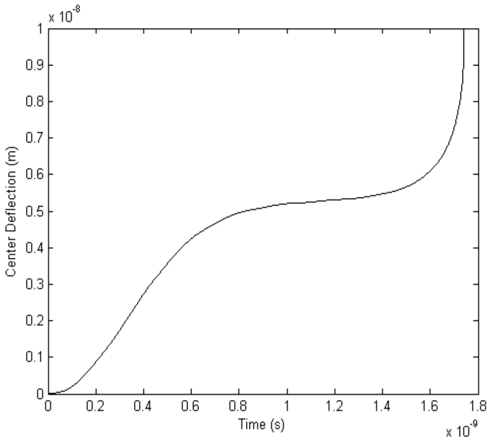


Fig. 8. Dynamic response of a fixed-fixed nano-switch subjected to the actuating step-wise DC voltage of $V_{DC} = 5.61$ v.

Fig. 9 and Fig. 10 show the time history and phase diagram for the nano-beam with damping ratio of $\varsigma = 0.1$. In these figures, continues and dashed curves relate to 5.66 v and 5.67 v step wise DC voltage respectively. As shown in these figures, the nano-beam under the actuating voltage of 5.66 v oscillates with reducing amplitude, however, by increasing the actuating voltage to 5.67 v, it collapses and pulls towards the fixed substrate.

Fig. 11 shows the pull in phenomena of the nano-beam with various value of damping ratio (ς). As shown in this figure, with increasing the damping ratio, the pull in voltage increases significantly. Fig. 12 also shows the variation of the pull-in voltage versus damping ratio (ς). As shown in this figure for $\varsigma \leq 1.8$ pull in voltage depends on the damping ratio and with increasing the damping ratio the pull-in voltage increases too. But above this level (for $\varsigma > 1.8$), pull-in voltage remains constant 6.25 v and can be regarded as damping-independent.

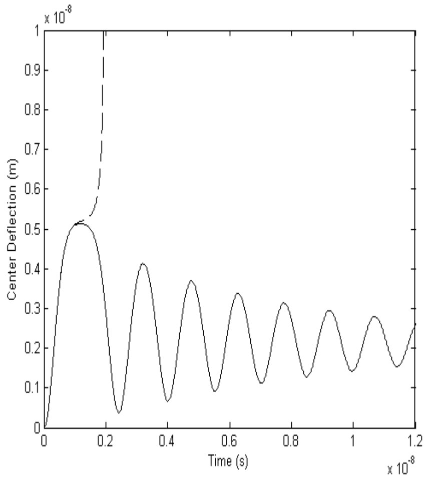


Fig. 9. Time history of center deflection (dashed and continues curves related to imposing 5.67 v and 5.66 v step wise DC voltage respectively).

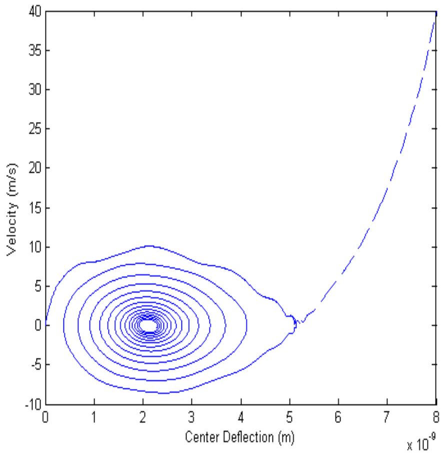


Fig. 10. Amplitude of center deflection versus velocity (dashed and continues curves related to imposing 5.67 v and 5.66 v step wise DC voltage respectively).

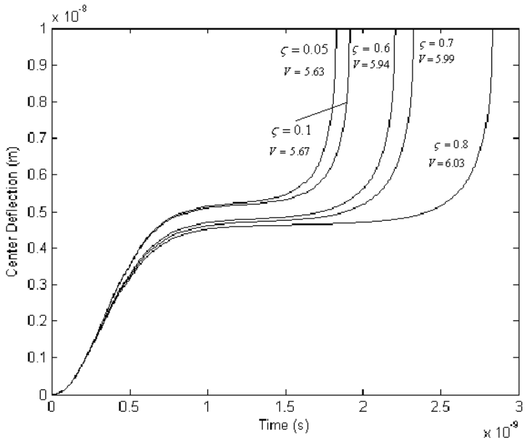


Fig. 11. Pull in time for various amount of damping ratio (ς).

