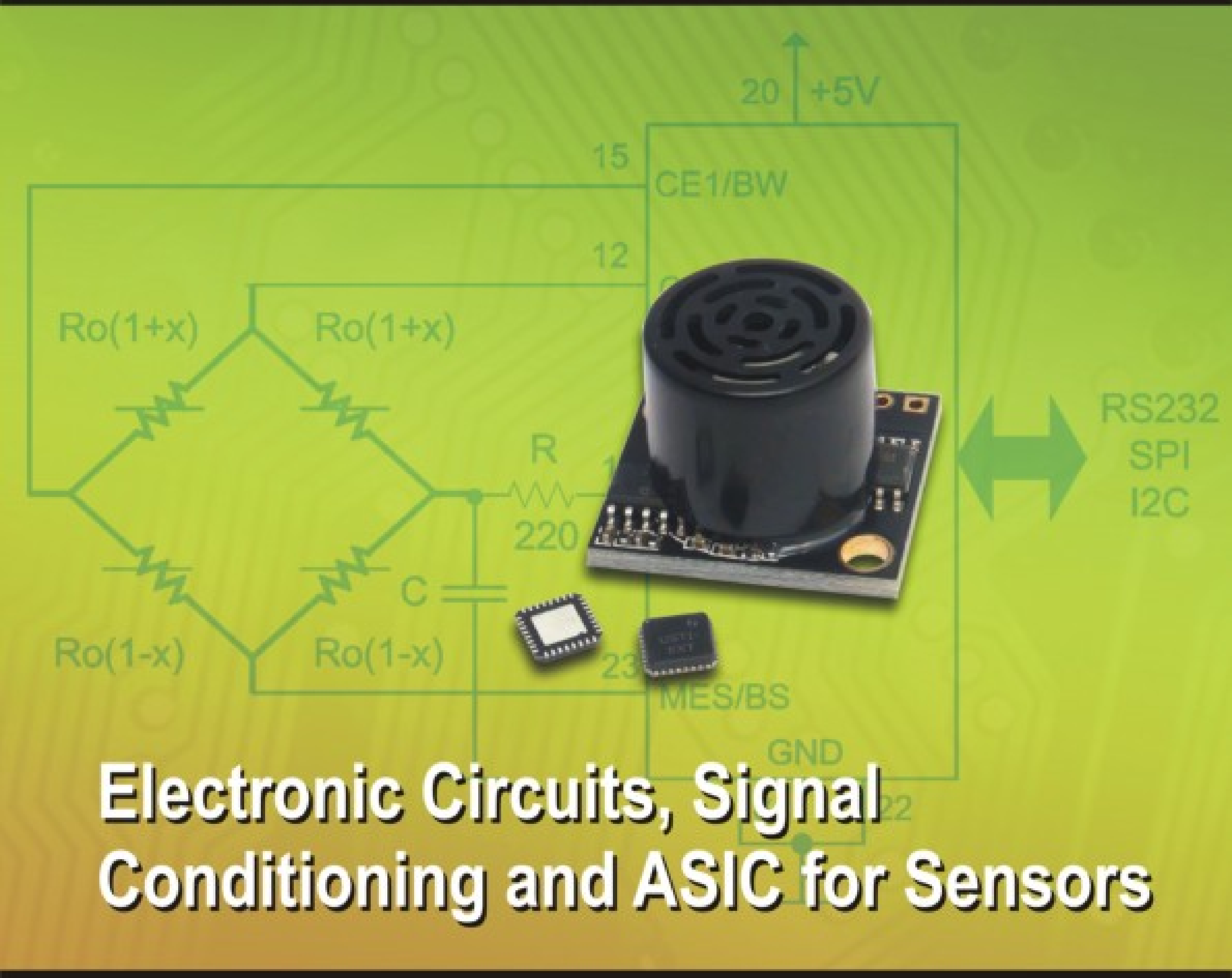
ISSN 1726-5479

SENSORS & TRANSDUCERS

vol. 141
6/12



Electronic Circuits, Signal Conditioning and ASIC for Sensors

International Frequency Sensor Association Publishing

IFSA

Sensors & Transducers

Volume 141, Issue 6, June 2012 www.sensorsportal.com ISSN 1726-5479

Sensors & Transducers Journal (ISSN 1726-5479) is a peer review international journal published monthly online by International Frequency Sensor Association (IFSA).
Available in electronic and on CD.

Sensors & Transducers Journal

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Volume 141
Issue 6
June 2012

www.sensorsportal.com

ISSN 1726-5479

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International Frequency Sensor Association (IFSA).

Sensors & Transducers Journal, Vol. 141, Issue 6, June 2012, pp. 92-105

Sensors & Transducers

ISSN 1726-5479

http://www.sensorsportal.com

Intelligent Robust Nonlinear Controller for MEMS Angular Rate Sensor

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Received: 12 March 2012 /Accepted: 25 June 2012 /Published: 30 June 2012

Abstract: This paper presents a control strategy for micro-electro-mechanical systems (MEMS) z-axis gyroscope, based on the coupling of the fuzzy logic control with the so-called sliding mode control (SMC) approach. An adaptive model reference state tracking controller which can estimate the angular velocity vector, and the damping and stiffness model coefficients in real-time is proposed. The motivation for using SMC in MEMS mainly relies on its appreciable features, such as design simplicity and robustness. The tracking performance of pure adaptive SMC is affected by high range chattering due to modeling uncertainties and exogenous inputs. In this paper, this problem is suitably circumvented by adopting an adaptive fuzzy sliding mode control (AFSMC) approach which results in chattering free tracking and improved estimation accuracy. For this proposed approach, we have used a fuzzy logic control to generate the reaching control signal. Therefore, the simple and intelligent Mamdani-type controller is resulted with high robustness against parametric uncertainties and exogenous disturbances. The stability of the system is guaranteed in the sense of the Lyapunov stability theorem. Numerical simulations using the nonlinear dynamic model of a MEMS z-axis gyroscope with uncertainties show the effectiveness of the approach in trajectory tracking problems and robustness in estimating the gyroscope parameters and also the angular velocity. The simulation results that are compared with the results of conventional adaptive SMC indicate that the control performance of the gyroscope system is satisfactory and the proposed AFSMC can achieve favorable tracking performance, and it is robust with regard to uncertainties and disturbances.

Keywords: Fuzzy control, Adaptive control, Sliding mode control, Chattering, MEMS gyroscope.

