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Estimation of Human Heart Activity Using Ensemble Kalman Filter

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Abstract: Heart beat measurement techniques come across various challenges. Electrocardiogram (ECG) obtained sometimes does not reveal complete information about electrochemical activity of human heart, because of which functioning of heart cannot be studied properly. In this paper Ensemble Kalman Filter (EnKF) is used to generate ECG signal efficiently with better accuracy such that the drawbacks of current techniques are eliminated. Here EnKF is applied to second order mathematical model of human heart, input applied to this mathematical model is a pacemaker signal. The initial values of heart muscle movements and electrochemical activity as a discrete data set are used and prediction steps are commenced. EnKF uses ensemble integration technique to model error statistics which helps obtaining more precise output. The results are obtained with negligible sum squared error, therefore the ECG obtained using EnKF can diagnose the disease related to heart with better accuracy.

Keywords: Heart model, Ensemble Kalman filter, Electrocardiogram, Non-linear systems, State estimation techniques.

1. Introduction

The existing heart beat prediction techniques has various drawbacks since the additive noise present in the measured input signal given to the electrocardiogram (ECG) machine is not properly modeled and is not completely eliminated. Presence of noise results into noticeable variations in readings from the desired output and large value of sum squared error, which has to be reduced. ECG is a time varying signals which represents the electrical activity of cardiac tissues. A single cycle of ECG reflects systole and diastole of heart. ECG is recorded by placing electrode on the skin. ECG is used to indicate the cardiac health such that heart problems can be detected.

The drawbacks of current heart model estimation systems are such that the recorded signal sometimes may not reflect the symptoms of heart diseases, that is sometimes ECG does not reflects any abnormality at all. In some instances, the ECG may be entirely normal despite the presence of an underlying cardiac conditions, this is because the systems estimation about the heart movements is not proper. Improper estimation is one of the cause of improper diagnosis of diseases related to heart.

Heart beat estimation models are proposed prior with different estimation techniques. Extended Kalman Filter (EKF) is one of the estimation techniques. The existing models for heart beat with different algorithms are given in [4, 6] and [9, 10]. EKF is used to reduce the noise in the ECG signal [3] but the EKF requires large dimensional space for matrix computation. The technique of estimation of EnKF circumvents the high computational cost of storing and propagating the background error covariance for a large model dimension but EKF technique cannot circumvents the high computational cost since it does not propagates an ensemble of state from which the required covariance information is obtained at the time of update.

In this paper the EnKF [7] is used for state estimation. The major difference in the output of Extended Kalman Filter and Ensemble Kalman Filter is that the approximation of states of model in EnKF [1, 2 and 7] is more proper as compared to EKF since the EKF uses a linearized equation for the error covariance propagation while the EnKF nonlinearly propagates a finite ensemble of model trajectories. The EKF cannot account for the wider range of model errors as well as it cannot account for the horizontal error correlations in large systems for computational reasons. All these drawbacks of EKF are eliminated by EnKF [2, 8].

In this paper second order heart beat model is used which is given in section II, to estimate the state of heart muscle movements and electrochemical activity of human heart. Which includes the pacemaker signal, which is acting as an input to the model. The model used here has been developed as a mathematical differential equation which expresses three main processes of heart muscle movements (a) stable equilibrium point, (b) threshold for triggering muscle movement and (c) coming back to initial position.

The heart model shows the nonlinear behavior therefore the EnKF is applied in section III, as for nonlinear systems results obtained with EnKF are more accurate. The EnKF uses ensemble of output of forecasted steps for the estimation of current state, it keeps updating the ensemble values for upcoming states and it updates its covariance matrix also. The approximation converges faster with EnKF [1]. The improper noisy estimation is improved using EnKF.

The output waveform is used to analyze the behavior of heart muscles and electrochemical activity of human heart. With EnKF the quality of estimation is enhanced. The MATLAB simulation is carried out in section IV. For this system, the results are obtained with small value of sum squared error. The output waveforms are given in section IV. Estimated values of length of muscle fiber and electrochemical activity are plotted against the true signal values. From the plots it can be observed that how the muscle fiber length varies with the change in the value of electrochemical activity with respect to time in presence of noise as well as in absence of noise along with the presence of sufficient deviation between the heart model considered for human body and heart model in EnKF. It is observed that the results obtained with the help of EnKF are more precise.

