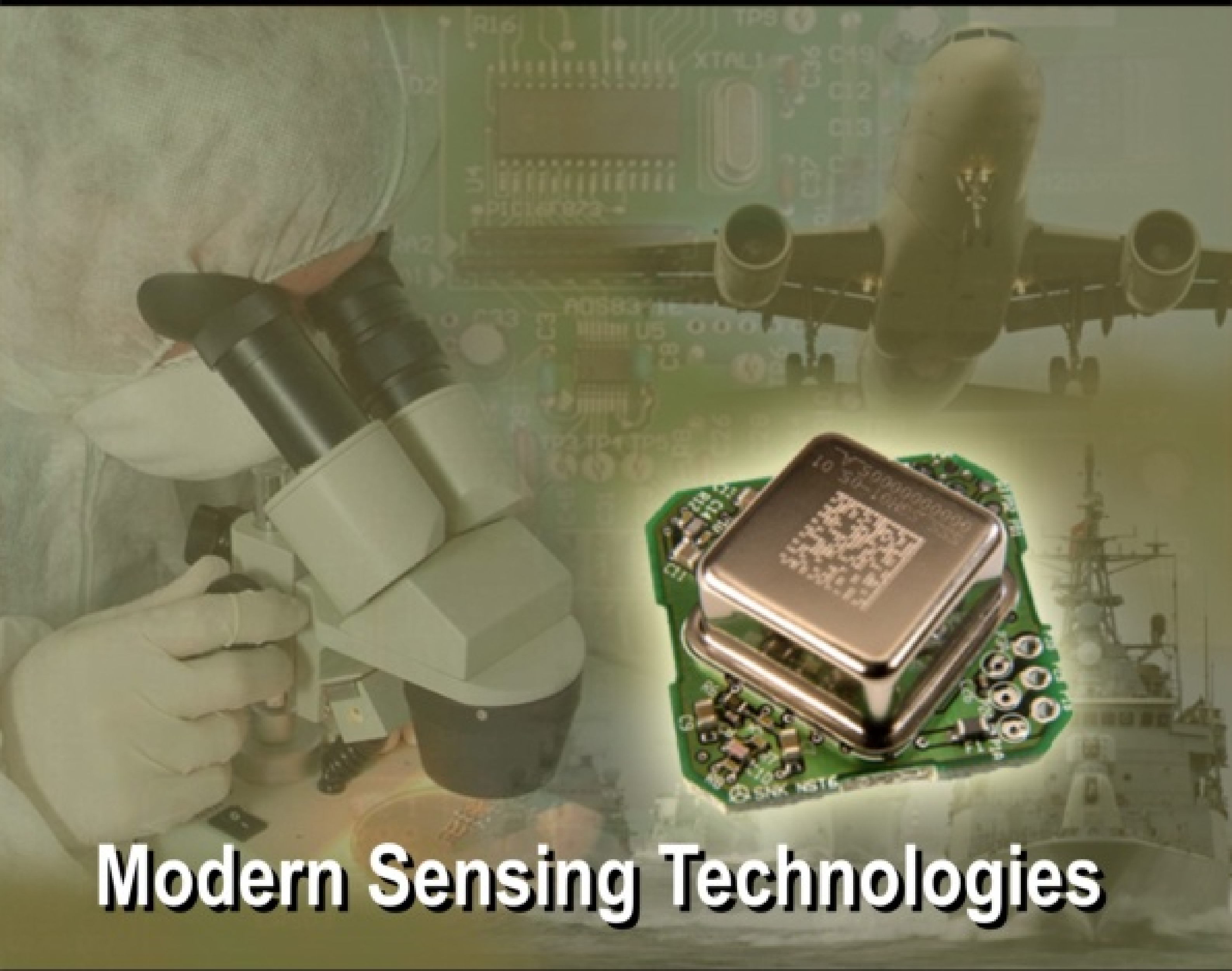
ISSN 1726-5749

SENSORS & TRANSDUCERS

4/08
vol. 90
Special



Modern Sensing Technologies

International Frequency Sensor Association Publishing

IFSA

Sensors & Transducers

Special Issue
April 2008

www.sensorsportal.com

ISSN 1726-5479

Editor-in-Chief: Sergey Y. Yurish
Guest Editors: Subhas Chandra Mukhopadhyay and Gourab Sen Gupta

Editors for Western Europe
Meijer, Gerard C.M., Delft University of Technology, The Netherlands
Ferrari, Vittorio, Universitá di Brescia, Italy

Editors for North America
Datskos, Panos G., Oak Ridge National Laboratory, USA
Fabien, J. Josse, Marquette University, USA
Katz, Evgeny, Clarkson University, USA

Editor South America
Costa-Felix, Rodrigo, Inmetro, Brazil

Editor for Eastern Europe
Sachenko, Anatoly, Ternopil State Economic University, Ukraine

