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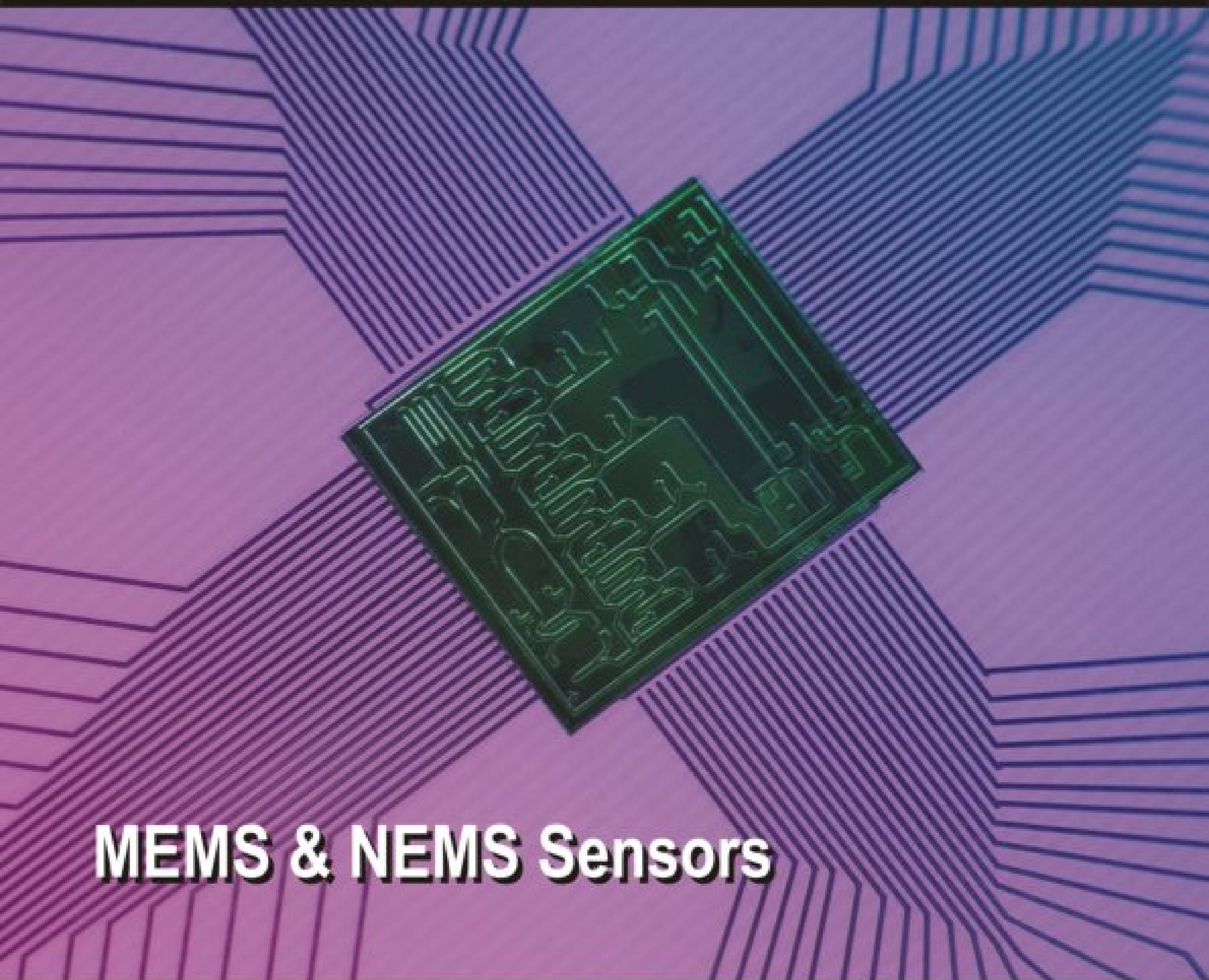