2. Mathematical Model of Human Heart Beat

Human heart pumps blood throughout the body. The electrical impulse is responsible for producing heartbeat. The sinus node produces this electrical impulse, therefore the sinus node acts as natural pacemaker of heart. The sinus node is located at the top of the right atrium. The electrical signal produced travels through the heart tissue due to which atria and ventricles get contracted and relaxed, then blood gets pumped towards the body, this act as orderly progression of depolarization. At the time of depolarization right atrium and right ventricle pumps oxygen-poor blood returning from body towards the lungs, so that blood will get re-oxygenated. The left atrium and left ventricle pumps this oxygenated blood throughout the body. The back flow of the blood is prevented by heart valves. The systemic circuit helps spreading blood in whole body.

The orderly pattern of depolarization gives rise to the characteristic ECG tracing. ECG conveys a large amount of information about the structure of the heart and the function of its electrical conduction system. The ECG is used to measure the rate and rhythm of heart beats, the size and position of the heart chambers, the presence of any damage to the heart's muscle cells or conduction system, the effects of cardiac drugs and the function of implanted pacemakers if any. The heart model [4, 6] and [9, 10] is developed as second order heart beat differential equation such that using it ECG can be traced

$$\epsilon * \dot{x}_1 = -(x_1^3 - Tx_1 + x_2)\,, \tag{1}$$

$$\dot{x}_2 = (x_1 - x_d) + (x_d - x_s) * u, \tag{2}$$

where x_1 represents the length of muscle fiber of heart, x_2 representsthe electrochemical activity, x_2 is the one of those factors which are responsible for producing heart muscle movements. ϵ represents the small positive constant, T represents the tension in muscle fiber, x_d represents the length of muscle fiber in diastole, x_s represents the length of muscle fiber in systole and u represents the pacemaker signal applied as the input signal. The model is developed such that it should satisfy the three main properties which are (a) the model exhibits the equilibrium state corresponding to diastole, (b) it also contains a threshold for triggering the electrochemical wave emanating from the pacemaker causing the heart to contract into systole and (c) it reflects the rapid return to the equilibrium state.

The first property is proved by taking general equations

$$\frac{dx_1}{dt} = f(x_1, x_2) \tag{3}$$

$$\frac{dx_2}{dt} = g(x_1, x_2) \tag{4}$$

This system is linearized around the initial values x_{10}, x_{20} and it is observed that for stability and to make the system free from undesired oscillations the Eigen values should be negative and real [6]. The another important point taken into consideration is that the rate of change of muscle fiber contraction depends at any particular instant on tension of the fiber and the chemical control changes at the rate directly proportional to the muscle fiber tension, this is proved by modifying the system as given

$$\epsilon * \frac{dx_1}{dt} = -T(x_1 - x_{10}) - (x_2 - x_{20}) \quad (5)$$

$$\frac{dx_2}{dt} = (x_1 - x_{10}) \quad (6)$$

The second property is proved by considering that the threshold is responsible for triggering the muscle movements. There are always two equilibrium states for the heart i.e. diastole and systole. An equation (5, 6) represents only one equilibrium state that is diastole. For systole the pacemaker input is considered as a threshold and given to the system of equation (5) and (6) therefore the system is modified [4, 6] as

$$\begin{aligned}\epsilon * \frac{dx_1}{dt} = {} & -T(x_1 - x_{10}) - (x_2 - x_{20}) \\ & - (x_1 - x_{10})^3 \\ & - 3x_{10}(x_1 - x_{10})^2\end{aligned} \quad (7)$$

$$\frac{dx_2}{dt} = x_1 - x_{10} \quad (8)$$

and above two equations for convenience are written as

$$\epsilon * \frac{dx_1}{dt} = -({x_1}^3 - Tx_1 + x_2), \quad (9)$$

where equation (9) is valid only for T > 0,

$$\frac{dx_2}{dt} = x_1 - x_{10} \quad (10)$$

The above model does not reflects the third essential property observed from the phase portraits [6], to satisfy the third property the equation is finally modeled as