Editor for Asia
Ohyama, Shinji, Tokyo Institute of Technology, Japan

Editorial Advisory Board

Abdul Rahim, Ruzairi, Universiti Teknologi, Malaysia
Ahmad, Mohd Noor, Nothern University of Engineering, Malaysia
Annamalai, Karthigeyan, National Institute of Advanced Industrial Science and Technology, Japan
Arcega, Francisco, University of Zaragoza, Spain
Arguel, Philippe, CNRS, France
Ahn, Jae-Pyoung, Korea Institute of Science and Technology, Korea
Arndt, Michael, Robert Bosch GmbH, Germany
Ascoli, Giorgio, George Mason University, USA
Atalay, Selcuk, Inonu University, Turkey
Atghiaee, Ahmad, University of Tehran, Iran
Augutis, Vygantas, Kaunas University of Technology, Lithuania
Avachit, Patil Lalchand, North Maharashtra University, India
Ayesh, Aladdin, De Montfort University, UK
Bahreyni, Behraad, University of Manitoba, Canada
Baoxian, Ye, Zhengzhou University, China
Barford, Lee, Agilent Laboratories, USA
Barlingay, Ravindra, RF Arrays Systems, India
Basu, Sukumar, Jadavpur University, India
Beck, Stephen, University of Sheffield, UK
Ben Bouzid, Sihem, Institut National de Recherche Scientifique, Tunisia
Binnie, T. David, Napier University, UK
Bischoff, Gerlinde, Inst. Analytical Chemistry, Germany
Bodas, Dhananjay, IMTEK, Germany
Borges Carval, Nuno, Universidade de Aveiro, Portugal
Bousbia-Salah, Mounir, University of Annaba, Algeria
Bouvet, Marcel, CNRS – UPMC, France
Brudzewski, Kazimierz, Warsaw University of Technology, Poland
Cai, Chenxin, Nanjing Normal University, China
Cai, Qingyun, Hunan University, China
Campanella, Luigi, University La Sapienza, Italy
Carvalho, Vitor, Minho University, Portugal
Cecelja, Franjo, Brunel University, London, UK
Cerda Belmonte, Judith, Imperial College London, UK
Chakrabarty, Chandan Kumar, Universiti Tenaga Nasional, Malaysia
Chakravorty, Dipankar, Association for the Cultivation of Science, India
Changhai, Ru, Harbin Engineering University, China
Chaudhari, Gajanan, Shri Shivaji Science College, India
Chen, Jiming, Zhejiang University, China
Chen, Rongshun, National Tsing Hua University, Taiwan
Cheng, Kuo-Sheng, National Cheng Kung University, Taiwan
Chiriac, Horia, National Institute of Research and Development, Romania
Chowdhuri, Arijit, University of Delhi, India
Chung, Wen-Yaw, Chung Yuan Christian University, Taiwan
Corres, Jesus, Universidad Publica de Navarra, Spain
Cortes, Camilo A., Universidad Nacional de Colombia, Colombia
Courtois, Christian, Universite de Valenciennes, France
Cusano, Andrea, University of Sannio, Italy
D'Amico, Arnaldo, Università di Tor Vergata, Italy
De Stefano, Luca, Institute for Microelectronics and Microsystem, Italy
Deshmukh, Kiran, Shri Shivaji Mahavidyalaya, Barshi, India
Kang, Moonho, Sunmoon University, Korea South
Kaniusas, Eugenijus, Vienna University of Technology, Austria
Katake, Anup, Texas A&M University, USA
Kausel, Wilfried, University of Music, Vienna, Austria
Dickert, Franz L., Vienna University, Austria
Dieguez, Angel, University of Barcelona, Spain
Dimitropoulos, Panos, University of Thessaly, Greece
Ding Jian, Ning, Jiangsu University, China
Djordjevich, Alexandar, City University of Hong Kong, Hong Kong
Donato, Nicola, University of Messina, Italy
Donato, Patricio, Universidad de Mar del Plata, Argentina
Dong, Feng, Tianjin University, China
Drljaca, Predrag, Instersema Sensoric SA, Switzerland
Dubey, Venketesh, Bournemouth University, UK
Enderle, Stefan, University of Ulm and KTB Mechatronics GmbH, Germany
Erdem, Gursan K. Arzum, Ege University, Turkey
Erkmen, Aydan M., Middle East Technical University, Turkey
Estelle, Patrice, Insa Rennes, France
Estrada, Horacio, University of North Carolina, USA
Faiz, Adil, INSA Lyon, France
Fericean, Sorin, Balluff GmbH, Germany
Fernandes, Joana M., University of Porto, Portugal
Francioso, Luca, CNR-IMM Institute for Microelectronics and Microsystems, Italy
Francis, Laurent, University Catholique de Louvain, Belgium
Fu, Weiling, South-Western Hospital, Chongqing, China
Gaura, Elena, Coventry University, UK
Geng, Yanfeng, China University of Petroleum, China
Gole, James, Georgia Institute of Technology, USA
Gong, Hao, National University of Singapore, Singapore
Gonzalez de la Rosa, Juan Jose, University of Cadiz, Spain
Granel, Annette, Goteborg University, Sweden
Graff, Mason, The University of Texas at Arlington, USA
Guan, Shan, Eastman Kodak, USA
Guillet, Bruno, University of Caen, France
Guo, Zhen, New Jersey Institute of Technology, USA
Gupta, Narendra Kumar, Napier University, UK
Hadjiloucas, Sillas, The University of Reading, UK
Hashsham, Syed, Michigan State University, USA
Hernandez, Alvaro, University of Alcala, Spain
Hernandez, Wilmar, Universidad Politecnica de Madrid, Spain
Homentcovschi, Dorel, SUNY Binghamton, USA
Horstman, Tom, U.S. Automation Group, LLC, USA
Hsiai, Tzung (John), University of Southern California, USA
Huang, Jeng-Sheng, Chung Yuan Christian University, Taiwan
Huang, Star, National Tsing Hua University, Taiwan
Huang, Wei, PSG Design Center, USA
Hui, David, University of New Orleans, USA
Jaffrezic-Renault, Nicole, Ecole Centrale de Lyon, France
Jaime Calvo-Galleg, Jaime, Universidad de Salamanca, Spain
James, Daniel, Griffith University, Australia
Janting, Jakob, DELTA Danish Electronics, Denmark
Jiang, Liudi, University of Southampton, UK
Jiao, Zheng, Shanghai University, China
John, Joachim, IMEC, Belgium
Kalach, Andrew, Voronezh Institute of Ministry of Interior, Russia
Rodriguez, Angel, Universidad Politecnica de Cataluna, Spain
Rothberg, Steve, Loughborough University, UK
Sadana, Ajit, University of Mississippi, USA