1. Introduction

MEMS gyroscope, which is one of the micro machined inertial sensors, and commonly used to measure the angular velocity in many areas including platform stabilization in space applications, activity monitoring in biomedical applications, sport equipment in consumer applications, robotics and machine and vibration monitoring in industrial applications, tracking and monitoring mechanical shock and vibration during transportation in automotive applications.

Vibrational MEMS gyroscopes are capable of vibrating along two orthogonal axes and energy is transferred from one axis (referred to as drive axis) to other axis (referred to as sense axis) through Coriolis forces, which is arising from a rotating reference frame and proportional to its rotation rate. The conventional mode of operation drives one of the axes of the gyroscope into a known oscillatory motion and then detects the Coriolis acceleration coupling along the sense axis, which is orthogonal to the drive axis. The response of the sense mode provides information about the unknown angular velocity. However, in practice, the fabrication imperfections and environmental variations are always present, resulting in parameter changes, cross stiffness and cross damping effects. Other noise sources such as time-varying system parameters, thermal and mechanical noise, and sensing circuitry noise also affect the performance. The angular velocity measurement and minimization of the cross coupling between two axes are challenging problems in vibrating gyroscopes. As a consequence, some kind of advanced control method is essential for improving the performance and stability of MEMS gyroscopes.

Sliding mode technique as a powerful robust control tool shows high robustness against parameter uncertainties and insensitivity to matched disturbances [1]. In this paper, the sliding mode controller (SMC) is gathered with an equivalent control term describing the behavior of the system when the trajectories stay over the sliding manifold. Furthermore, the variable structure control term enforces the trajectories to reach the sliding manifold. Using the tracking capabilities of adaptive control methods in the presence of parameter uncertainties, adaptive SMCs are developed [2]. It has been proven that fuzzy logic can approximate any nonlinear function to any desired accuracy because of the universal approximation property [3]. Drawback of chattering phenomenon can be avoided by using fuzzy logic.

An adaptive control law for merging parameter identification solution and SMC, has been proposed and analytically studied by Andrievsky [4]. Batur *et al.* [5] has developed a sliding mode control for MEMS gyroscope system combined with a force balancing control strategy to identify the angular velocity. Leland [6] has presented an adaptive controller for tuning the natural frequency of the drive axis of a vibratory gyroscope. Park and Horowitz [7] have reported an adaptive controller for a MEMS gyroscope which drives both axes of vibration and controls the entire operation of the gyroscope. Fei and Batur [8] have presented an adaptive sliding mode control for vibratory gyroscope. Yau [9] has proposed a robust fuzzy sliding mode control scheme for the synchronization of two chaotic nonlinear gyroscopes subjected to uncertainties and external disturbances.

The proposed controller in this paper is an adaptive SMC with a proportional and integral sliding surface which is enhanced by a fuzzy control system to decrease the chattering effects, significantly. The designed Mamdani type fuzzy system is based on the knowledge and experiences of expert engineers and operators. Therefore, independent of the gyroscope complex dynamics, a simple *if then* rule base is generated which is more suitable for real time applications. In this intelligent adaptive SMC, the angular velocity and all the unknown parameters of the gyroscope are estimated unbiased. The superiority of the proposed adaptive fuzzy SMC (AFSMC) to the adaptive SMC is revealed through simulation results.

The paper is organized as follows; the dynamics of MEMS vibratory rate gyroscope is described in Section 2. In Section 3, the adaptive fuzzy sliding mode controller with 7×7 fuzzy rule table, triangular membership functions and proportional and integral sliding surface, is developed. In section 4, we study the simulation results. And finally, Section 5 presents some conclusions remarks.

2. Dynamics of MEMS Vibratory Rate Gyroscope

The schematic of a z-axis MEMS gyroscope is shown in Fig. 1. This vibratory gyroscope includes a proof mass suspended by springs, an electrostatic actuation and sensing mechanisms for forcing an oscillatory motion and sensing the position and velocity of the proof mass. The nonlinearity of the effective spring has been introduced by Asokanthan and Wang [10]; therefore, the cubic type stiffness is added to the linear stiffness terms for representing the overall effective stiffness coefficient. It is assumed that the table where the proof mass is mounted moves with a constant velocity and the gyroscope rotates at a slowly changing angular velocity about z axis; The Coriolis force is generated in a direction perpendicular to the driving and rotational axes.

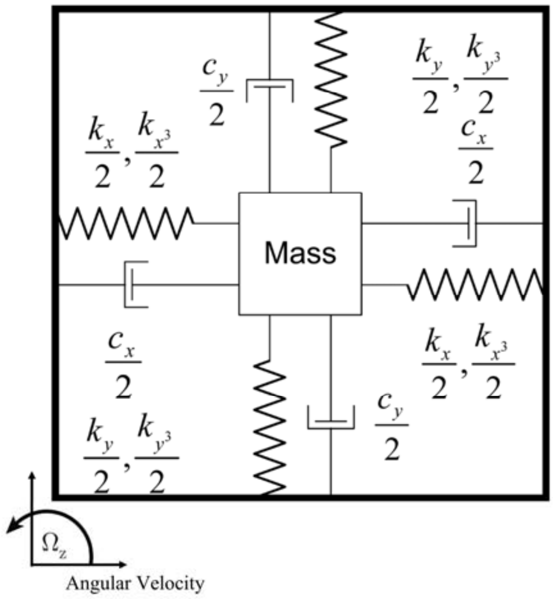

Fig. 1. Dynamical model of MEMS z-axis gyroscope.

By using Lagrange's equation and with the assumptions stated above, the dynamics of gyroscope become as:

$$m\ddot{x} + d_x^*\dot{x} + \left(k_x^* - m\Omega_z^{*2}\right)x + k_{x^3}^* x^3 + k_{xy}^* y + d_{xy}^*\dot{y} - m\dot{\Omega}_z^* y - 2m\Omega_z^*\dot{y} = F_x^*, \quad (1)$$

$$m\ddot{y} + d_x^*\dot{y} + \left(k_x^* - m\Omega_z^{*2}\right)y + k_{y^3}^* y^3 + k_{xy}^* x + d_{xy}^*\dot{x} - m\dot{\Omega}_z^* x - 2m\Omega_z^*\dot{x} = F_y^*. \quad (2)$$

The origin for *x*-*y* coordinates is at the center of the proof mass in the absence of the applied force. The fabrication imperfections are considered in the coupling spring and damping coefficients, k_{xy}^* and d_{xy}^*. The spring and damping terms along *x* and *y* axes, k_x^*, $k_{x^3}^*$, k_y^*, $k_{y^3}^*$, d_x^* and d_y^* are almost known. However, small unknown variations may occur in the corresponding nominal values. The accurate value of the proof mass, m is determined. F_x^* and F_y^* are the control forces along the x and y directions, respectively.