$$\epsilon * \frac{dx_1}{dt} = -(x_1^3 - Tx_1 + x_2) \quad (11)$$

$$\frac{dx_2}{dt} = (x_1 - x_d) + (x_d - x_s) * u\,, \quad (12)$$

where u [6] is the control variable associated with the pacemaker which is defined as u =1 for $x_{20} \le x_2 \le x_{21}$ and for those values of x_1 for which

$$(x_1^3 - Tx_1 + x_2) > 0 \quad (13)$$

and all values of x_1 and u = 0 otherwise, where x_{21} is the first value of x_2 obtained after x_{20}, which is the final second order equation to which the EnKF is applied. Here the pacemaker signal is acting as input signal u, which is generated either by natural pacemaker or by artificial cardiac pacemaker attached with unhealthy heart.

3. Ensemble Kalman Filter Applied to Heart Model

The Ensemble Kalman Filter [2] is an estimator to predict the statistics of noise in system and for predicting the system's state by using ensemble integration. EnKF is used for those systems which shows nonlinear behavior. EnKF is used for the system which has large data samples obtained after discretization of partial differential equation, therefore EnKF uses sample covariance instead of covariance matrix. EnKF always assumes the probability distributions involved are Gaussian. Ensemble is a sample which is independent identically distributed random variables and its probability distribution is represented by the mean and covariance, therefore it is assumed that the ensemble is normally distributed. The ensemble covariance is computed from all ensemble members together, which introduces dependence and the EnKF formula [8] is a nonlinear function of the ensemble, which destroys the normality of the ensemble distribution.

For finding the error statistics and state estimate for current time t_n where n represents the time instant, EnKF uses the ensemble of previous states and corresponding outputs. If it is assumed that there are q forecasted state estimates at n sec, this initial or forecasted states are used to estimate the output at n^{th} state and then ensemble matrix gets updated which is then used to calculate the output and next state. The ensemble [7] is written as

$$x_n^f \triangleq (x_n^{f_1}), (x_n^{f_2}), \dots, (x_n^{f_q}), \quad (14)$$

where f indicates the ensemble member number, $x_n^{f_i}$ is obtained after applying $x_{n-1}^{f_i}$ to the system model where $x_{n-1}^{f_i}$ is calculated as

$$(initial\ true\ state\ value) + f_0\,, \quad (15)$$

where initial value matrix has initial values of x_1 and x_2. Where f_0 represents deviation from true state values.

In this paper the ensemble size that is forecasted estimates states is chosen as q = 40. Initially these 40 values are applied to system model which considers those values as initial state and then for each such value current states are calculated, which acts as modified ensemble whose mean is calculated. The ensemble mean [1, 2] and [5] is calculated using all these values

$$\bar{x}_n^f \triangleq \frac{1}{q}\sum_{i=1}^{q} x_n^{f_i}, \quad (16)$$

which is denoted as

$$\bar{x}_n^f \in R^{m\times q} \quad (17)$$

therefore 40 different values of output will get generated after processing it as per EnKF algorithm, average of this output values gives the current estimate.

The estimated state value for current state using Ensemble Kalman Filter algorithm is given by

$$\bar{x}_n^a = \frac{1}{q}\sum_{i=1}^{q} x_n^{a_i} \quad (18)$$

$$x_n^{a_i} = x_n^{f_i} + \hat{K}_n\left[ym_n - h\left(x_n^{f_i}\right)\right] \quad (19)$$

and i varies from 1 to q, where $\hat{K}_n$ is filter gain calculated as ratio of error covariance matrices,

$$\hat{K}_n = \hat{P}_{xy_n}^f(\hat{P}_{yy_n}^f)^{-1} \quad (20)$$

Further the elements of sample covariance matrix are found out, Ensemble Kalman Filter's state error covariance matrix is given by