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Research on Nonlinear Vibration in Micro-Machined Resonant Accelerometer

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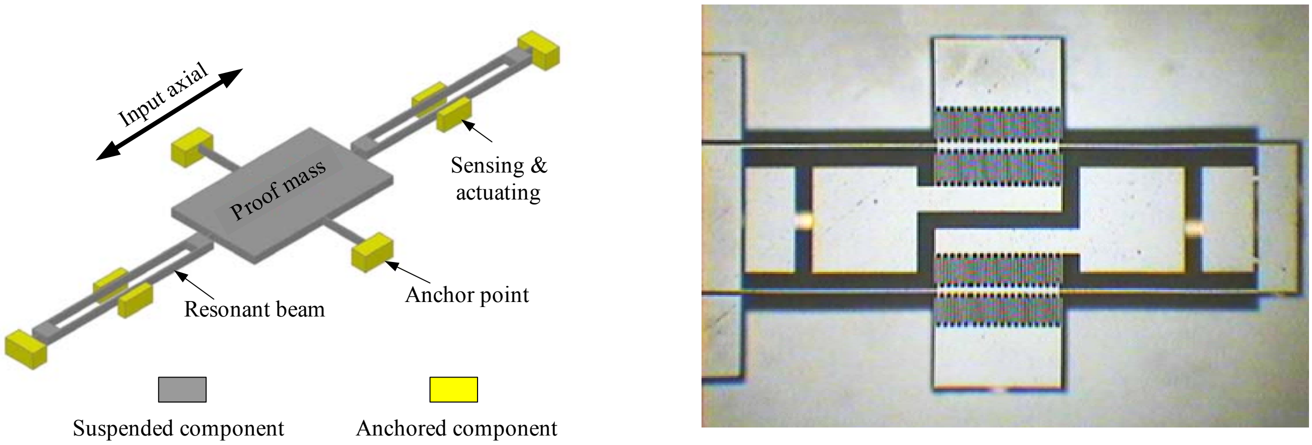
Abstract: This paper analyzes the nonlinear vibration of the Micro-machined Resonant Accelerometer (MRA) by system modeling and simulation. The optimization designs to weaken the influence of the nonlinear vibration are proposed. With decreasing the mechanism dimension of the MRA, the effects of the nonlinear vibration become more obviously. The nonlinear vibration affects the precision and system stability of the micro-machined resonant accelerometer. The simulation results of the nonlinear system are well consistent with the experiments, and predict that the optimizing structure will effectively weaken the nonlinear effect. *Copyright © 2012 IFSA.*

Keywords: Micro-machined resonant accelerometer, Nonlinear vibration, Duffing equation, Optimizing structure.

1. Introduction

Micro-machined Resonant Accelerometer is a novel micro-machined accelerometer based on force-frequency characteristics of resonant beams. As the Schematic of MRA shown in Fig. 1(a), when input acceleration act on the proof mass, the inertia force along the sensitive axis causes the nature frequency of resonant beams on both sides of proof mass shifting in the opposite directions. Sensing the nature frequency of the resonant beams and outputting the differential frequency make it possible to have high precision and strong anti-jamming capability.

Previous works have proved the feasibility of the system [1] and some system optimizations have done to achieve a higher sensitivity. The SEM picture of MRA is shown in Fig. 1(b). In section 2 of this paper, we build the model of the nonlinear vibration in two ways: one is the dynamic-static method based on the Newton's second Law, and the other is based on expressions of energy and variation, called the method of Lagrange equation. We analyses the influence of the nonlinear vibration in section 3 and propose the methods to reduce the bad influence of nonlinear vibration in section 4, and the new structure is being implemented.



(a) Schematic of MRA

(b) SEM picture of MRA

Fig. 1. Schematic and SEM picture of MRA.