Dividing gyroscope dynamics (1) and (2) by the reference mass results in the following vector forms as:

$$\ddot{q}^* + \frac{D^*}{m}\dot{q}^* + \frac{K_1^*}{m}q^* + \frac{K_3^*}{m}q^{*3} = \Omega_z^{*2}q^* + \dot{S}^*q^* + 2S^*\dot{q}^* + u^*, \tag{3}$$

where

$$q^* = \begin{bmatrix} x^* \\ y^* \end{bmatrix},\ u^* = \begin{bmatrix} u_x^* \\ u_y^* \end{bmatrix},\ D^* = \begin{bmatrix} d_x^* & d_{xy}^* \\ d_{xy}^* & d_y^* \end{bmatrix},\ K_1^* = \begin{bmatrix} k_x^* & k_{xy}^* \\ k_{xy}^* & k_y^* \end{bmatrix},\ K_3^* = \begin{bmatrix} k_{x^3}^* & 0 \\ 0 & k_{y^3}^* \end{bmatrix},\ S^* = \begin{bmatrix} 0 & \Omega_z^* \\ -\Omega_z^* & 0 \end{bmatrix}$$

The variation of the angular rate is assumed negligible. Nondimensionalizing the equations of motion of a gyroscope is useful because the numerical simulation is easy, even under the existence of large two time-scale differences in gyroscope dynamics [7]. Using the non-dimensional time, $t^* = \omega_0 t$ and dividing both sides of (3) by reference frequency and length, ω_0^2 and q_0 gives the final form of the non-dimensional equation of motion as follows:

$$\frac{\ddot{q}^*}{q_0} + \frac{D^*}{m\omega_0}\frac{\dot{q}^*}{q_0} + \frac{K_1^*}{m\omega_0^2}\frac{q^*}{q_0} + \frac{K_3^*}{m\omega_0^2}\frac{q^{*3}}{q_0^3} = \frac{\Omega_z^{*2}}{\omega_0^2}\frac{\dot{q}^*}{q_0} + 2\frac{S^*}{\omega_0}\frac{\dot{q}^*}{q_0} + \frac{u^*}{m\omega_0^2 q_0}, \tag{4}$$

where new parameters are defined as follows:

$$q = \frac{q^*}{q_0},\quad d_{xy} = \frac{d_{xy}^*}{m\omega_0},\quad \Omega_z = \frac{\Omega_z^*}{\omega_0},\quad u_x = \frac{u_x^*}{m\omega_0^2 q_0},\quad u_y = \frac{u_y^*}{m\omega_0^2 q_0},\quad \omega_x = \sqrt{\frac{k_x}{m\omega_0^2}},\quad \omega_y = \sqrt{\frac{k_y}{m\omega_0^2}},$$
$$\omega_{xy} = \frac{k_{xy}}{m\omega_0^2},\ \delta_x = \frac{k_{x^3}q_0^2}{m\omega_0^2},\ \delta_y = \frac{k_{y^3}q_0^2}{m\omega_0^2}. \tag{5}$$

Consequently, the non-dimensional representation of (1) and (2) becomes:

$$\ddot{q} + D\dot{q} + K_1 q + K_3 q^3 = \Omega_z^2\dot{q} + 2S\dot{q} + u, \tag{6}$$

where

$$q = \begin{bmatrix} x \\ y \end{bmatrix},\ u = \begin{bmatrix} u_x \\ u_y \end{bmatrix},\ D = \begin{bmatrix} d_x & d_{xy} \\ d_{xy} & d_y \end{bmatrix},\ K_1 = \begin{bmatrix} \omega_x & \omega_{xy} \\ \omega_{xy} & \omega_y \end{bmatrix},\ K_3 = \begin{bmatrix} \delta_x & 0 \\ 0 & \delta_y \end{bmatrix},\ S = \begin{bmatrix} 0 & \Omega_z \\ -\Omega_z & 0 \end{bmatrix}$$

3. Adaptive Fuzzy Sliding Mode Controller

This section presents the AFSMC approach with a proportional and integral sliding surface for MEMS gyroscopes. The AFSMC makes the driving axes of the MEMS gyroscope track the desired trajectories generated by a reference model. Furthermore, the input angular velocity along axis z will be estimated. The linear dynamics (5) is rewritten in the following state space model:

$$\dot{X} = AX + Bu, \tag{7}$$

$$A=\begin{bmatrix}0 & 1 & 0 & 0\\ -\omega_x^2 & -d_x & -\omega_{xy} & -(d_{xy}-2\Omega_z)\\ 0 & 0 & 0 & 1\\ -\omega_{xy} & -(d_{xy}+2\Omega_z) & -\omega_y^2 & -d_y\end{bmatrix},\ B=\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}^T,\ u=\begin{bmatrix}u_x\\ u_y\end{bmatrix},\ X=\begin{bmatrix}x\\ \dot{x}\\ y\\ \dot{y}\end{bmatrix} \tag{8}$$

The control target for a MEMS gyroscope is to maintain the proof mass to oscillate in x and y directions at frequencies ω_1 and ω_2 and at amplitudes A_1 , A_2, respectively. These requirements can be expressed as $x_d = A_1\sin(\omega_1 t)$ and $y_d = A_2\cos(\omega_2 t)$. Equivalently, the control objective can be stated in terms of a reference model or desired trajectory as:

$$\ddot{q}_d + K_d q_d = 0, \tag{9}$$

where $K_d = diag\{\omega_1^2 \quad \omega_2^2\}$. Similar to (7), the reference model can be written as the following state space model.

$$\dot{X}_d=\begin{bmatrix}0 & 1 & 0 & 0\\ -\omega_1^2 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & -\omega_2^2 & 0\end{bmatrix}X_d \equiv A_d X_d, \tag{10}$$

Considering unknown parametric uncertainty ΔA and uncertain extraneous disturbance, $f(t)$, the model (7) is represented as:

$$\dot{X}(t) = (A+\Delta A)X(t) + Bu(t) + f(t), \tag{11}$$

We make the following assumptions:

Assumption 1. Considering both matched and unmatched sections of $\Delta A(t)$ and $f(t)$, there exist unknown matrices, $D(t)$ and $G(t)$ of appropriate dimensions, such that:

$$\Delta A(t) = BD(t) + \Delta\tilde{A}(t), \tag{12}$$

$$f(t) = BG(t) + \tilde{f}(t), \tag{13}$$

where $BD(t)$ and $\Delta\tilde{A}(t)$ are matched and unmatched uncertainties, respectively; similarly, $BG(t)$ and $\tilde{f}(t)$ are matched and unmatched disturbances, respectively. From this assumption, (11) can be rewritten as:

$$\dot{X}(t) = AX(t) + Bu(t) + Bf_m(X,t) + f_u(X,t), \tag{14}$$

where $Bf_m(X,t)$ represents the system matched lumped uncertainty and disturbance which is given by:

$$f_m(X,t) = D(t)X(t) + G(t), \tag{15}$$

The term $f_u(X,t)$ represents the lumped unmatched uncertainty and disturbance which is given by:

$$f_u(X,t) = \Delta\tilde{A}(t)X(t) + \tilde{f}(t), \tag{16}$$

Assumption 2. The matched and unmatched lumped uncertainty and external disturbance f_m and f_u are bounded by known positive parameters α_m , α_u such as $\| f_m(X,t)\| \le \alpha_m$ and $\| f_u(X,t)\| \le \alpha_u$, where $\| x \|$ denotes the Euclidean norm when x is a vector: $\|x\| = \sqrt{x^T x}$.

Assumption 3. There exists a constant matrix K^* such that the condition, $A + BK^{*T} = A_d$ is always satisfied (K^* is a 4×2 matrix) [8].

3.1. Adaptive Sliding Mode Controller

The tracking error and its derivative are given as:

$$e(t) = X(t) - X_d(t), \tag{17}$$

$$\dot{e}(t) = A_d e(t) + (A - A_d)X(t) + Bu(t) + Bf_m(X,t) + f_u(X,t), \tag{18}$$

The proportional-integral sliding surface, $s(t) = 0$ is defined as:

$$s(t) = \lambda e(t) - \int_0^t \lambda (A_d + BK_e) e(\tau) d\tau, \tag{19}$$

where λ is the constant matrix such that λB is nonsingular. The constant K_e satisfies that $A_d + BK_e$ is a Hurwitz matrix.

The time derivative of the sliding surface is as:

$$\dot{s}(t) = \lambda(A - A_d)X(t) + \lambda Bu(t) + \lambda Bf_m(X,t) + \lambda f_u(X,t) - \lambda BK_e e(t), \tag{20}$$

Setting, $\dot{s}(t) = 0$ to solve the equivalent control, u_{eq} gives:

$$\begin{aligned} u_{eq} &= -(\lambda B)^{-1}\lambda(A - A_d)X(t) + K_e e(t) - f_m(X,t) - (\lambda B)^{-1}\lambda f_u(X,t) \\ &= K^{*T}X(t) + K_e e(t) - f_m(X,t) - (\lambda B)^{-1}\lambda f_u(X,t) \end{aligned}, \tag{21}$$

The adaptive version of the control algorithm is proposed as:

$$u(t) = K^T(t)X(t) + K_e e(t) - \rho(\lambda B)^{-1}\frac{s(t)}{\|s(t)\|}, \tag{22}$$

where $K(t)$ is the estimate of K^*. The last component of the control signal is designed to address the matched and unmatched disturbances. This component is given as

$$u_s(t) = \begin{bmatrix} u_{s1} \\ u_{s2} \end{bmatrix} = -\rho(\lambda B)^{-1}\frac{s(t)}{\|s(t)\|}, \tag{23}$$

where ρ is the constant. We define the estimation error as

$$\tilde{K}(t) = K(t) - K^*, \tag{24}$$

Substituting (24) and (22) into (14) yields

$$\dot{X}(t) = A_d X(t) + B\tilde{K}^T(t)X(t) + BK_e e(t) + Bf_m(X,t) + f_u(X,t) - B\rho(\lambda B)^{-1}\frac{s(t)}{\|s(t)\|}, \tag{25}$$

The tracking error equation now becomes:

$$\dot{e}(t) = (A_d + BK_e)e(t) + B\tilde{K}^T(t)X(t) + Bf_m(X,t) + f_u(X,t) - B\rho(\lambda B)^{-1}\frac{s(t)}{\|s(t)\|}, \tag{26}$$

The dynamics of the sliding surface $s(t)$ is given as:

$$\dot{s}(t) = \lambda B\tilde{K}^T(t)X(t) + \lambda Bf_m(X,t) + \lambda f_u(X,t) - \rho\frac{s(t)}{\|s(t)\|}, \tag{27}$$

Define a Lyapunov function as:

$$V(t) = \frac{1}{2}s^T(t)s(t) + \frac{1}{2}tr\left[\tilde{K}(t)M^{-1}\tilde{K}^T(t)\right], \tag{28}$$

where $M = diag\{m_1 \quad m_2\}$ is a positive definite matrix. Taking the time derivative of $V(t)$ yields as:

$$\begin{aligned}\dot{V}(t) &= s^T(t)\dot{s}(t) + tr\left[\tilde{K}(t)M^{-1}\tilde{K}^T(t)\right] \\ &= s^T(t)\left[\lambda B\tilde{K}^T(t)X(t) + \lambda Bf_m(X,t) + \lambda f_u(X,t) - \rho\frac{s(t)}{\|s(t)\|} + tr\left[\tilde{K}(t)M^{-1}\dot{\tilde{K}}^T(t)\right]\right] \\ &= -s^T(t)\rho\frac{s(t)}{\|s(t)\|} + s^T(t)\lambda Bf_m(X,t) + s^T(t)\lambda f_u(X,t) + s^T(t)\lambda B\tilde{K}^T X(t) + tr\left[\tilde{K}(t)M^{-1}\dot{\tilde{K}}^T(t)\right]\end{aligned}, \tag{29}$$