$$E_n^f \in R^{m\times q} \quad (21)$$

with ensemble error matrix as

$$E_n^f \triangleq \left[x_n^{f_1} - \bar{x}_n^f, \ldots\ldots.., x_n^{f_q} - \bar{x}_n^f\right] \quad (22)$$

and the output error matrix is given by

$$E_{y_n}^a \triangleq \left[y_n^{f_1} - \bar{y}_n^f, \ldots\ldots.., y_n^{f_q} - \bar{y}_n^f\right], \quad (23)$$

where output matrix is calculated for one of the values out of two states i.e. x_1 or x_2, which is getting estimated in correspondence with measured signal. For n^{th} state ensemble of estimated states at $(n-1)^{th}$ state is used for calculations. The approximated values [2] are

$$\hat{P}_n^f \triangleq \frac{1}{q-1}E_n^f * \left[E_n^f\right]^T \quad (24)$$

$$\hat{P}_{xy_n}^f \triangleq \frac{1}{q-1}E_n^f * \left[E_{y_n}^f\right]^T \quad (25)$$

$$\hat{P}_{yy_n}^f \triangleq \frac{1}{q-1}E_{y_n}^f * [E_{y_n}^f]^T \quad (26)$$

from equations (25) and (26) EnKF gain is obtained. The approximated error covariance matrix is

$$\hat{P}_n^a \triangleq \frac{1}{q-1}E_n^a * (E_n^a)^T \quad (27)$$

Perturbed measured value is obtained as given below, when true state values are applied to heart model then the output gets generated which has processed values of x_1 and x_2. The processed values then gets added with the state noise w_n [7]. w_n is sampled from a normal distribution with zero average and covariance Q_n. The sample error covariance matrix calculated from w_n converges to Q_n as $q \to \infty$.

$$y1_n = y(output\ at\ nth\ state) + w_n \quad (28)$$

Therefore $y1_n$ contains the processed values of x_1 and x_2. At the time of measurement if any one value out of this two state values is measured then in that value measurement noise v_n [7] gets added.

$$ym_n = y1_n + v_n, \quad (29)$$

where v_n is a zero mean random variable which has normal distribution and covariance R_n. The sample error covariance matrix obtained from v_n converges to R_n as $q \to \infty$.

Substituting all computed values in equation (19) the estimated value at n^{th} state is found out. The current state value is given as

$$x_n^{a_i} = x_n^{f_i} + \hat{K}_n\left[y1_n + v_n - h\left(x_n^{f_i}\right)\right] \quad (30)$$

for all i varies from 1 to q and finally current output [1, 2] and [5] is estimated as

$$\bar{x}_n^a = \frac{1}{q}\sum_{i=1}^{q} x_n^{a_i} \quad (31)$$

The computation burden in case of EKF due to approximation of the nonlinearity $f(x,u)$ and $h(x)$ [7] in evaluation of the filter gain $\hat{K}_n$ is reduced in EnKF, therefore no need to calculate the Jacobian of $f(x,u)$ and $h(x)$ thus EnKF has less numbers of computations.

4. Matlab Simulation and Discussion

Performance of the proposed EnKF after applying it to the heart model is observed using MATLAB simulation. The parameters estimated through this simulation are length of muscle fiber of heart (x_1) and electrochemical activity (x_2). The model of heart given in equations (1, 2) is chosen for analysis. Estimator is tracking the true value of parameters efficiently, is observed from the results of different case studies. The value of sum squared error obtained between true value and estimated value of the parameters is very small, sum squared error for different case studies is given in Table 2.

Table 1 shows the values [10] used for MATLAB simulation of heart model. The pacemaker signal [10] is used as input signal u. Initial true state values given to model are considered as -0.3 for x_1 and 0.01 for x_2.

The set of differential equations (1, 2) is solved in MATLAB using values as per Table 1. The solution of this equation gives true state values, which further gets added with the state noise w_n and measurement noise v_n due external environmental factors affecting. Noise values are given as

$$w_n = \begin{bmatrix} c1 \\ c2 \end{bmatrix} * random\ noise \tag{32}$$

$$v_n = c3 * random\ noise \tag{33}$$

Table 1. Set Parameters for Heart Model.

