Sensors & Transducers, Vol. 175, Issue 7, July 2014, pp. 299-305

Sensors & Transducers

An Efficient Track-Before-Detect Algorithm Based on Complex Likelihood Ratio in Radar Systems

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Received: 17 July 2014 /Accepted: 30 June 2014 /Published: 31 July 2014

Abstract: In this paper, we presented a track-before-detect (TBD) method which can cope with range-Doppler ambiguity for medium pulse repetition frequency (MPRF) radars. The target state evolutions in the ambiguous range-Doppler domain are considered as a hybrid system with the range-Doppler ambiguous number deemed as mode variable. Phase information of measurement is used to provide a detection sensitivity improvement, and the reduction of computation is acquired by using complex likelihood instead of the envelope likelihood. Finally, a dynamic programming algorithm is used to estimate target state and ambiguous number of every PRF, and the true trajectory of target is backtracked after resolving the ambiguity. Simulation results show that the proposed method achieves 1.2 dB performance improvement at 50 % detection rate compared with the present method.

Keywords: State estimation, Target tracking, Range-Doppler ambiguity, Dynamic programming, Track-before-detect.

1. Introduction

Early detection and trajectory estimation of low observable targets from medium pulse repetition frequency (MPRF) radars is a well-known problem. The classical approach for radar detection and tracking take thresholded measurements as an input. When the energy reflected by a target is too low, no plot can be constructed and this target will therefore be declared lost. To overcome this problem a method, called track-before-detect (TBD), has been developed, see [1]. The TBD approach allows simultaneous detection and tracking using unthresholded data and show superior detection performance over the conventional methods. The typical track-before-detect algorithm includes direct maximum likelihood [2], the Hough transform [3-4], the dynamic programming algorithm [5-6] and particle filter [7-8].

Because of the small amount of calculation, the dynamic programming-based (DP) track-before-detect (TBD) algorithm has been applied widely in the engineering.

For MPRF radars, the range and Doppler are ambiguous. The most common method for resolving range and Doppler ambiguities involves using multiple PRFs. This has the effect of changing the apparent target range estimated by pulse burst, and allows for some ambiguity resolution in either Doppler or range. The classical solution to the PRF selection problem is to perform PRF staggering. But due to the range-Doppler ambiguity isn't estimated and compensated, the energy of targets will not be effectively accumulated and acquired by making use of the conventional TBD algorithms. There are some studies for dim target detection on radar with range and Doppler ambiguity. In [6] the authors represent the ambiguous Doppler by the apparent Doppler and the ambiguity number, propose a DP-TBD method for joint estimation of the target trajectory and the Doppler ambiguity sequence. In [9] a multiple PRF TBD detection algorithm based on particle filter is presented. In [10] a new pulse repetition frequency (PRF) selection method is proposed for the TBD algorithm, and adaptive adjustment of pulse repetition frequency mode was proposed in [11], in which influence of range-Doppler ambiguity can be removed.

In this paper, we presented a DP-TBD method which can cope with range-Doppler ambiguity and eclipsing effects for medium pulse repetition frequency radars. The target state evolutions in the ambiguous range-Doppler domain are considered as a hybrid system with the range-Doppler ambiguous number deemed as mode variable. We also using phase information to provide a detection sensitivity improvement, and the complex likelihood was found to be an order of magnitude cheaper to evaluate than the existing envelope likelihood. And then for every single PRF radar data, a dynamic programming algorithm is used to estimate target state, ambiguous number and obtain the maximum target track energy. Finally, the range-Doppler ambiguity is resolving on the accumulation of track energy and the true trajectory of target is backtracked. Simulation shows that the proposed method can improve the detection performance of dim target with range-Doppler ambiguity.

2. Problem Definition

Consider a set of MPRFs used for range-Doppler ambiguity resolving denoted by $PRF_i\left(i=1,2,...,I\right)$, and suppose the i^{th} MPRF in this set, PRF_i is selected. $R_{\max}$ is assumed as the maximum range of interest and the corresponding maximum unambiguous range R_u^i is given by

$$R_u^i=\frac{c}{2PRF_i}\,, \tag{1}$$

where c is the speed of light.