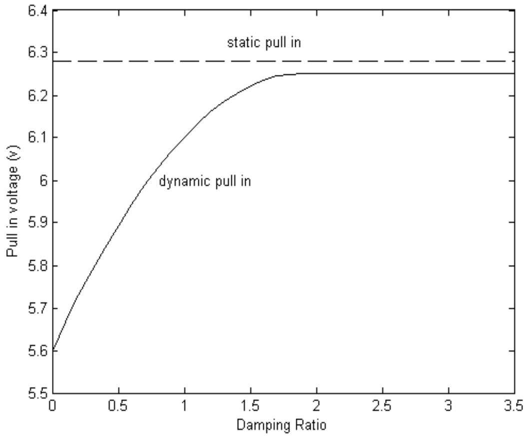


Fig. 12. Pull in voltage versus damping ratio(ς).

6. Conclusion

The governing equations to study the static deflection of a nano-beam switch subjected to the electrostatic forces were presented. Due to the nonlinearity of these equations, they were linearized and solved using SSLM and Galerkin weighted residual method. The pull-in phenomena have been investigated for a nano-beam based switch with different geometries. It was shown that by decreasing the thickness of the nano-beam, decreasing the primary gap of the nano-switch and increasing the length of the nano-beam, the role of the vdW and Casimir forces increase significantly. The dynamic response of the nano-beam has also been studied by solving the governing equations at the presence of the damping effect using Galerkin based reduced order model. The dynamic response and the dynamic pull-in phenomena of the nano-beam have been obtained. The results show that by increasing the value of the step-wise DC voltage to a critical value, the nano-beam collapses, whereas below this level the beam oscillates. The results also show that the undamped dynamic pull-in voltage without considering Casimir and vdW forces is about 90.8 % of the static pull-in voltage. It has also shown that the undamped dynamic pull-in voltage at the presence of Casimir and vdW forces is about 89 % of the static pull-in voltage. It was shown that by increasing the damping ratio to a certain value, dynamic pull in voltage increases too. But for damping ratios beyond this critical level, the dynamic pull-in voltage remains constant and is equal to the static pull-in voltage.

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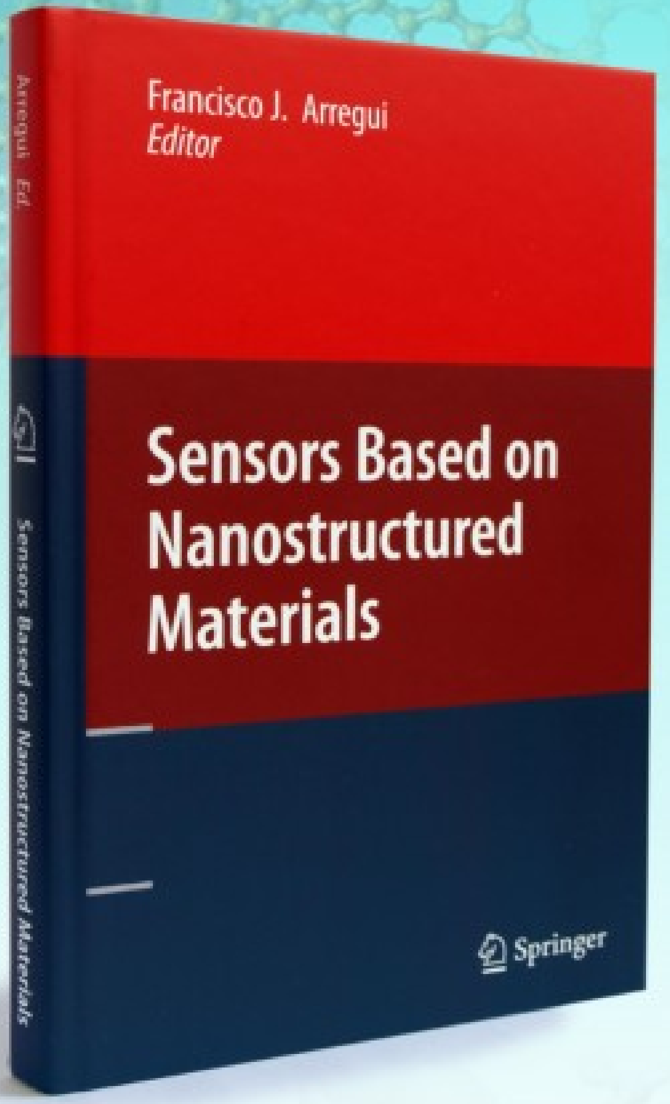
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