Kavasoglu, Nese, Mugla University, Turkey
Ke, Cathy, Tyndall National Institute, Ireland
Khan, Asif, Aligarh Muslim University, Aligarh, India
Kim, Min Young, Koh Young Technology, Inc., Korea South
Ko, Sang Choon, Electronics and Telecommunications Research Institute, Korea South
Kockar, Hakan, Balikesir University, Turkey
Kotulska, Malgorzata, Wroclaw University of Technology, Poland
Kratz, Henrik, Uppsala University, Sweden
Kumar, Arun, University of South Florida, USA
Kumar, Subodh, National Physical Laboratory, India
Kung, Chih-Hsien, Chang-Jung Christian University, Taiwan
Lacnjevac, Caslav, University of Belgrade, Serbia
Lay-Ekuakille, Aime, University of Lecce, Italy
Lee, Jang Myung, Pusan National University, Korea South
Lee, Jun Su, Amkor Technology, Inc. South Korea
Lei, Hua, National Starch and Chemical Company, USA
Li, Genxi, Nanjing University, China
Li, Hui, Shanghai Jiaotong University, China
Li, Xian-Fang, Central South University, China
Liang, Yuanchang, University of Washington, USA
Liawruangrath, Saisunee, Chiang Mai University, Thailand
Liew, Kim Meow, City University of Hong Kong, Hong Kong
Lin, Hermann, National Kaohsiung University, Taiwan
Lin, Paul, Cleveland State University, USA
Linderholm, Pontus, EPFL - Microsystems Laboratory, Switzerland
Liu, Aihua, University of Oklahoma, USA
Liu Changgeng, Louisiana State University, USA
Liu, Cheng-Hsien, National Tsing Hua University, Taiwan
Liu, Songqin, Southeast University, China
Lodeiro, Carlos, Universidade NOVA de Lisboa, Portugal
Lorenzo, Maria Encarnacio, Universidad Autonoma de Madrid, Spain
Lukaszewicz, Jerzy Pawel, Nicholas Copernicus University, Poland
Ma, Zhanfang, Northeast Normal University, China
Majstorovic, Vidosav, University of Belgrade, Serbia
Marquez, Alfredo, Centro de Investigacion en Materiales Avanzados, Mexico
Matay, Ladislav, Slovak Academy of Sciences, Slovakia
Mathur, Prafull, National Physical Laboratory, India
Maurya, D.K., Institute of Materials Research and Engineering, Singapore
Mekid, Samir, University of Manchester, UK
Melnyk, Ivan, Photon Control Inc., Canada
Mendes, Paulo, University of Minho, Portugal
Mennell, Julie, Northumbria University, UK
Mi, Bin, Boston Scientific Corporation, USA
Minas, Graca, University of Minho, Portugal
Moghavvemi, Mahmoud, University of Malaya, Malaysia
Mohammadi, Mohammad-Reza, University of Cambridge, UK
Molina Flores, Esteban, Benemérita Universidad Autónoma de Puebla, Mexico
Moradi, Majid, University of Kerman, Iran
Morello, Rosario, DIMET, University "Mediterranea" of Reggio Calabria, Italy
Mounir, Ben Ali, University of Sousse, Tunisia
Mukhopadhyay, Subhas, Massey University, New Zealand
Neelamegam, Periasamy, Sastra Deemed University, India
Neshkova, Milka, Bulgarian Academy of Sciences, Bulgaria
Oberhammer, Joachim, Royal Institute of Technology, Sweden
Ould Lahoucin, University of Guelma, Algeria
Pamidighanta, Sayanu, Bharat Electronics Limited (BEL), India
Pan, Jisheng, Institute of Materials Research & Engineering, Singapore
Park, Joon-Shik, Korea Electronics Technology Institute, Korea South
Penza, Michele, ENEA C.R., Italy
Pereira, Jose Miguel, Instituto Politecnico de Setebal, Portugal
Petsev, Dimiter, University of New Mexico, USA
Pogacnik, Lea, University of Ljubljana, Slovenia
Post, Michael, National Research Council, Canada
Prance, Robert, University of Sussex, UK
Prasad, Ambika, Gulbarga University, India
Prateepasen, Asa, Kingmoungut's University of Technology, Thailand
Pullini, Daniele, Centro Ricerche FIAT, Italy
Pumera, Martin, National Institute for Materials Science, Japan
Radhakrishnan, S. National Chemical Laboratory, Pune, India
Rajanna, K., Indian Institute of Science, India
Ramadan, Qasem, Institute of Microelectronics, Singapore
Rao, Basuthkar, Tata Inst. of Fundamental Research, India
Raoof, Kosai, Joseph Fourier University of Grenoble, France
Reig, Candid, University of Valencia, Spain
Restivo, Maria Teresa, University of Porto, Portugal
Robert, Michel, University Henri Poincare, France
Rezazadeh, Ghader, Urmia University, Iran