As illustrated in Fig. 1, assume the ambiguous range measurement r_k^i , $r_k^i<R_u^i$ at time k . All possible ranges $\tilde{r}_k^i$ are generated by

$$\tilde{r}_k^i=r_k^i+(m_{r,k}^i-1)R_u^i\,, \tag{2}$$

where $m_{r,k}^i\in\left\{1,\cdots,M_r^i+1\right\}$ is the range ambiguous number. The maximum range ambiguous number M_r^i is given by

$$M_r^i=Int(R_{\max}/R_u^i) \tag{3}$$

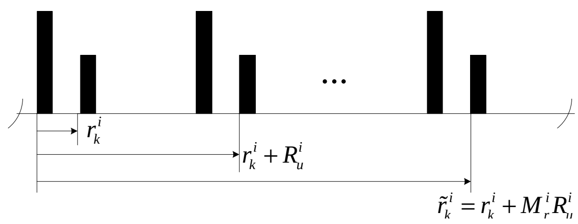

Fig. 1. All possible ranges from one range measurement.

$D_{\max}$ is assumed as the maximum Doppler of interest and the corresponding maximum unambiguous Doppler D_u^i is given by

$$D_u^i=\frac{\lambda PRF_i}{2}\,, \tag{4}$$

where $\lambda=c/f_c$ is the wavelength of the transmitted energy.

Assume the ambiguous Doppler measurement d_k^i , $d_k^i\le D_u^i$ at time k . All possible Doppler $\tilde{d}_k^i$ are generated by

$$\tilde{d}_k^i=d_k^i+(m_{d,k}^i-1)D_u^i\,, \tag{5}$$

where $m_{d,k}^i\in\left\{1,\cdots,M_d^i+1\right\}$ is the Doppler ambiguous number. The maximum Doppler ambiguous number M_d^i is given by

$$M_d^i=Int(D_{\max}/D_u^i) \tag{6}$$

2.1. Target Model

The kinematic component of the target state at time k , $\tilde{x}_k$, consists of range, range rate and range acceleration. Writing $\tilde{x}_k=\left(\tilde{r}_k^i,\tilde{d}_k^i,a_k^i\right)^{\mathrm{T}}$, where the superscript T stands for the transpose of a matrix or a vector, the time evolution of $\tilde{x}_k$ is modeled as

$$\tilde{x}_k=F_k\tilde{x}_{k-1}+G_kw_k\,, \tag{7}$$

where w_k is the range acceleration increment, which is closely related to Gaussian noise with zero mean and covariance Q. The two matrices required to specify this equation are given by

$$F_k=\begin{vmatrix}1 & T & T^2/2\\ 0 & 1 & T\\ 0 & 0 & 1\end{vmatrix}\qquad G_k=\begin{vmatrix}T^2/2\\ T\\ 1\end{vmatrix}, \tag{8}$$

where T is the fixed sampling period.

Defining the target state in ambiguous range-Doppler domain as $x_k = \left(r_k^i, d_k^i, a_k^i\right)^T$. Using (2) and (5), the state evolution can be written as

$$x_k = F_k x_{k-1} + B_k + C_k + G_k w_k \tag{9}$$

The transition matrix about ambiguous number of range-Doppler is given by

$$B_k = \begin{vmatrix} (m_{r,k-1}^i - m_{r,k}^i)R_u^i \\ 0 \\ 0 \end{vmatrix}, \quad C_k = \begin{vmatrix} m_{d,k-1}^i D_u^i T_R \\ (m_{d,k-1}^i - m_{d,k}^i)D_u^i \\ 1 \end{vmatrix} \tag{10}$$

The range-Doppler ambiguity number $m_{r,k}^i, m_{d,k}^i$ can be considered as the mode variable. In the following, we use the range-Doppler ambiguous number and mode interchangeably.

It should be noted that in (10), we have assumed $m_{r,k}^i - m_{r,k-1}^i \in \{-1,0,1\}$, $m_{d,k}^i - m_{d,k-1}^i \in \{-1,0,1\}$ that is, the range-Doppler ambiguous number transitions only occur in adjacent ambiguity numbers.