2. Modeling of the Nonlinear Vibration

With the decreasing of the mechanism dimensions of the Micro-machined Resonant Accelerometer, the axial tension caused by the relatively large displacement of the resonant beam cannot be neglected. [2] The resonant beam can be simplified as an axially loaded elastic slender beam with a cross section A, momentum of inertia I and length L. Denoting the density of the material with ρ, the Young's Modulus with E, the initial constant axial load N and the transversal displacement of the resonant beam with $y(x,t)$, x being the axial coordinate and t the time.

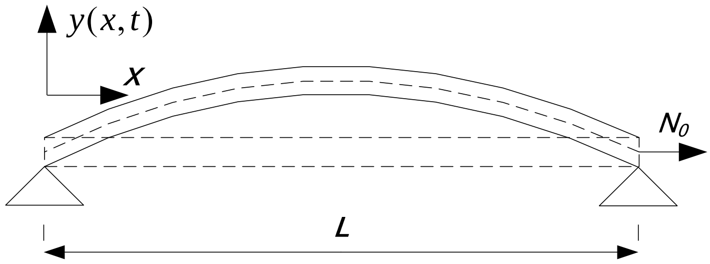
There are several hypotheses for the resonant beam as follows: (1) the beam is a Euler-Bernoulli beam without regard to the effects of the shearing deformation and the momentum of inertia; (2) variation of the cross section during vibration is neglected; (3) the stretching of the beam during vibration is small but finite that the linear stress-strain relationship is still applicable. [4].

2.1. Dynamic-static Method

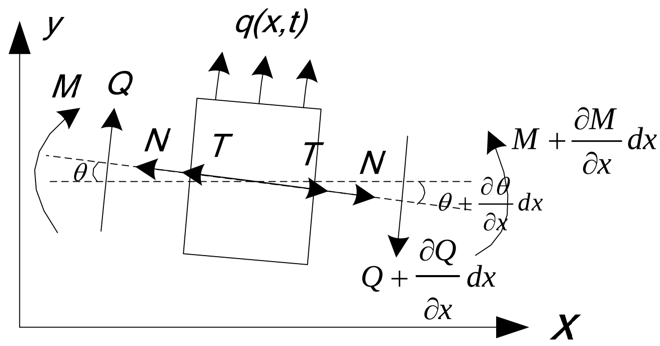
With the above hypotheses, we build a simplified model of the resonant beam as showed in Fig. 2(a) and analyze the differential segment as showed in Fig. 2(b). The differential equation for the transverse displacement is obtained as equation (1):

$$\rho A\ddot{y}+\left(EIy''\right)''-Ny''=q\left(y\right), \tag{1}$$

where $\dot{y}=\frac{\partial y}{\partial t}, \ddot{y}=\frac{\partial^2 y}{\partial t^2}, y'=\frac{\partial y}{\partial x}, y''=\frac{\partial^2 y}{\partial x^2}$. The equation (1) is a normal vibration equation of the resonant beam and several solutions are approached. The frequency solution of the first mode as showed in equation (2) indicates that, resonant frequency increases when N is a tensile load while decreases when N is a compressive load.



(a) Single-degree-of-freedom model of MRA oscillator.



(b) Force analysis of the resonant beam.

Fig. 2. The simplified modal and force analysis of the MRA oscillator.

$$f = f_0 \sqrt{1 + \alpha \frac{N_0 L^2}{EI}}, \quad f_0 = \frac{c^2}{2\pi L^2} \sqrt{\frac{EI}{\rho A}} \tag{2}$$

f_0 is the first mode frequency of the resonant beam without axial load N. The coefficient c and α depend on the boundary condition. These three equations are always available when discuss the macro resonant beams. While in the micro-machined resonant accelerometer, the high Q makes it easy to get relatively large amplitude of the vibration. Thus the axial tension caused by the relatively large displacement of the resonant beams cannot be neglected.[3] Denoting the axial tension caused by the vibration displacement with T, and then equation (1) should be changed into (3):

$$\rho A \ddot{y} + \left(EIy''\right)'' - Ny'' - Ty'' = q\left(y\right), \tag{3}$$