Parameters	Values
T	1 mg
(x_d)	1.024 micrometer
(x_s)	-1.3804 micrometer
ϵ	0.2
x_1 (initial)	-0.3
x_2 (initial)	0.01
Ensemble size	EnKF members ≥ twice the number of states

The constants $c1$, $c2$ and $c3$ show the amplitude of respective noise levels. $(c1 * random\ noise)$ gets added in x_1 and $(c2 * random\ noise)$ gets added in x_2. The solution of equations (1, 2) which is added with state noise is known as true value, this true value gets added with measurement noise, this sum is known as measured value which act as input to EnKF. This measured value is estimated by EnKF.

EnKF always have number of members greater than or equal to twice the number of states. Initial EnKF members are formed as true initial value added with deviation, given by

$$EnKF(member) = \begin{bmatrix} -0.3 \\ 0.01 \end{bmatrix} + f_0, \tag{34}$$

where f_0 is the deviation, here deviation value used for simulation are

$$f_0 = \begin{bmatrix} -0.4 \\ -0.01 \end{bmatrix} \tag{35}$$

This forecasted ensemble member is considered as initial estimation of parameters. Here second order differential equation is used therefore it has two states to be found, so the ensemble of size greater than or equal to twice the number of states is used for estimation. If EnKF is considered with more than two members then those members are declared as given in equation (14).

Following case studies are done after simulation of heart model with EnKF in absence of noise as well as in presence of noise. The graphs are plotted between true value against estimated value.

- Case study 1: The state noise and measurement noise values are assumed negligible. The ensemble size is considered 40. Time steps are considered as 0.01 sec and graph is plotted for 1 sec. The result is shown in Fig. 1.

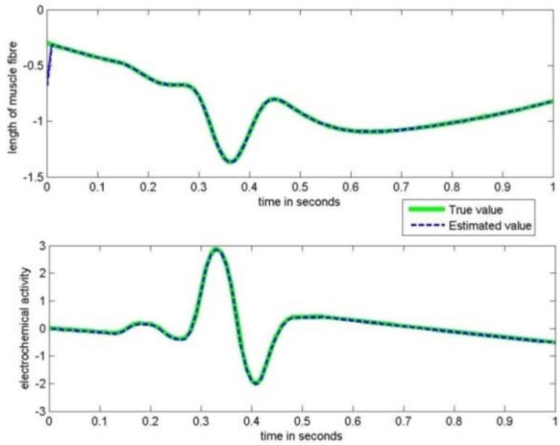

Fig. 1. Simulation result for case study 1.

- Case study 2: The state noise and measurement noise values are assumed negligible. The ensemble size is considered with 80 members. Time steps are considered as 0.01 sec and graph is plotted for 1 sec. The result is shown in Fig. 2.

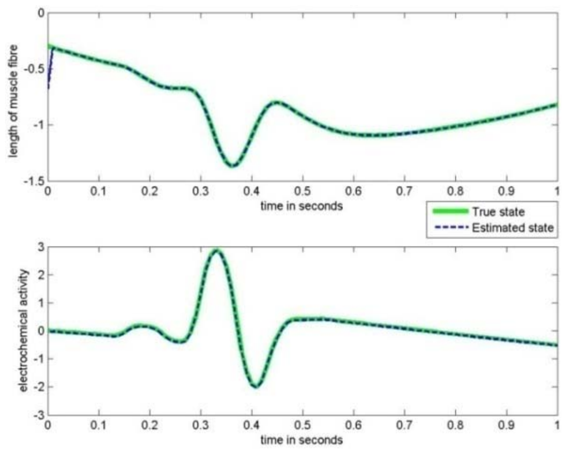

Fig. 2. Simulation result for case study 2.

- Case study 3: The state noise and measurement noise values are assumed negligible. The ensemble size is considered with 40 members. Time steps are considered as 0.0001 sec and graph is plotted for 1 sec. The result is shown in Fig. 3.
- Case study 4: The small amount of state noise and measurement noise values are added. The ensemble size is considered with 40 members. Time steps are considered as 0.01 sec and graph is plotted for 1 sec. The result is shown in Fig. 4.
- Case study 5: The large amount of state noise and Same measurement noise values as used in case study 4 are added. The ensemble size is considered with 40 members. Time steps are considered as 0.01 sec and graph is plotted for 1 sec. The result is shown in Fig. 5.