Royo, Santiago, Universitat Politecnica de Catalunya, Spain
Sadeghian Marnani, Hamed, TU Delft, The Netherlands
Sandacci, Serghei, Sensor Technology Ltd., UK
Sapozhnikova, Ksenia, D.I.Mendeleyev Institute for Metrology, Russia
Saxena, Vibha, Bhbha Atomic Research Centre, Mumbai, India
Schneider, John K., Ultra-Scan Corporation, USA
Seif, Selemani, Alabama A & M University, USA
Seifter, Achim, Los Alamos National Laboratory, USA
Sengupta, Deepak, Advance Bio-Photonics, India
Shearwood, Christopher, Nanyang Technological University, Singapore
Shin, Kyuho, Samsung Advanced Institute of Technology, Korea
Shmaliy, Yuriy, Kharkiv National University of Radio Electronics, Ukraine
Silva Girao, Pedro, Technical University of Lisbon, Portugal
Singh, V. R., National Physical Laboratory, India
Slomovitz, Daniel, UTE, Uruguay
Smith, Martin, Open University, UK
Soleymanpour, Ahmad, Damghan Basic Science University, Iran
Somani, Prakash R., Centre for Materials for Electronics Technol., India
Srinivas, Talabattula, Indian Institute of Science, Bangalore, India
Srivastava, Arvind K., Northwestern University, USA
Stefan-van Staden, Raluca-Ioana, University of Pretoria, South Africa
Sumriddetchka, Sarun, National Electronics and Computer Technology Center, Thailand
Sun, Chengliang, Polytechnic University, Hong-Kong
Sun, Dongming, Jilin University, China
Sun, Junhua, Beijing University of Aeronautics and Astronautics, China
Sun, Zhiqiang, Central South University, China
Suri, C. Raman, Institute of Microbial Technology, India
Sysoev, Victor, Saratov State Technical University, Russia
Szewczyk, Roman, Industrial Research Institute for Automation and Measurement, Poland
Tan, Ooi Kiang, Nanyang Technological University, Singapore,
Tang, Dianping, Southwest University, China
Tang, Jaw-Luen, National Chung Cheng University, Taiwan
Teker, Kasif, Frostburg State University, USA
Thumbavanam Pad, Kartik, Carnegie Mellon University, USA
Tian, Gui Yun, University of Newcastle, UK
Tsiantos, Vassilios, Technological Educational Institute of Kaval, Greece
Tsigara, Anna, National Hellenic Research Foundation, Greece
Twomey, Karen, University College Cork, Ireland
Valente, Antonio, University, Vila Real, - U.T.A.D., Portugal
Vaseashta, Ashok, Marshall University, USA
Vazques, Carmen, Carlos III University in Madrid, Spain
Vieira, Manuela, Instituto Superior de Engenharia de Lisboa, Portugal
Vigna, Benedetto, STMicroelectronics, Italy
Vrba, Radimir, Brno University of Technology, Czech Republic
Wandelt, Barbara, Technical University of Lodz, Poland
Wang, Jiangping, Xi'an Shiyou University, China
Wang, Kedong, Beihang University, China
Wang, Liang, Advanced Micro Devices, USA
Wang, Mi, University of Leeds, UK
Wang, Shinn-Fwu, Ching Yun University, Taiwan
Wang, Wei-Chih, University of Washington, USA
Wang, Wensheng, University of Pennsylvania, USA
Watson, Steven, Center for NanoSpace Technologies Inc., USA
Weiping, Yan, Dalian University of Technology, China
Wells, Stephen, Southern Company Services, USA
Wolkenberg, Andrzej, Institute of Electron Technology, Poland
Woods, R. Clive, Louisiana State University, USA
Wu, DerHo, National Pingtung University of Science and Technology, Taiwan
Wu, Zhaoyang, Hunan University, China
Xiu Tao, Ge, Chuzhou University, China
Xu, Lisheng, The Chinese University of Hong Kong, Hong Kong
Xu, Tao, University of California, Irvine, USA
Yang, Dongfang, National Research Council, Canada
Yang, Wuqiang, The University of Manchester, UK
Ymeti, Aurel, University of Twente, Netherland
Yu, Haihu, Wuhan University of Technology, China
Yufera Garcia, Alberto, Seville University, Spain
Zagnoni, Michele, University of Southampton, UK
Zeni, Luigi, Second University of Naples, Italy
Zhong, Haoxiang, Henan Normal University, China
Zhang, Minglong, Shanghai University, China
Zhang, Qintao, University of California at Berkeley, USA
Zhang, Weiping, Shanghai Jiao Tong University, China
Zhang, Wenming, Shanghai Jiao Tong University, China
Zhou, Zhi-Gang, Tsinghua University, China
Zorzano, Luis, Universidad de La Rioja, Spain
Zourob, Mohammed, University of Cambridge, UK