2.2. Measurement Model

The measurement is the reflected complex response on range-Doppler domain for radar. The range and Doppler domains are divided into N_r and N_d cells, respectively. Let z_k be a stacked vector of all the pixel responses of the kth frame.

$$z_k = \left\{ z_k^{(p,q)} \mid p = 1,\dots,N_r, q = 1,\dots,N_d \right\} \tag{11}$$

The measurement per range-Doppler cell is defined by

$$z_k^{(p,q)} = \begin{cases} \exp\{j\phi_k\} h^{(p,q)}(x_k) + n_k^{(p,q)}, & \text{target} \\ n_k^{(p,q)}. & \text{no target} \end{cases}, \tag{12}$$

where the target signal at the pixel (p,q), given that the target state is x_k, is denoted by $h^{(p,q)}(x_k)$. n_k is a stacked vector of the pixel noise signals. Assume that n_k is an independent identically distributed complex Gaussian noise sequence with zero mean and a known covariance matrix R. Defining the unknown phase shift of target signal generated is ϕ_k, it is assumed to be uniformly distributed over $[0, 2\pi)$.

Let $s_k = \exp\{j\phi_k\}$, the pdf of measurement when a target is present is as follows

$$p(z_k \mid \mathrm{x}_k, \phi) = \frac{1}{\pi^{N*L}|R|} \exp\left\{ -\frac{1}{2}\left(z_k - s_k h(\mathrm{x}_k)\right)^{\mathrm{H}} R^{-1}\left(z_k - s_k h(\mathrm{x}_k)\right) \right\} \tag{13}$$

and when no target exists is

$$p(z_k \mid H_0) = \frac{1}{\pi^{N*L}|R|} \exp\left\{ -\frac{1}{2} z_k^{\mathrm{H}} R^{-1} z_k \right\}, \tag{14}$$

where R is the covariance matrix of Gaussian noise.

In order to calculate conveniently, the likelihood ratio is defined as follow.

$$\begin{aligned} &L(z_k \mid \mathrm{x}_k, \phi_k) \\ &= \exp\left\{ -\frac{1}{2}\left(z_k - s_k h(\mathrm{x}_k)\right)^{\mathrm{H}} R^{-1}\left(z_k - s_k h(\mathrm{x}_k)\right) + \frac{1}{2} z_k^{\mathrm{H}} R^{-1} z_k \right\} \\ &= \exp\left\{ \frac{1}{2} z_k^{\mathrm{H}} R^{-1} s_k h(\mathrm{x}_k) + \frac{1}{2} s_k^* h(\mathrm{x}_k)^{\mathrm{H}} R^{-1} z_k - \frac{1}{2} s_k^* h(\mathrm{x}_k)^{\mathrm{H}} R^{-1} s_k h(\mathrm{x}_k) \right\} \\ &= \exp\left\{ \frac{1}{2} h(\mathrm{x}_k)^{\mathrm{H}} R^{-1} h(\mathrm{x}_k) \right\} \\ &\times \exp\left\{ \frac{1}{2} s_k z_k^{\mathrm{H}} R^{-1} h(\mathrm{x}_k) + \frac{1}{2} s_k^* h(\mathrm{x}_k)^{\mathrm{H}} R^{-1} z_k \right\} \end{aligned} \tag{15}$$

defining:

$$\begin{cases} \xi = \Xi \exp\{j\theta_k\} = h(\mathrm{x}_k)^{\mathrm{H}} R^{-1} z_k \\ \Gamma = \exp\left\{ \frac{1}{2} h(\mathrm{x}_k)^{\mathrm{H}} R^{-1} h(\mathrm{x}_k) \right\} \end{cases}, \tag{16}$$

where Ξ and ξ denote the magnitude and phase of θ_k respectively. Using (16), the likelihood ratio can be written as

$$\begin{aligned} &L(z_k \mid \mathrm{x}_k, \phi_k) \\ &= \Gamma \exp\left\{ \frac{1}{2} s_k \xi^* + \frac{1}{2} s_k^* \xi \right\} \\ &= \Gamma \exp\left\{ \frac{1}{2}(\cos\phi_k + j\sin\phi_k)\xi^* + \frac{1}{2}(\cos\phi_k - j\sin\phi_k)\xi \right\} \\ &= \Gamma \exp\left\{ \frac{1}{2}\cos\phi_k[\xi^* + \xi] + \frac{1}{2}\sin\phi_k[\xi^* - \xi] \right\} \\ &= \Gamma \exp\{\Xi\cos\phi_k\cos\theta_k + \Xi\sin\phi_k\sin\theta_k\} \\ &= \Gamma \exp\{\Xi\cos(\phi_k - \theta_k)\} \end{aligned} \tag{17}$$