With the linear stress-strain hypothesis, the additional axial load T can be obtained as equation (4):

$$T = EA\frac{\Delta L}{L} \approx \frac{EA}{2L}\int_0^L y'^2 dx, \tag{4}$$

Denote $\frac{EA}{2L}$ by k_x and substitute the expression of T in equation (3), then the nonlinear vibration equation for the resonant beam is obtained:

$$\rho A\ddot{y} + \left(EIy''\right)'' - Ny'' - k_x y'' \int_0^L \frac{1}{2}\left(y'\right)^2 dx = q\left(y\right), \tag{5}$$

Equation (5) is a forth order partial differential equation which could not acquire an accurate analytic solution. Next part we introduce the method of Lagrange equation to obtain the approximate solution of the equation.

2.2. Method of Lagrange Equation

Known that the system has certain mode of vibration which is time-independent, we introduce the normal form of solution as equation (6), where $\varphi(x)$ is the first order mode shape and f is the natural frequency of the harmonic vibration $Y(t)$ [3]:

$$y(x,t) = \varphi(x)Y(t) = \varphi(x)Y_{\max} \sin\left(2\pi ft + \theta\right), \tag{6}$$

Then the Lagrange Function expressed with kinetic energy T and potential energy U is:

$$\mathrm{L} = \mathrm{T} - \mathrm{U} = \frac{1}{2}\dot{Y}^2 \int_0^L \rho A\varphi^2 dx - [\frac{1}{2}Y^2 \int_0^L EI\left(\varphi''\right)^2 dx + \frac{1}{2}NY^2 \int_0^L \left(\varphi'\right)^2 dx + \frac{1}{4}k_x Y^4 \left(\int_0^L \left(\varphi'\right)^2 dx\right)^2], \tag{7}$$

With the Lagrange Function (7), we got the Lagrange Equation of the nonlinear system (8):

$$\frac{d}{dt}\frac{\partial \mathrm{L}}{\partial \dot{Y}} - \frac{\partial \mathrm{L}}{\partial Y} = 0, \tag{8}$$

$$\ddot{Y}\int_0^L \rho A\varphi^2 dx + Y\int_0^L EI\left(\varphi''\right)^2 dx + NY\int_0^L \left(\varphi'\right)^2 dx + k_x Y^3 \left(\int_0^L \left(\varphi'\right)^2 dx\right)^2 = 0, \tag{9}$$

Equation (9) can be simplified as (10) which is a typical nonlinear Duffing Equation:

$$M\ddot{Y} + k_1 Y + k_3 Y^3 = 0, \tag{10}$$

k_3 is called the third order nonlinear coefficient of the system. Different from the linear solution, the frequency of the resonant beam depend upon the vibration amplitude [4]:

$$f = f_0\left(1 + \frac{3}{8}\frac{k_3}{k_1}Y^2{}_{\max}\right), \tag{11}$$

3. Properties of the Nonlinear Vibration Beam

From the Duffing Equation we can obtain the relationship between the resonant frequency f, harmonic driving force $F = F_0 \cos\omega t$, and amplitude $Y_{\max}$ is:

$$\left(\frac{F_0}{k_1}\right)^2 = \left(2\left(1 - \frac{f}{f_0}\right)Y_{\max} + \frac{3}{4}\frac{k_3}{k_1}Y^3{}_{\max}\right)^2, \tag{12}$$

For $k_3 > 0$, the resonance curve shows a hard spring effect as show in Fig. 3. The figure also shows that the bistable and non-steady state of the nonlinear system.