Sensors & Transducers Journal (ISSN 1726-5479) is a peer review international journal published monthly online by International Frequency Sensor Association (IFSA). Available in electronic and CD-ROM. Copyright © 2008 by International Frequency Sensor Association. All rights reserved.

Sensors & Transducers Journal

Contents

Volume 90
Special Issue
April 2008

www.sensorsportal.com

ISSN 1726-5479

Special Issue on Modern Sensing Technologies

Editorial

Sensors for Medical/Biological Applications

Wireless Sensors

Capacitive Sensors

Sensors Signal Processing

Gas Sensors

Image Sensors

Miscellaneous

Authors are encouraged to submit article in MS Word (doc) and Acrobat (pdf) formats by e-mail: editor@sensorsportal.com
Please visit journal’s webpage with preparation instructions: http://www.sensorsportal.com/HTML/DIGEST/Submition.htm

Sensors & Transducers Journal, Vol. 90, Special Issue, April 2008, pp. 139-149

Sensors & Transducers

ISSN 1726-5479

http://www.sensorsportal.com

Kalman Filter for Indirect Measurement of Electrolytic Bath State Variables: *Tuning Design and Practical Aspects*

Carlos A. Braga, Joâo V. da Fonseca Neto, Nilton F. Nagem, Jorge A. Farid and Fábio Nogueira da Silva
Federal University of Maranhão, Av. dos Portugueses, S/N, Centro, CEP: 65.001-970, São Luís-MA, Brasil
E-mail: jviana@dee.ufma.br, fndasilva@hotmail.com

Received: 15 October 2007 /Accepted: 20 February 2008 /Published: 15 April 2008

Abstract: The development *Kalman* filter tuning model based on *QR* duality principle of the gain is the main issue of this article. The filter design is oriented to measure the most important state variable of the electrolytic bath, the percentual of alumina. The technical solution encloses on line evaluation of the *Kalman* filter working with a real production pot. The main goal is to compute a set of filter gains that represents the behavior of the alumina inside the cell. The design and analysis of the *Q* and *R* covariances matrices are exercised in order to find a pattern of the reduction cell resistance variations that may be associated with the Al_2O_3 concentration. The filter bandwidth tuning is performed by increasing or decreasing the filter bandpass from the *Q* and *R* variations. This research goes in the direction of practical aspects limits of the indirect measurement system implementations. The robustness of the filter is evaluated in terms of observability, roundoff and modeling errors. *Copyright © 2008 IFSA.*

Keywords: *Kalman* Filter Tuning, Reduction, Indirect Measurement, Electrolytic Bath and QR duality principle

---

1. Introduction

In this paper are presented some results of a research directed to the development of an alumina concentration indirect measurement system, [1]. The procedure to evaluate the best gain is based on *QR* covariance matrices duality principle, [2] and indirect measurement algorithm was coded in application software considering the computing restrictions of a real-time control system. The main idea stands to present the *Kalman* filter skills in pattern recognition of Al_2O_3 concentration, considering a strong interaction between Al_2O_3 and reduction cell electrical resistance variations.

The proposed problem solution is focused on the design of the digital *Kalman* filter gains to indirect measurement of electrolytic bath state variables. The filtered and predicted states are used to estimate the alumina concentration in the electrolytic bath, [9]. The standard formulations of the *Kalman* filter are discussed in [3] and [4]. The *Kalman* filter theory is strongly dependent on the plant model. Consequently, the identification of suitable model to represent the plant is essential to predict the states of the smelter pot.