Because of ϕ_k is uniformly distributed over $[0, 2\pi)$, then

$$\begin{aligned} &L(z_k \mid \mathrm{x}_k) \\ &= \int_0^{2\pi} L(z_k \mid \mathrm{x}_k, \phi_k)\, p(\phi_k) d\phi_k \\ &= \frac{\Gamma}{2\pi} \int_0^{2\pi} \exp\{\Xi\cos(\phi_k - \theta_k)\} d\phi_k \\ &= \exp\left\{ \frac{1}{2} h(\mathrm{x}_k)^{\mathrm{H}} R^{-1} h(\mathrm{x}_k) \right\} I_0\left(2\left|h(\mathrm{x}_k)^{\mathrm{H}} R^{-1} z_k\right|\right) \end{aligned} \tag{18}$$

If the noise is spatially uncorrelated with variance σ^2, then R is the identity matrix scaled by σ^2 and the likelihood simplifies to

$$L(z_k \mid \mathrm{x}_k) = \exp\left\{-\frac{h(\mathrm{x}_k)^{\mathrm{H}} h(\mathrm{x}_k)}{\sigma^2}\right\} I_0\left(\frac{2\left|h(\mathrm{x}_k)^{\mathrm{H}} z_k\right|}{\sigma^2}\right) \quad (19)$$

The above expression is implicitly dependent on the target amplitude. One way to remove this dependence is to marginalize over it as in

$$\bar{L}(z_k \mid \mathrm{x}_k) = \int_0^\infty L(z_k \mid \mathrm{x}_k)\, p(I_k) dI_k \quad (20)$$

3. The DP-TBD for MPRF Radar

Fig. 2 is the flowchart of the proposed method for MPRF radars.

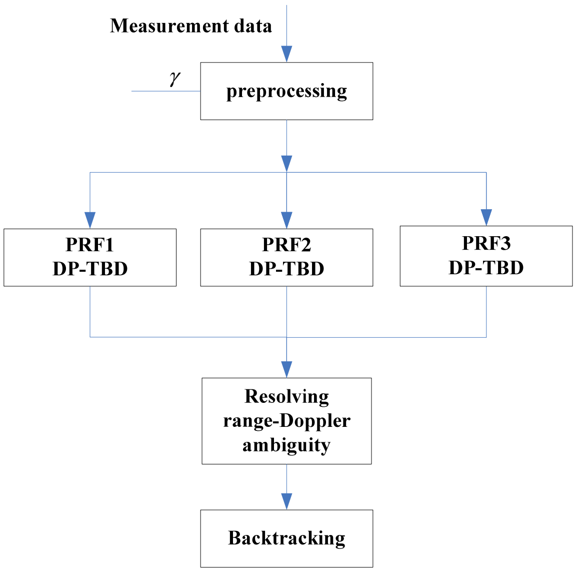

Fig. 2. Flowchart of the proposed method.

From Fig. 2, it can be seen that the proposed method functions in staggering multiple PRFs mode, and it consists of four steps. First of all, In order to reduce the data storage and computation, the radar measurement data have to be preprocessed. Let γ is threshold of preprocessing, and set the likelihood ratio of target data to ζ when $\left|z_k^{(p,q)}\right| < \gamma$. And then for every single PRF radar data, dynamic programming algorithm is used to estimate target state, ambiguous number and obtain the maximum target track energy. Finally, the range-Doppler estimation is disambiguated on the accumulation of track energy and the trajectory of target is backtracked.

3.1. Merit Function

In our situation, given the set of unthresholded apparent range-Doppler data maps up to the kth visit, $Z_{1:k} = (z_1, z_2, ..., z_k)$, we are focusing on finding the optimal target track $X_{1:k} = (x_1, x_2, ..., x_k)$ in apparent range-Doppler maps, and the corresponding range-Doppler ambiguous number sequence is

$$M_{r,1:k} = \left(m_{r,1}^i, m_{r,2}^i, ..., m_{r,k}^i\right),\ M_{d,1:k} = \left(m_{d,1}^i, m_{d,2}^i, ..., m_{d,k}^i\right).$$

The joint MAP estimates of $\hat{X}_{1:k}, \hat{M}_{r,1:k}$ and $\hat{M}_{d,1:k}$ is given by

$$(\hat{X}_{1:k}, \hat{M}_{r,1:k}, \hat{M}_{d,1:k}) = \arg\max p(X_{1:k}, M_{r,1:k}, M_{d,1,k} \mid Z_{1:k}) \quad (21)$$