To make $\dot{V}(t) \le 0$, the adaptive law is proposed as:

$$\dot{\tilde{K}}^T(t) = \dot{K}^T(t) = -MB^T\lambda^T s(t)X^T(t), \tag{30}$$

The initial gain $K(0)$ is used arbitrary. The adaptive law (30) results as:

$$\begin{aligned}\dot{V}(t) &= -\rho\|s(t)\| + s^T(t)\lambda B f_m(X,t) + s^T(t)\lambda f_u(X,t) \\ &\le -\rho\|s(t)\| + \|s(t)\|\,\|\lambda B\|\,\|f_m(X,t)\| + \|s(t)\|\,\|\lambda\|\,\|f_u(X,t)\| \\ &\le -\rho\|s(t)\| + \|s(t)\|\,\|\lambda B\|\,\alpha_m + \|s(t)\|\,\|\lambda\|\,\alpha_u \\ &= -\|s(t)\|\left(\rho - \|\lambda B\|\alpha_m - \|\lambda\|\alpha_u\right)\end{aligned}, \quad (31)$$

Considering, $\rho \ge \|\lambda B\|\alpha_m + \|\lambda\|\alpha_u + \eta$, in which η is a positive fixed value, $\dot{V}(t)$ becomes negative semi-definite, i.e., $\dot{V}(t) \le -\eta\|s(t)\|$. This implies that the trajectory reaches the sliding surface in finite time and remains on the sliding surface. From (20), $e(t)$ will also asymptotically converge to zero. Furthermore, using LaSalle's invariant set theorem, $\lim_{t\to\infty} s(t) = 0$.

3.2. Fuzzy Controller Design

Fuzzy Control (FC) has supplanted conventional technologies in many applications. One major feature of fuzzy logic is its ability to express the amount of ambiguity in human thinking. Thus, when the mathematical model of the process does not exist, or exists but with uncertainties, FC is an alternative way to deal with the unknown process.

In this paper, in order to eliminate the chattering problem, a fuzzy inference engine is used to remove the discontinuity of the sign function of the SMC at the reaching phase. Combining the FC with the adaptive SMC in equation (22) results in developing the proposed AFSMC methodology. The main advantage of this method is that the robust behavior of the system is guaranteed. The second advantage of the proposed AFSMC is that the performance of the system in the sense of removing chattering is improved in comparison with the same SMC technique without using FC.

The reaching law is designed as:

$$u_r(t) = K_f u_f(t), \quad (32)$$

where K_f is the normalization factor of the output variable, and $u_f(t)$ is the output of the AFSMC, which is determined in accordance with the normalized outputs of the ASMC, i.e. $s(t)$ and $\dot{s}(t)$. The fuzzy control rules can be represented as the mapping of the input linguistic variables $s(t)$ and $\dot{s}(t)$ to the output linguistic variable $u_f(t)$ as follows [11]:

$$u_f(t) = AFSMC\big(s(t), \dot{s}(t)\big), \quad (33)$$

where $s(t)$ is the two-dimensional vector with components $s_1(t)$ and $s_2(t)$; accordingly, $u_f(t)$ is the two-dimensional vector with components $u_{f1}(t)$ and $u_{f2}(t)$. Also $AFSMC(s(t), \dot{s}(t))$ denotes the functional characteristics of the fuzzy logic controller (FLC) as shown in Fig. 2.

In this FLC with two inputs and seven linguistic variables for each input, there are 49 possible *if then* rules with all combinations of the inputs. The set of linguistic variable which imply inputs $s(t)$, $\dot{s}(t)$ and output $u_f(t)$ with 49 rules in the rule base have been classified as: negative big (NB), negative medium (NM), negative small (NS), zero (Z), positive small (PS), positive medium (PM) and positive big (PB).

The linguistic fuzzy rules are defined heuristically in the following generic form:

$$R^{(l)}: if\ \ s(t)\ is\ A_1^l\ and\ \dot{s}(t)\ is\ A_2^l\ then\ u_f(t)\ is\ B^l$$

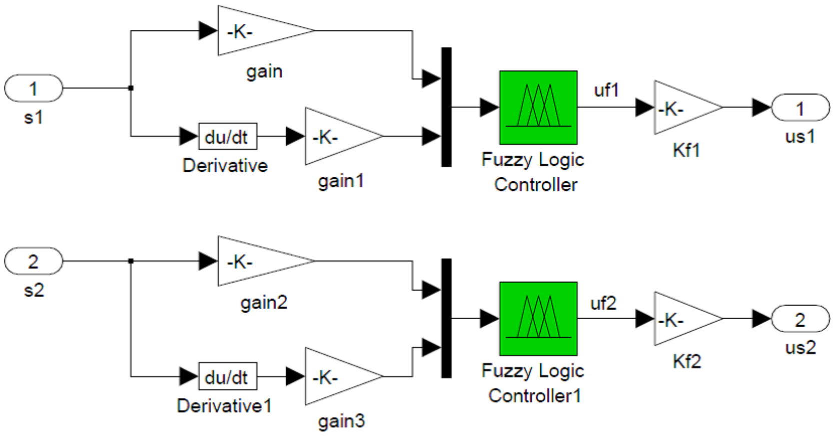

Fig. 2. Fuzzy inference system of AFSMC**.**

where, A_1^l, A_2^l and B^l are the membership functions in the l-th *if then* rule corresponding to the input and output fuzzy sets, respectively. In the fuzzy inference engine, the intersection minimum and the center average defuzzification operations are used [12]*.*

The *if then* rules of the MEMS gyroscope system are represented in Table 1. In the fuzzy rule table, the rules are designed such that the following points are taken under consideration:

1. The output fuzzy sets are normalized in the interval (-1, 1), then $\left|u_f(t) = AFSMC(s(t), \dot{s}(t))\right| \leq 1$.
2. When the product $s(t)\dot{s}(t)$ is positive, the membership value of $u_f(t)$ are set such that its sign is the same as that of $s(t)$, therefore the following inequality is insured:

$$s^T(t)u_f(t) = s^T(t)\ AFSMC(s(t), \dot{s}(t)) \leq |s(t)|, \tag{34}$$

This condition is required to achieve the system stability.

Table 1. Rule Base of AFSMC.