The correlated research used to develop methods to adjust gains based on the noise covariances matrices are reported by [5]. A specific method, called the QR duality principle, [2], is incorporated in the QR estimation methods, [5] and [6], to improve the process model. Evolutionary computation methods have been applied to evaluate the best QR matrices selections, [7].

The paper is organized in Sections and one Appendix to describe the tuning method and its results to obtain the alumina indirect measurement. In Section 2 is presented the characterization of indirect measurement system designed to evaluate the Al_2O_3 in the electrolytic bath, the system basic components are the reduction cell model, the state space observers based on *Kalman* Filter and the measurements of current and voltage. The QR dual mathematical approximation for *Kalman* filter tuning and a general procedure for its bandwidth tuning are discussed in Section 3. The procedure for QR duality principle gains design of the SFK is presented in Section 4, tuning issues such as state space pot model stability and observability are computed. In section 5 the real time implementation of the gain adjustment is implemented in process computer and its results are evaluated to show flexibility of the *Kalman* filter theory, when is applied in the stochastic state space observers gain computation. The concluding remarks are pointed out in Section 6, the results are related to the *Kalman* filter evaluation of the proposed methodology that is oriented to indirect measurement.

2. Indirect Measurement System

The reduction cell model, state observers, standard *Kalman* filtering (SKF) theory and direct measurements devices are the scientific and technical basis of the proposed indirect measurement system. The diagram of Fig. 1 represents the proposed system of the plant and state observer, the SKF theory is used for tuning the state observer gains and the devices (sensors and current transformers) are used to measure the reduction cell voltages and currents.

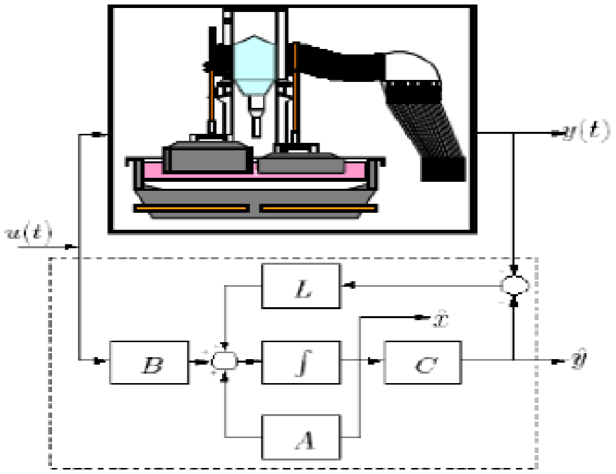


Fig. 1. The Reduction Cell and State Observer Block Diagram.

2.1. Reduction Cell Model

The reduction cell behavior model is a third order discrete linear stochastic system described, in the state variables by,

$$x_{k+1} = Ax_k + \Gamma\, \upsilon_k \tag{1}$$

$$y_k = Cx_k + w_k\,, \tag{2}$$

where $x_k \in \Re^n$ represents the resistance, slope and curvature. The $y_k \in \Re^n$ output represents the measured signal. $A \in \Re^{nxn}$ represents the system dynamics, $\Gamma \in \Re^{nxn}$ is the related noise input. $v_k \in \Re^n$ and $w_k \in \Re^q$ are the zero mean Gaussian white noise sequences with the known variances Q and R, respectively.

2.2. State Observers

The state observer main idea is the replacement of conventional measurements systems by a device based on hardware and software. The observer designed has two main issues that are the reduction cell model identification and gain adjustment. In a large sight, the state observers appliance is related with existence of noisy measurements, hard access and high cost of sensors. As seen in Fig. 1, the observers inputs are the measured output and control signals. The observer dynamic is given by,

$$x = Ax + Bu + L_{estim}(y - Cx)\,, \tag{3}$$

where L_{estim} is the observer gains that can be computed by deterministic or stochastic methods. In our case, the gains are computed by the *Kalman* filter theory and our system is driven by noise.

2.3. Direct Measurements

The reduction cell direct measurements are performed by the hardware devices, sensors of voltage and current connected to reduction cell. The sensors output signals are digitalized and processed in the process computer. For practical purposes, the measured output signals are associated with Eq. (2), y_k is the k^{th} output signal that can be represented by the state vector trajectory, Cx_k, and its noise signal, w_k.

2.4. Standard Kalman Filtering

A linear, unbiased and minimum error variance estimation of the reduction cell states, x_k, at instants k, is obtained by the SKF basic equations, [8] that assembles a recursive scheme given by

$$P^P = AP^FA^T + \Gamma Q\Gamma^T \tag{4}$$

$$x^P{}_{k+1} = Ax^P{}_k \tag{5}$$

$$K_K = P^PC^T + (C\,P^PC^T + R)^{-1} \tag{6}$$

$$x^F{}_{k+1} = x^P{}_{k+1} + K_K(y^m{}_{k+1} - Cx^P{}_{k+1} \tag{7}$$

$$P^F = P^P - K_K\, CP^P, \tag{8}$$

where P_P is the state covariance matrix in prediction stage. Eqs. (4-5) are concerned with the prediction stage that is separated from the correction stage, Eqs. (7-8), by the gain update given by Eq. (6). The optimal state estimation, based on the measured sequences, is represented by Eq. (7).