Defining the Merit function as follow

$$I_k(x_k, m_{r,k}^i, m_{d,k}^i) = \max p(X_{1:k}, M_{r,1:k}, M_{d,1,k} \mid Z_{1:k}) \quad (22)$$

From Bayes' theorem, the merit function can be written as

$$\begin{aligned} I_k(x_k, m_{r,k}^i, m_{d,k}^i) &= \max\left\{\frac{p(z_k \mid x_k) p(x_k \mid x_{k-1}, m_{r,k}^i, m_{d,k}^i, m_{r,k-1}^i, m_{d,k-1}^i)}{p(z_k \mid Z_{1:k-1})} \times\right. \\ & p(m_{r,k}^i \mid m_{r,k-1}^i, m_{d,k-1}^i, x_{k-1})\, p(m_{d,k}^i \mid m_{d,k-1}^i, x_{k-1}) \times \\ & \left. I_k(x_{k-1}, m_{r,k-1}^i, m_{d,k-1}^i) \right\} \end{aligned} \quad (23)$$

The function $p(z_k \mid Z_{1:k-1})$ is used for normalization purposes only and can be dropped. Similarly, the likelihood $p(z_k \mid x_k)$ can also be substituted by the likelihood ratio $\bar{L}(z_k \mid x_k)$ without affecting the maximum value. The merit function can be redefined as

$$\begin{aligned} I_k(x_k, m_{r,k}^i, m_{d,k}^i) = \ln \bar{L}(z_k \mid \mathrm{x}_k) + \\ \max_{\mathrm{x}_k}\{\ln p(x_k \mid x_{k-1}, m_{r,k}^i, m_{d,k}^i, m_{r,k-1}^i, m_{d,k-1}^i) + \\ \ln p(m_{r,k}^i \mid m_{r,k-1}^i, m_{d,k-1}^i, x_{k-1}) + \\ \ln p(m_{d,k}^i \mid m_{d,k-1}^i, x_{k-1}) + \\ I_k(x_{k-1}, m_{r,k-1}^i, m_{d,k-1}^i)\ \} \end{aligned} \quad (24)$$

3.2. Dynamic Programming TBD Algorithm

Assuming K group data is accumulated, the algorithm steps are as follows.

Step 1. Preprocessing.

For $1 \le k \le K$, Let γ is threshold of preprocessing, then

$$\begin{cases} \ln \bar{L}(z_k \mid x_k) = \ln \bar{L}(z_k \mid x_k) \ \text{when} \ \left|z_k^{(p,q)}\right| \ge \gamma \\ \ln \bar{L}(z_k \mid x_k) = \zeta \qquad\qquad \text{when} \ \left|z_k^{(p,q)}\right| < \gamma \end{cases} \tag{25}$$

The likelihood ratio $\bar{L}(z_k \mid x_k) > \zeta$ when $\left|z_k^{(p,q)}\right| \ge \gamma$.

Step 2. Initialization.

Initializing $I_1(\mathrm{x}_1)$ for all states $x_1 = \left(r_1^i, d_1^i, a_1^i\right)^T$

$$\begin{cases} I_1(x_1, m_{r,1}^i, m_{d,1}^i) = \ln \bar{L}(z_1 \mid x_1) \\ \Psi_1(x_1) = 0 \end{cases}, \tag{26}$$

where $\Psi_k(x_k)$ is used to store the state transfer between groups.

Step 3. Recursion.

For $2 \le k \le K$, calculating the merit function $I_k(x_k, m_{r,k}^i, m_{d,k}^i)$ and record the state transition $\Psi_k(x_k)$.

$$\Psi_k(x_k) = \arg \max_{x_{k-1} \in S_d} I_{k-1}(x_{k-1}, m_{r,k-1}^i, m_{d,k-1}^i) \tag{27}$$

Step 4. Merit function transfer.

For all $x_{k-1} = \left(r_{k-1}^i, d_{k-1}^i, a_{k-1}^i\right)^T$; s.t. $I_{k-1}(.) > \zeta$ and $I_k(.) = \zeta$ where $x_k \in \xi(x_{k-1})$ then

$$\begin{cases} I_k(.) = I_{k-1}(.) \\ \Psi_k(x_k) = x_{k-1} \\ x_k = o(x_{k-1}) \end{cases}, \tag{28}$$

where $\xi(x_{k-1})$ denotes all the states x_k for which an origin from x_{k-1} is possible. $o(x_{k-1})$ denotes the state x_k which has most probability to be originated from state x_{k-1} .