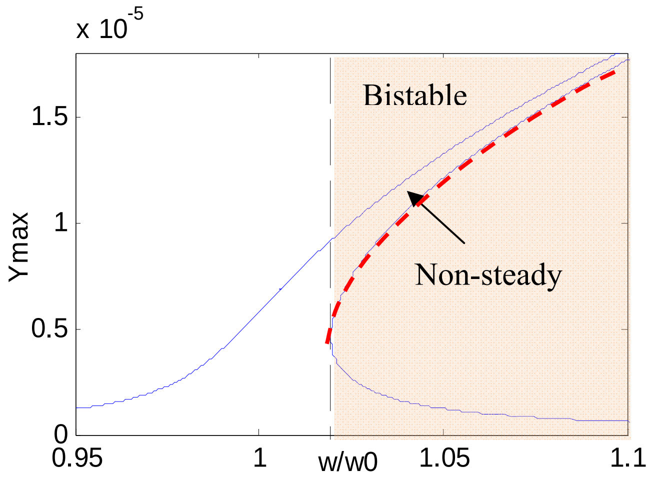
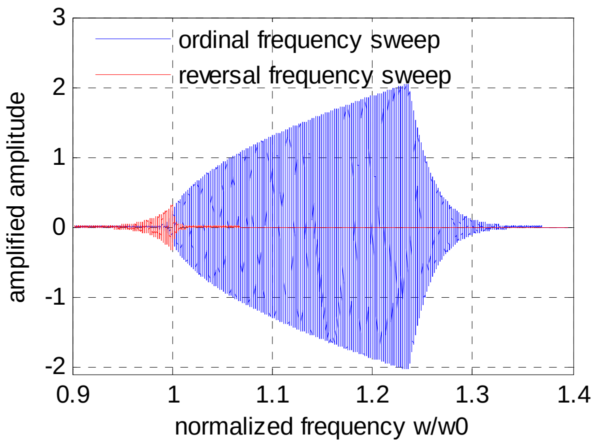
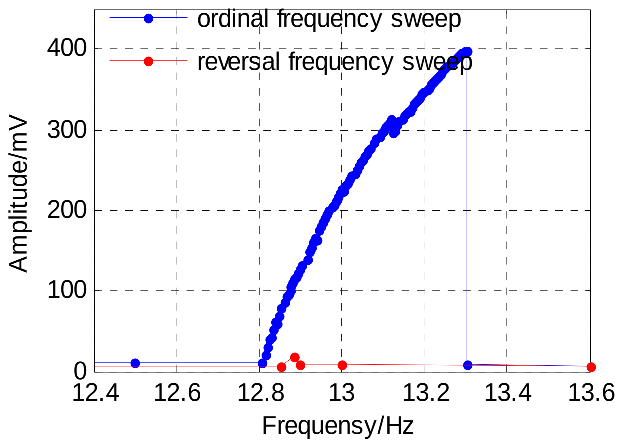


Fig. 3. Vibration amplitude vs. Driving frequency.

There are two important effect of this nonlinear system while open-loop frequency sweeping: amplitude jumping and frequency-amplitude hysteresis.[5] As show in Fig. 4, when ordinal frequency sweeping, the amplitude increases gradually but jumps to a relatively low value from Y_1 when at the frequency f_1; while reversal frequency sweeping, the amplitude jumps to a relatively high value Y_2 at the frequency f_2 and then decreases gradually. Since $f_1 > f_2$, $Y_1 > Y_2$, the frequency-amplitude hysteresis loop is formed. We simulated the nonlinear by Matlab Simulink, as showed in Fig. 4(a), and the open loop experiment result of the real resonant accelerometer beam showed in Fig. 4(b) proves the validity of the nonlinear system model.