$u_f(t)$		$s(t)$						
		NB	**NM**	**NS**	**Z**	**PS**	**PM**	**PB**
$\dot{s}(t)$	**NB**	NB	NB	NM	NS	PS	PS	PS
	NM	NB	NM	NM	NS	PS	PS	PS
	NS	NM	NM	NS	NS	PS	PS	PM
	Z	NM	NM	NS	Z	PS	PM	PM
	PS	NM	NS	NS	PS	PS	PM	PM
	PM	NS	NS	NS	PS	PM	PM	PB
	PB	NS	NS	NS	PS	PM	PB	PB

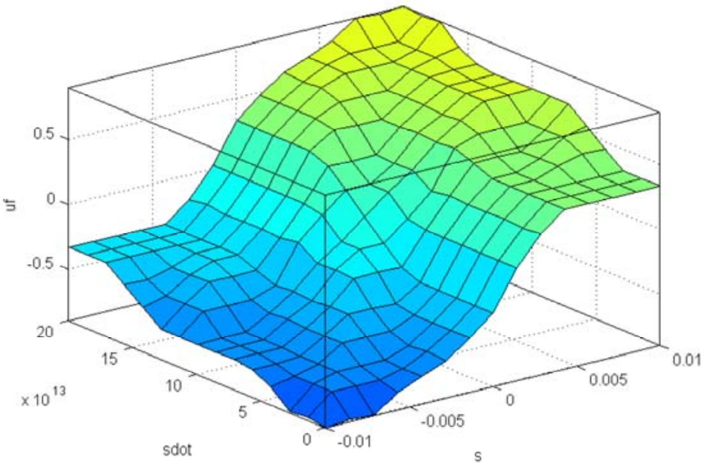

Fig. 3. Control surface generated by a 7×7 rule table in Table 1.

The AFSMC law has been represented as:

$$u(t) = u_{eq}(t) + u_r(t) = u_{eq}(t) + K_f u_f(t) \\ = u_{eq}(t) + AFSMC(s(t), \dot{s}(t)) \quad , \tag{35}$$

Using the Lyapunov function, the stability of system (17) with the control law (35), is proved based on the Lyapunov's direct method.

4. Simulation Results

In this section, the performance of the proposed AFSMC on a lumped MEMS gyroscope system is evaluated using simulation results for tracking and estimating the angular rate, Ω_z. The modeling uncertainties, ΔA are considered as: ± 3 % change in the spring and damping coefficients from corresponding nominal values, ± 2 % changes in the magnitude of coupling terms d_{xy} and ω_{xy}, and ± 5% variations of Ω_z. It is assumed that ΔA only has unmatched terms while, the external disturbance, $f(t)$ includes both matched and unmatched sections which are modeled as random variables with zero mean and unity variance. Parameters of the MEMS gyroscope are used as follows [5, 13].

The unknown angular velocity, Ω_z is assumed to have an amplitude of 5.0 rad/s and the initial condition on K matrix is as, $K(0) = 0.95K^*$. The desired motion trajectories are $x_d = \sin(\omega_1 t)$ and $y_d = \sin(\omega_2 t)$, where $\omega_1 = 4.17\,kHz$ and $\omega_2 = 5.11\,kHz$. The sliding mode matrix K_e in (19), λ, the sliding mode gain, ρ and the adaptive gain M in (30) are used as follows:

$$K_e = \begin{bmatrix} -9900 & -10\,000 & 1000 & 19000 \\ -990 & -990 & -990 & -990 \end{bmatrix},\ \lambda = \begin{bmatrix} 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix},\ \rho = \begin{bmatrix} 19500 & 0 \\ 0 & 19500 \end{bmatrix},\ M = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

Initial values for the reference model states are as, $[3\times10^{-6}, 4\times10^{-4}, 3\times10^{-6}, 4\times10^{-4}]^T$ all in meters or meter per seconds.

The simulated control tracking errors of AFSMC are shown in Fig. 4, and the estimated angular rates are shown in Fig. 5 using ASMC and AFSMC. The Control force by ASMC and AFSMC are shown in Fig. 6 and Fig. 7. Fig. 8 demonstrates that the estimates of controller parameters converge to their true values with persistently exciting driving signals on both axes. According to the figures, the chattering free tracking and estimation performance of the AFSMC is significantly superior compared with that of the pure adaptive kind. It can be observed that the adaptive fuzzy sliding mode system eliminates chattering significantly compared with the adaptive sliding mode force as shown in Fig. 6 and Fig. 7; Due to chattering phenomenon the control force of ASMC is not applicable to system but applying of the control force of AFSMC is possible.

In other words, using the control law (35) and the parameter adaptation law (30), the estimation of all unknown parameters of MEMS gyroscope in particular the input angular rate converges to the true values, and the tracking errors asymptotically converge to zero.

Table 2. The MEMS Gyroscope Parameters used in simulation.

Parameter	Value	Unit
m	0.57×10^{-8}	kg
d_x	0.429×10^{-6}	Ns/m
d_{xy}	0.0429×10^{-6}	Ns/m
d_y	0.687×10^{-6}	Ns/m
k_x	80.98	N/m
k_{x3}	3.56×10^{12}	N/m^3
k_{xy}	5	N/m
k_y	71.62	N/m
k_{y3}	3.56×10^{12}	N/m^3
w_0	1	kHz
q_0	1×10^{-6}	m
m	0.57×10^{-8}	kg
d_x	0.429×10^{-6}	Ns/m
d_{xy}	0.0429×10^{-6}	Ns/m
d_y	0.687×10^{-6}	Ns/m

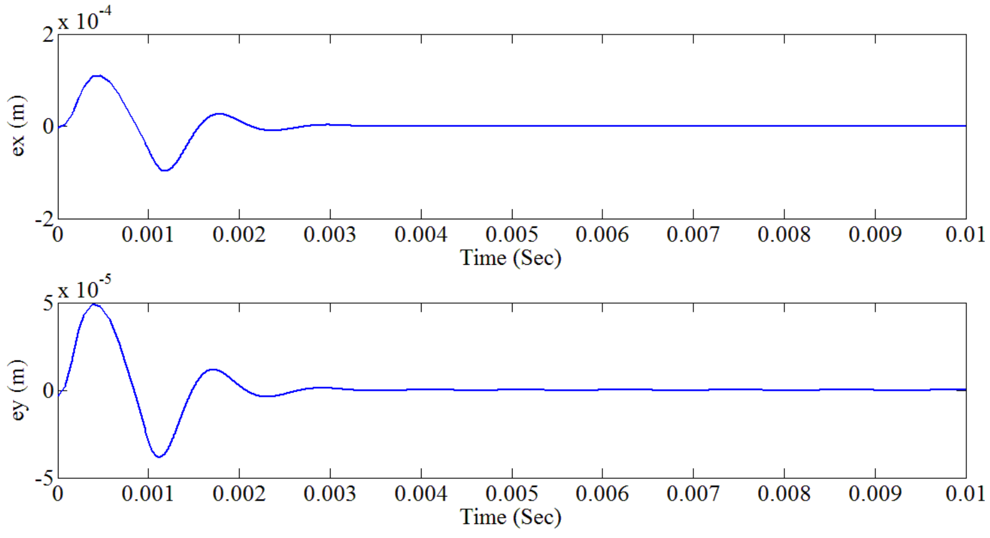

Fig. 4. Tracking errors of AFSMC.