Step 5. Termination.

For a detection threshold V_T , a detection result is declared

$$\left\{\hat{x}_K\right\} = \left\{\hat{x}_K\!: \ I_K(x_K, m_{r,K}^i, m_{d,K}^i) > V_T\right\} \tag{29}$$

Step 6. Resolving range-Doppler ambiguity.

The range estimation is disambiguated on the accumulation of track energy, the Doppler speed of target is compared.

Step 7. Backtracking.

For $\hat{x}_K$, for $k = K-1,...,1$

$$(\hat{x}_k, \hat{m}_{r,k}^i, \hat{m}_{d,k}^i) = \Psi_{k+1}(x_{k+1}) \tag{30}$$

Using Eq. (2) and (5), calculate the real target state $\tilde{x}_k = \left(\tilde{r}_k^i, \tilde{d}_k^i, a_k^i\right)^T$.

4. Simulation

In our simulation, we consider a scenario where one target has to be tracked by radar that can use staggering three PRFs mode. The target travels at initial range 65 km, with velocity v=3 Mach and heading a=235º. The carrier frequency of the radar is f=3 GHz, with the signal bandwidth B=5 MHz and the revisit interval T =1 s. The frequencies of multiple PRFs are 7 kHz, 10 kHz and 13 kHz, the correspondingly the maximum unambiguous range are 21.4 km, 15 km and 11.5 km respectively, and unambiguous Doppler are 175 m/s, 250 m/s and 325 m/s respectively. The number of pulses in train is 64. The number of Monte-Carlo experiments is 100.

Fig. 3 shows that the true trajectory of target and the trajectory of target ambiguous range in multiple PRFs mode. From Fig. 3, we can see that the range ambiguous number transition of PRF1 has occurred at time k=3 and of PRF2 at time k=14. By the proposed method the transition of range ambiguous number is accurately detected, so as to realize the accurate estimation of target motion trajectory. Fig. 4 show that the root mean squared error (RMSE) on the target range in multiple PRFs mode. From Fig. 4, we can see that RMSE of three PRF are all less than 40 m, the error is less than two range cell.

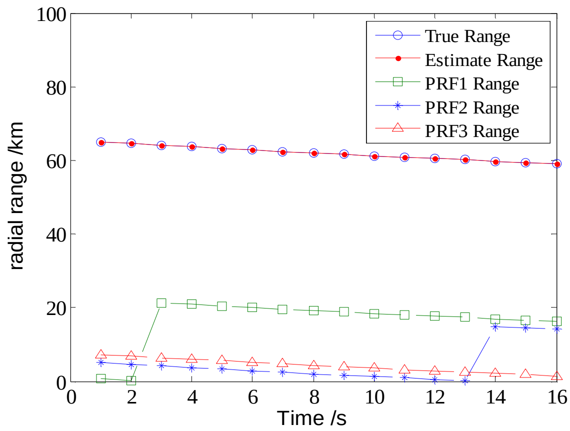

Fig. 3. The trajectory of target.

Fig. 5 shows that the true trajectory of target Doppler speed and the trajectory of target ambiguous Doppler in multiple PRFs mode. From Fig. 5, we can see that the Doppler ambiguous number transition of PRF1 has occurred at time k=14. By the proposed method, Doppler ambiguous number is accurately detected, so as to realize the accurate estimation of target Doppler trajectory.

Fig. 6 show that the root mean squared error (RMSE) on the target Doppler in multiple PRFs mode. From Fig. 6, we can see that RMSE of three PRF are all less than 10 m/s, the error is less than three Doppler cells.

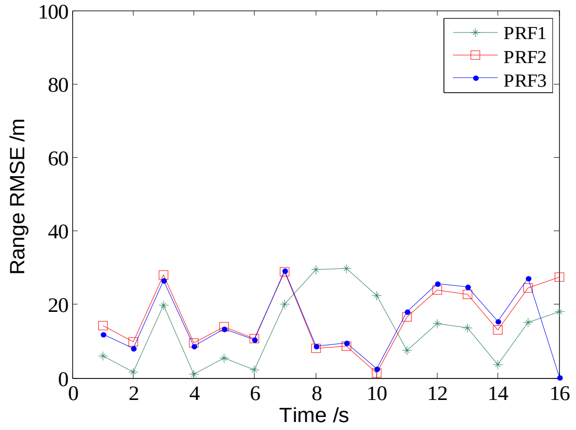

Fig. 4. RMSE on the target range.