3. Kalman Filtering QR Tuning

Aiming to develop a procedure to support metaheuristics or Bayesian inference to guide the Q and R matrices selection of *Kalman* filter gain. The SKF basic equations, Eqs. (4)-(8), can be represented in terms of its parameters functionality to insert theduality principles in matrices selection procedure. The $\boldsymbol{P^P}$ state prediction covariance matrix, Eq. (4), is strongly related with the K_K gain and Q covariance,

$$P^P = f_1\,(P^F, A, B, Q) \tag{9}$$

The gain mapping,

$$K_K = f_2\,(P^P, C, R) \tag{10}$$

As can be evaluated by Eq. (10), the K_K *Kalman* gain is dependent only of three parameters, the P^P predicted state is a mapping of P^F , A, Q and ¡. If the P^P covariance matrix is replaced by its quadratic form, Eq. (4), in Eq. (6) a new mapping for K_K is given by

$$K_K = f_{\mathrm{K}}\,(A, P^F, \Gamma, Q, C, R) \tag{11}$$

The observers gains, Eq. (11), are assembled to explicitly represent the relation between Q and R matrices. Considering $BQB^T \gg APA^T$ and $R \gg CPC^T$. If $C \equiv I$ and $B = I$, the $K_k \approx Q/R$ approximation for *Kalman's* gain is and its simplified mapping is given by,

$$K_K = f_{\mathrm{K}}^{\mathrm{approx}}\,(Q, R) \tag{12}$$

Relation (12) shows the strong influence of Q and R covariances matrices on the $\boldsymbol{f_{\mathrm{K}}^{\mathrm{approx}}}$ mapping for the gain adjustment, if certain restrictions are respected in system modeling design. The proposed procedure is the basic procedure chosen to guide the search, independently of the method to evaluate these matrices.

4. QR Duality Gains Design

In this section is presented the Q and R matrices general procedure to tune de *Kalman* filter gains. These matrices are implemented in the process computer to make the performance evaluation of the filter. The results are compared with other filters, such as $\alpha\beta\delta$ and polinomial type, with the purpose of evaluate the robustness of SKF in presence of operation changes.

4.1. Reduction Cell Modeling

The reduction cell modeling is performed in a third order state space description. Three states variables related with resistance of reduction cell dynamics are represented in state space description. Replacing the values of the dynamic system matrix A and output matrix C, in Eq. (1),

$$
\begin{aligned}
\dot{x} &= \begin{bmatrix} 1.00 & 1.00 & 0.00 \\ 0.00 & 1.00 & 1.00 \\ 0.00 & 0.00 & 1.00 \end{bmatrix} \begin{bmatrix} x_k^R \\ x_k^{slope} \\ x_k^{curv} \end{bmatrix} + \zeta \\
y &= \begin{bmatrix} 1.00 & 0.00 & 0.00 \end{bmatrix} \begin{bmatrix} x_k^R \\ x_k^{slope} \\ x_k^{curv} \end{bmatrix} + \nu \ ,
\end{aligned}
\tag{13}
$$

where $[x_k^R x_k^{slope} x_k^{curv}]$ are the state variables that represent the filtered resistance, the resistance and curvature, respectively. y_k represents the resistance calculated from the pot voltage and line current. The model eigenvalues are in the unit circle contour, the computed eigenvalues are 1, 1 and 1. The observability matrix rank is 3, this value guarantees the construction of the model states by pot the resistance measurements.

4.2. Voltage and Current Measurements

The measured values of pot voltage and line current, as well as the calculated resistance, are shown at a sampling time of 1 second. The filtered resistance and estimated slope and curvature behavior can be seen in Fig. 2.