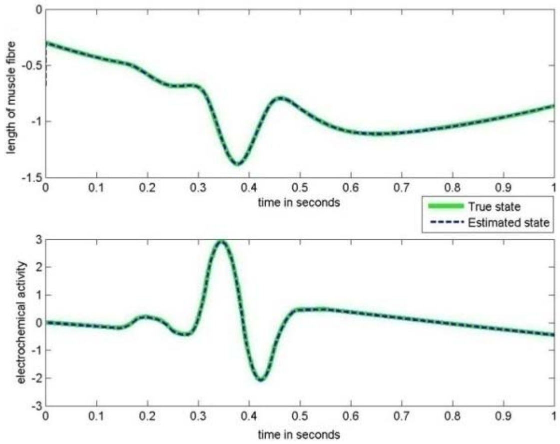

Fig. 3. Simulation result for case study 3.

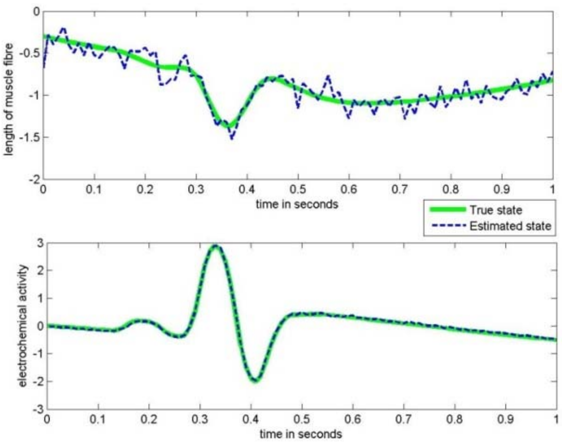

Fig. 4. Simulation result for case study 4.

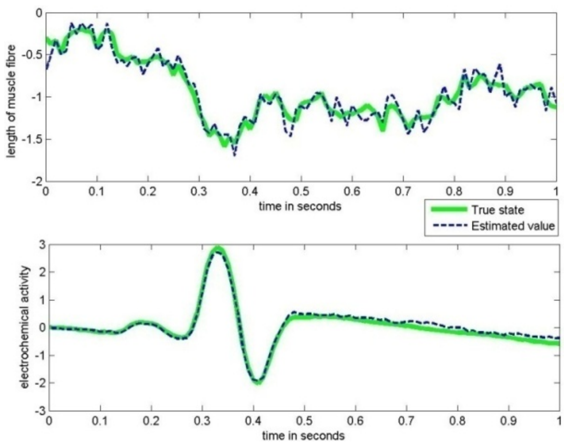

Fig. 5. Simulation result for case study 5.

- Case study 6: The small amount of state noise and large amount of measurement noise values are added. The ensemble size is considered with 40 members. Time steps are considered as 0.01 sec and graph is plotted for 1 sec. The result is shown in Fig. 6.
- Case study 7: Negligible amount of state noise and measurement noise values are added. The ensemble size is considered with 40 members. Time steps are considered as 0.01 sec and graph is plotted for 4 sec. The result is shown in Fig. 7.
- Case study 8: The small amount of state noise and measurement noise values are added. The ensemble size is considered with 40 members. Time steps are considered as 0.01 sec and graph is plotted for 4 sec. The result is shown in Fig. 8.

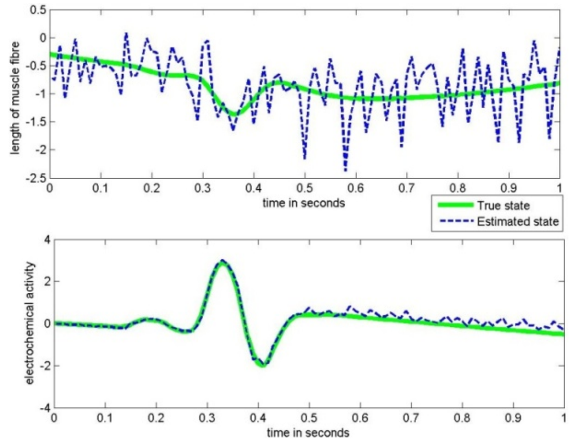

Fig. 6. Simulation result for case study 6.