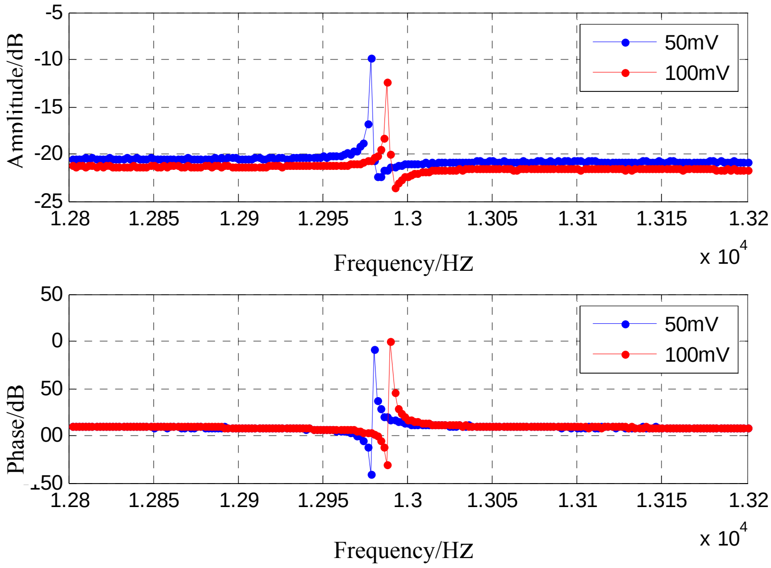
(a) The result of the nonlinear system simulation.



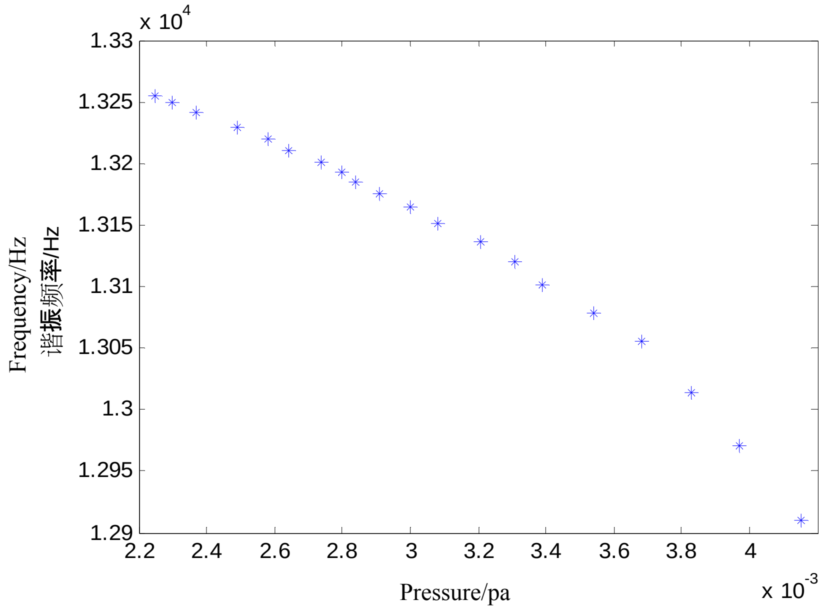
(b) The result of the nonlinear system experiment.

Fig. 4. The results of simulation and experiment show the frequency characteristics of the nonlinear system.

Since we have known that the frequency of the resonant beam depend upon the vibration amplitude in the nonlinear system, the driving force and system damping all make an effect on the resonant frequency, as showed in Fig. 5. The experiment results show that, frequency increases while the driving amplitude increases, and decreases while the pressure of the experiment environment increases, which are well consistent with the model analyses. Also we can see the figure of merit Q becomes slightly lower while the driving amplitude increase, which may indicate the instability of the nonlinear vibration.



(a) Resonant frequency varied with driving amplitude.



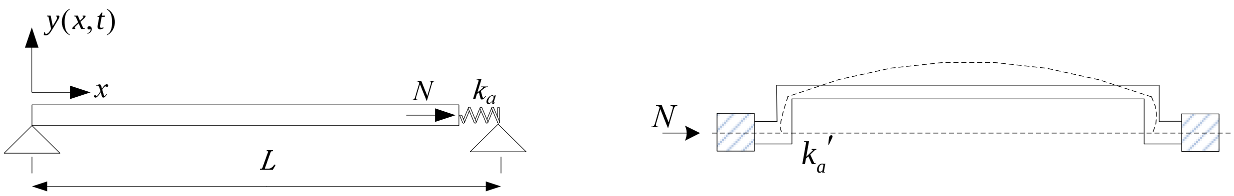
(b) Resonant frequency varied with environment pressure.