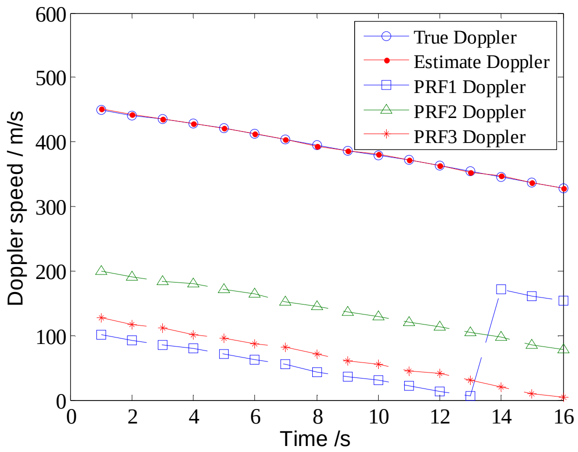

Fig. 5. The Doppler speed of target.

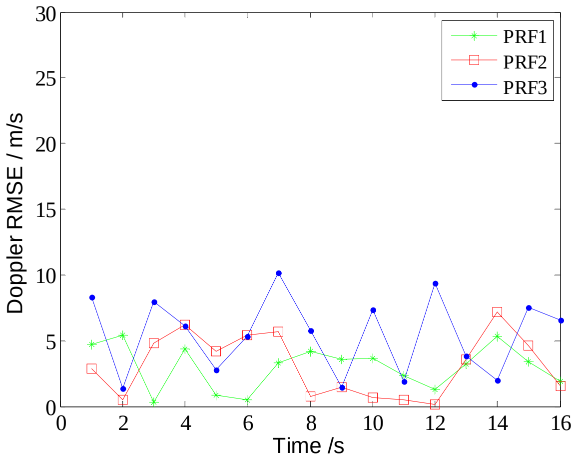

Fig. 6. RMSE on the target Doppler.

Fig. 7 shows the detection probability comparison of the proposed method and the residual lookup table method. From Fig. 7, we can see that the proposed method achieves 1.2 dB performance improvement at 50 % detect rate. This is because in the case of low SNR, the target energy of a PRF is lower than the pretreatment threshold, the correctness rate is decreased when the ambiguity is resolved by making use of residual look-up table method, resulting in the decreased detection probability of target. In contrast, the correctness rate of resolving the range-Doppler ambiguity is improved through accumulation of target energy by the proposed method.

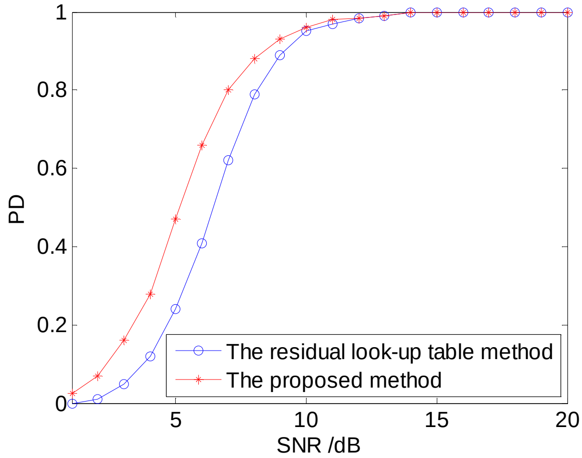

Fig. 7. Detection probability versus SNR.

5. Conclusions

In this paper, we presented a DP-TBD method which can cope with range-Doppler ambiguity and eclipsing effects for medium pulse repetition frequency radars. Exploiting the target state evolutions in the ambiguous range-Doppler maps as a hybrid system, we have proposed a track-before-detect method for joint MAP estimation of target's trajectory in the ambiguous states and the corresponding ambiguous number sequence. We also using phase information to provide a detection sensitivity improvement, and the complex likelihood was found to be an order of magnitude cheaper to evaluate than the existing envelope likelihood. Simulation shows that the probability of target detection PD achieves an approximately 1.2 dB performance improvement at 50 % detect rate, because of the correctness rate of resolving the range-Doppler ambiguity is improved through accumulation of target energy by the proposed method.

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