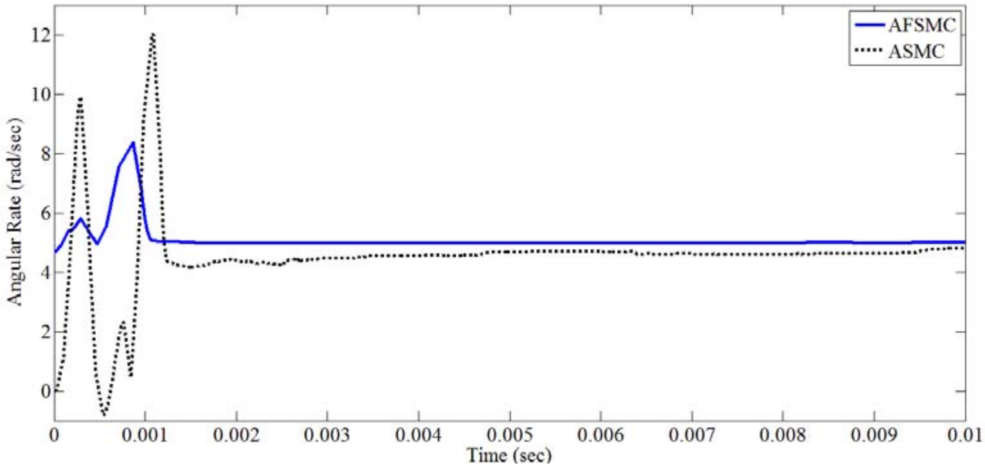

Fig. 5. Adaptation of angular velocity by fuzzy and pure ASMC.

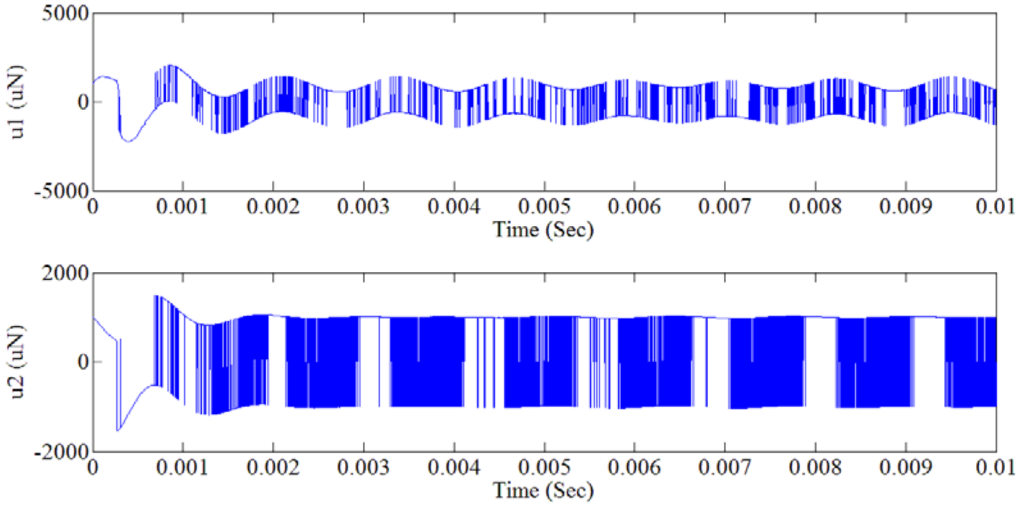

Fig. 6. Control force by ASMC.

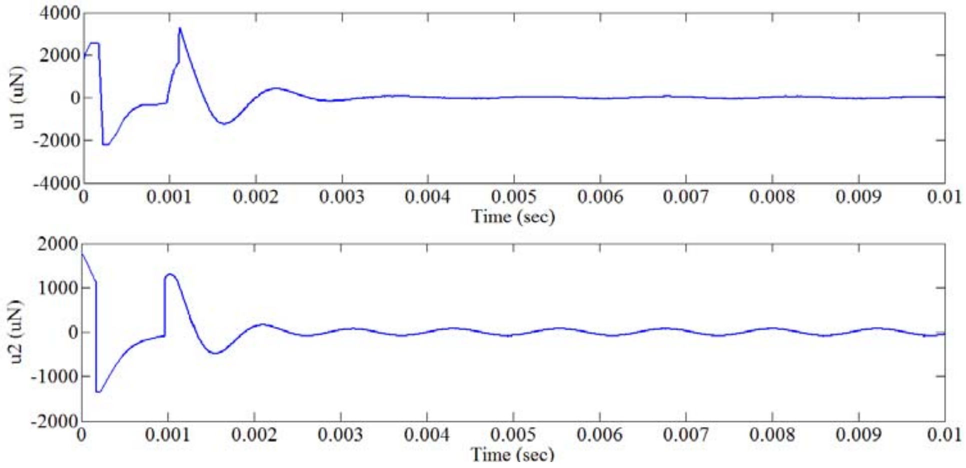

Fig. 7. Control force by AFSMC.