Fig. 5. Resonant frequency is susceptible to driving amplitude and environment pressure.

4. Methods to Weaken the Nonlinear Vibration

As we know, the nonlinear vibration is produced by the axial tension, caused by the relatively large displacement of the resonant beam. When the amplitude of the resonant beam is much smaller than the beam dimension, the axial tension caused by the displacement can be neglected. So one way to weaken the nonlinear vibration is to decrease the amplitude of the resonant beam, which means to decrease the driving force and make the force stable [6].

In order to weaken the axial tension caused by the displacement, we could add a spring in series as showed in Fig. 6(a). For single beam, the physical structure could be designed as shown in Fig. 6(b). The elastic coefficient of the spring in series is $k_a = \frac{EA}{12l}\left(\frac{w}{l}\right)^2$, where A is the cross- section area of the spring, w is the width and l is the length of the spring. The third order nonlinear coefficient $k_3 = \frac{k_a k_x}{k_a + k_x}\left(\int_0^L (\varphi')^2\, dx\right)^2$ will be decreased remarkably while the slender proportion $\frac{w}{l}$ decreased. The Simulink result showed in Fig. 7 indicates that, when the nonlinear coefficient k_3 becomes $1/10$ of the original value, the influence of the amplitude to the frequency become much smaller than before.[7]



(a) The model of resonant beam with spring in series.

(b) Physical structure of the single beam with spring in series.

Fig. 6. The model and structure of resonant beam with spring in series

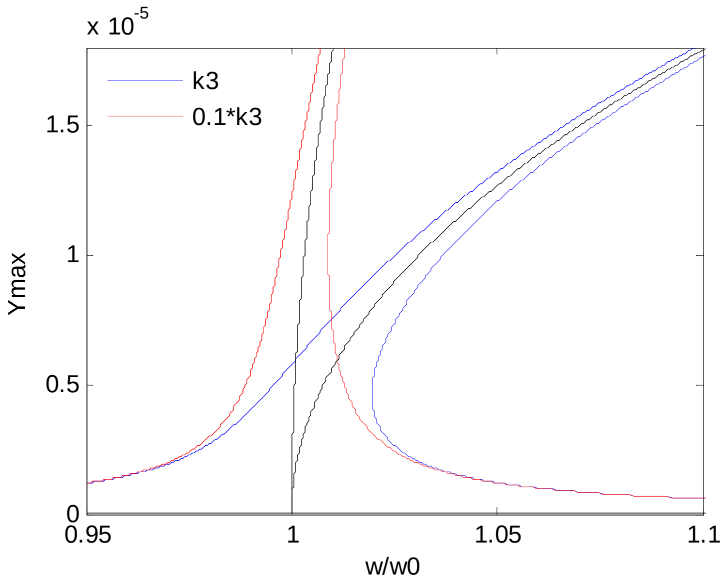


Fig. 7. The optimizing structure reduces the nonlinearity of the Vibration amplitude VS Driving frequency.

5. Conclusions

We build a model with the consideration of axial tension caused by the large displacement of the resonant beams, and then simulate and analyze the influence of the nonlinear effects. The simulation results of the system characteristics are well consistent with the experiments, which verify the existence of nonlinear vibration and the validity of the nonlinear system model. With the simulation and analyses, we propose an optimization designation to weaken the influence of the nonlinear vibration, and the new structure is being implemented. By optimizing the structure and experiment condition, the nonlinear vibration could be reduced effectively.

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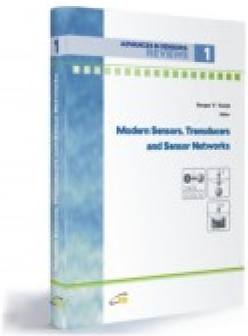

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