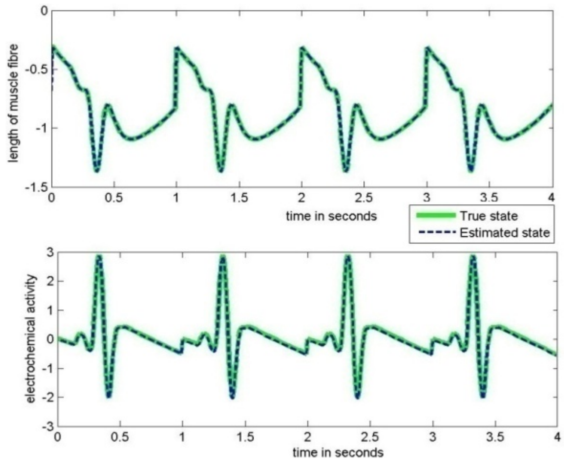

Fig. 7. Simulation result for case study 7.

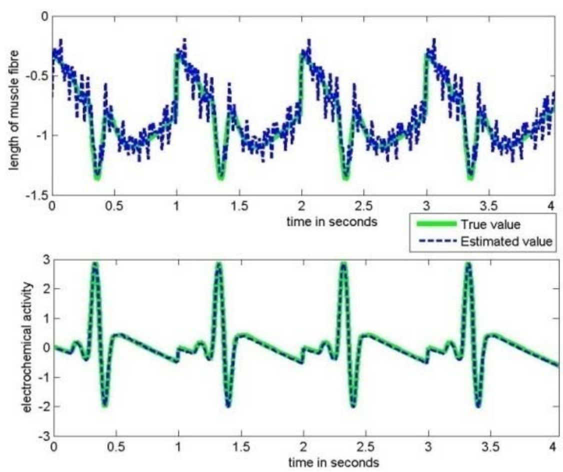

Fig. 8. Simulation result for case study 8.

The values of constants c1, c2 and c3 in presence and absence of noise along with sum squared error obtained for all this case studies are given in Table 2. The sum squared error (SSE) is calculated as,

$$\mathrm{SSE} = \sum_{i=1}^{n} (y1_i - \bar{x}_i)^2 \,, \tag{36}$$

where i represents the time instant.

Table 2. Values Used for Simulation.

Case study number	C1	C2	C3	Sum squared error
1	0	0	0	0.0307
2	0	0	0	0.0491
3	0	0	0	0.7871
4	10^{-3}	10^{-4}	0.1	1.1787
5	10^{-1}	10^{-2}	0.1	2.0054
6	10^{-3}	10^{-4}	0.5	31.7522
7	0	0	0	0.0017
8	10^{-3}	10^{-4}	0.1	0.0049

5. Conclusion

The observation of case studies 1, 2 and 3 reveals that in absence of noise EnKF is tracking the true values very efficiently with sum squared error approximately zero. In case study 2 the size of ensemble is increased but with increased size sum squared error is also got sufficient increment thus ensemble size should not be increased a lot. For case studies 4 and 5 measurements noise is kept constant and state noise is varied, with increased value of state noise small increment in sum squared error is observed.

In case study 6 the measurement noise is further increased. From case studies 6, 4 it is noted that when state noise is constant and measurement noise is increased the increment in sum squared error is very large. Therefore it is concluded that sum squared error is influenced more by the value of measurement noise as compared to state noise. For multiple cycles case studies 7, 8 are done and corresponding sum squared error are noted. From case studies it is concluded that EnKF is estimating the parameters successfully in absence of noise as well as in presence of noise within the proper noise bounds. Therefore EnKF applied to heart model can be used to diagnose the disease related to heart with better accuracy.

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