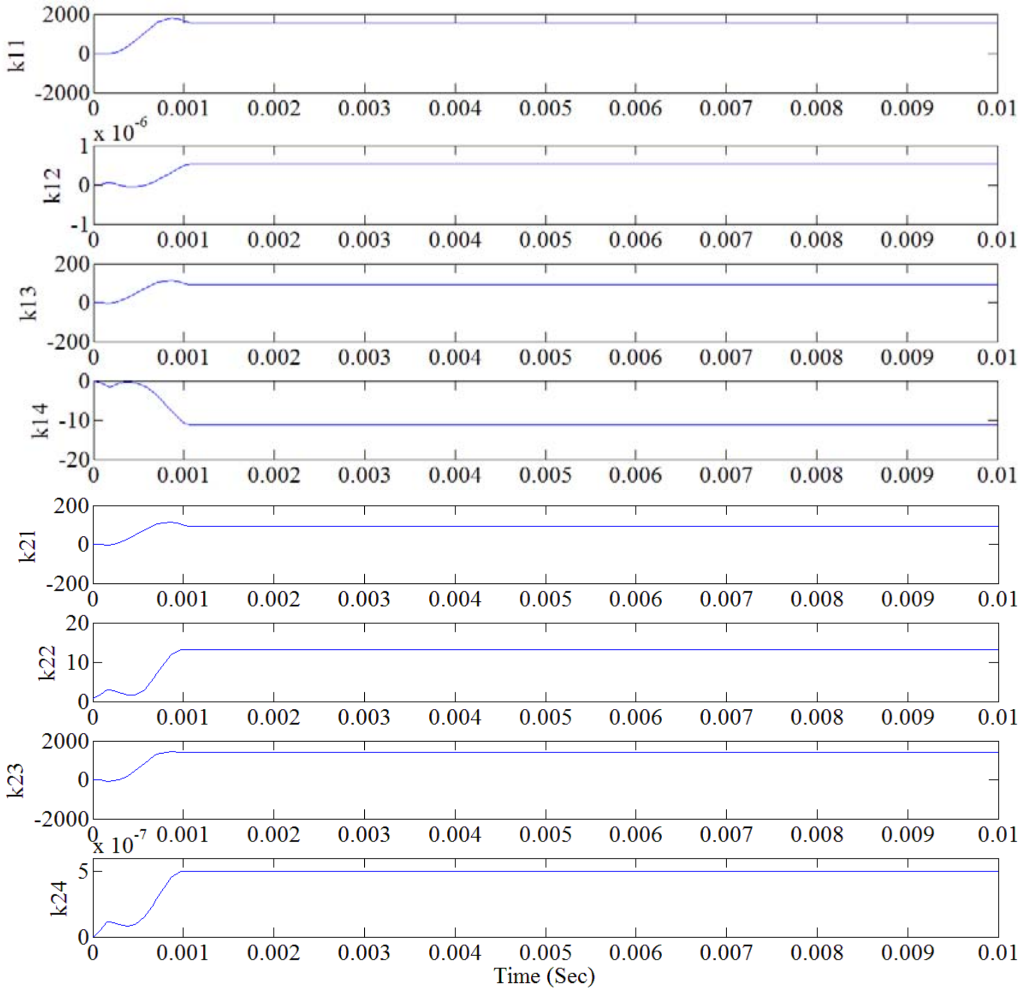

Fig. 8. Adaptation of gyroscope parameters by AFSMC.

5. Conclusions

The present paper has described the development of a model reference adaptive fuzzy sliding mode control (AFSMC) where it has been successfully applied for the control of the axes of the MEMS gyroscope on the desired directions and to estimate the unknown angular velocity. The proposed robust non-chattering AFSMC based on the fuzzy control (FC) scheme possesses great benefit in practical applications. It takes the advantages of robustness of SMC and chattering elimination of FC. Due to using triangular membership functions and the proposed Mamdani type fuzzy system, the chattering phenomenon that frequently appears in the conventional SMC is also eliminated without deteriorating the system robustness, and accurate estimation and tracking performances are resulted. The proposed approach could also prove a success in a challenging domain of MEMS gyroscopes where the dynamics of MEMS gyroscope is expressed by nonlinear, time varying, coupled differential equations. The AFSMC with proportional and integral sliding action can handle both matched and unmatched uncertainties provided that the upper bounds of these uncertainties are available. In situations where there is precise tracking of fast trajectories for non-linear systems with high nonlinearities and large uncertainties, the existing conventional sliding mode control (SMC) schemes are inadequate. The performance noticed for conventional controllers is not very appreciable.

Generally speaking, it can be said that the performance of adaptive fuzzy inference system based controller is better than the conventional approach in the control of MEMS gyroscopes. Simulation results showed the effectiveness of the proposed control methodology. It has been verified that the proposed adaptive fuzzy logic sliding mode controller has superior tracking performance and robustness in estimating the gyroscope parameters and also the angular velocity in the presence of matched and unmatched input disturbances and model variations.

References

[1]. V. I. Utkin, Sliding modes in control optimization, *Springer-Verlag*, Berlin, 1992.

[2]. S. Sastry, M. Bodsom, Adaptive control: stability, convergence and robustness, *Prentice Hall Inc.*, New Jersey, 1989.

[3]. K. M. Passino, S. Yurkovitch, Fuzzy Control, *Addison Wesley Longman*, CA, 1998.

[4]. B. R. Andrievsky, A. L. Fradkov, A. A. Stotsky, Shunt compensation for indirect sliding-mode adaptive control, in *Proceedings of the* 13^{th} *IFAC World Congress*, San Francisco, USA, 1996, pp. 193-198.

[5]. C. Batur, T. Sreeramreddy, Q. Khasawneh, Sliding mode control of a simulated MEMS gyroscope, *ISA Transactions*, Vol. 45, Issue 1, 2006, pp. 99-108.

[6]. R. Leland, Adaptive mode tuning for vibrational gyroscopes, *IEEE Transactions on Control System Technology*, Vol. 11, Issue 2, 2003, pp. 242-247.

[7]. S. Park, R. Horowitz, New adaptive mode of operation for MEMS gyroscopes, *ASME Transaction Dynamic Systems, Measurement and Control*, Vol. 126, 2004, pp. 800-810.

[8]. J. Fei, C. Batur, A novel adaptive sliding mode control with application to MEMS gyroscope, *ISA Transactions*, Vol. 48, 2009, pp. 73-78.

[9]. H. T. Yau, Chaos synchronization of two uncertain chaotic nonlinear gyros using fuzzy sliding mode control, *Mechanical Systems and Signal Processing*, Vol. 54, 2008, pp. 69-78.

[10]. S. F. Asokanthan, T. Wang, Nonlinear instabilities in a vibratory gyroscope subjected to angular speed fluctuations, *Nonlinear Dynamics*, Vol. 54, 2008, pp. 69-78.

[11]. M. R. Moghanni, J. Keighobadi, A. Ghanbari, Fuzzy adaptive sliding mode controller for MEMS vibratory rate gyroscope, in *Proceedings of the* 18^{th} *IFAC World Congress*, Milan, Italy, 28 August - 2 September 2011, pp. 4192-4197.

[12]. J. Keighobadi, M. B. Menhaj, M. Kabganian, Feedback- linearization and fuzzy controllers for trajectory tracking of wheeled mobile robots, *Kybernetes*, Vol. 39, No. 1, 2010, pp. 83-106.

[13]. F. Braghin, F. Resta, E. Leo, G. Spinola, Nonlinear dynamics of vibrating MEMS, *Sensors and Actuators*, Vol. 134, 2007, pp. 98-108.

Guide for Contributors

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