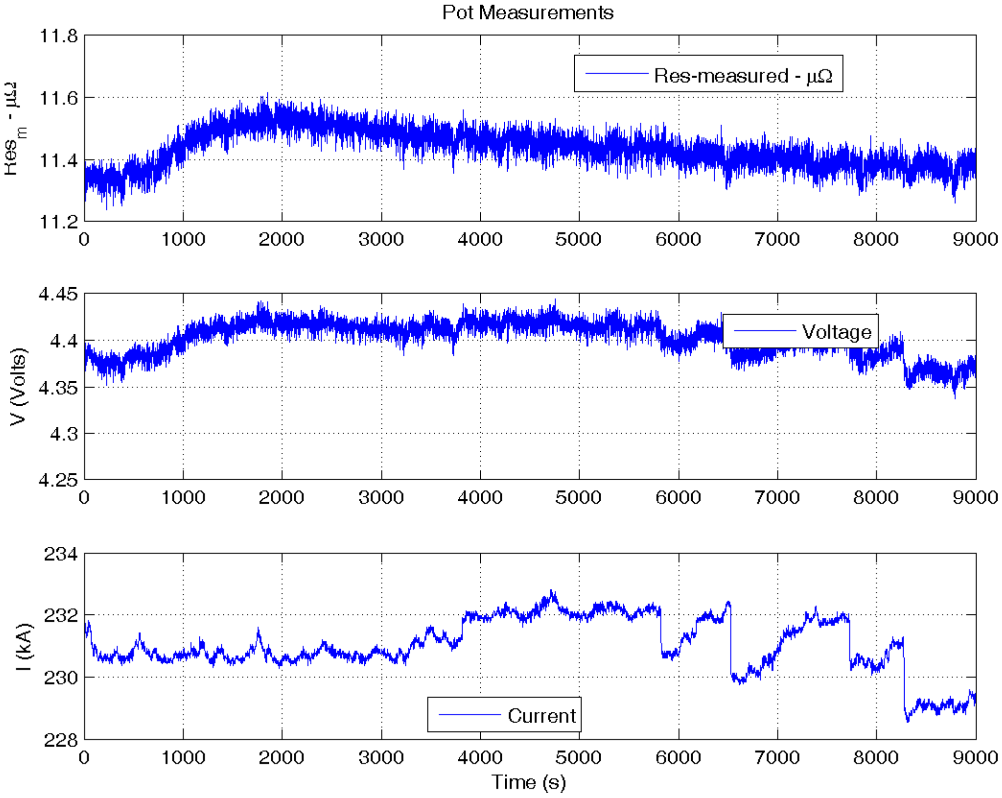


Fig. 2. Voltage, current and resistance measurements in the bath.

The reduction cell voltage and **line** current are represented as the reduction resistance to be used as the only measured state variable, equation (2)**,** in **s**ection 2. This resistance is used as the measured output of the *Kalman* filter, equation (7)**,** in **s**ection 2, to predict the reduction cell slope and curvature states.

A statistical analysis of the incoming inputs (voltage and current) and calculated resistance helps to understand the distribution of the values around the mean, and it is useful to understand the nature of the variations during the different stages of the normal operation.

The computed resistance and measured signals means, deviation, maximums and minimums, Table 1, and related with its time behavior. As can be observed, the pot is working under normal operational conditions.

Table 1. Resistance, Voltage and Current Statistics.

	Measurements		
	Resistance	**Voltage**	**Current**
	μΩ	**V**	**kA**
Mean	11.4421	4.3687	227.991
Max	11.627	4.423	233.542
Min	11.159	4.273	220.677
Std	0.083	0.0248	1.279
Cov	0.007	0.0006	1.637

The pot operation mode, Fig. 3, and histograms, Fig. 4, reflects the resistance behaviour during different feed control.

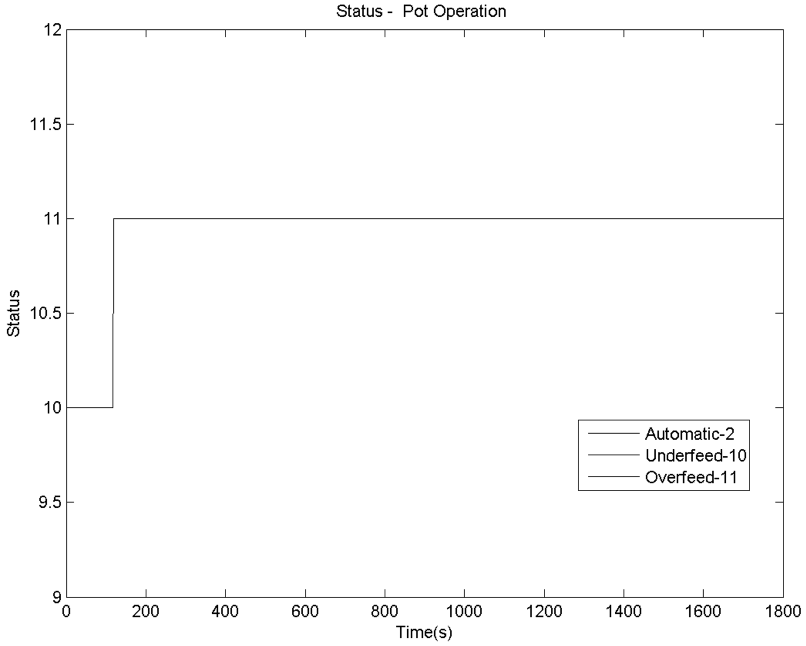


Fig. 3. Reduction Cell Status of Operation.

4.3. Gains Adjustment

A procedure to tune the *Kalman* filter gains is based on duality relation, equation (12), following the guidelines suggested by [2], and afterwards it is compared with other digital filter implementations. The variations matrices are associated with the bandwidth of *Kalman*'s filter and its stability. The procedure strategy is based on the functional mapping,

$$K_n = f(Q_n, R) \tag{14}$$

The values of R matrix are kept fixed for gains adjustment and variations are applied in covariances of matrix Q. The mapping of equation (14) for a sequence of five Q covariances deviations is presented in Table 2.

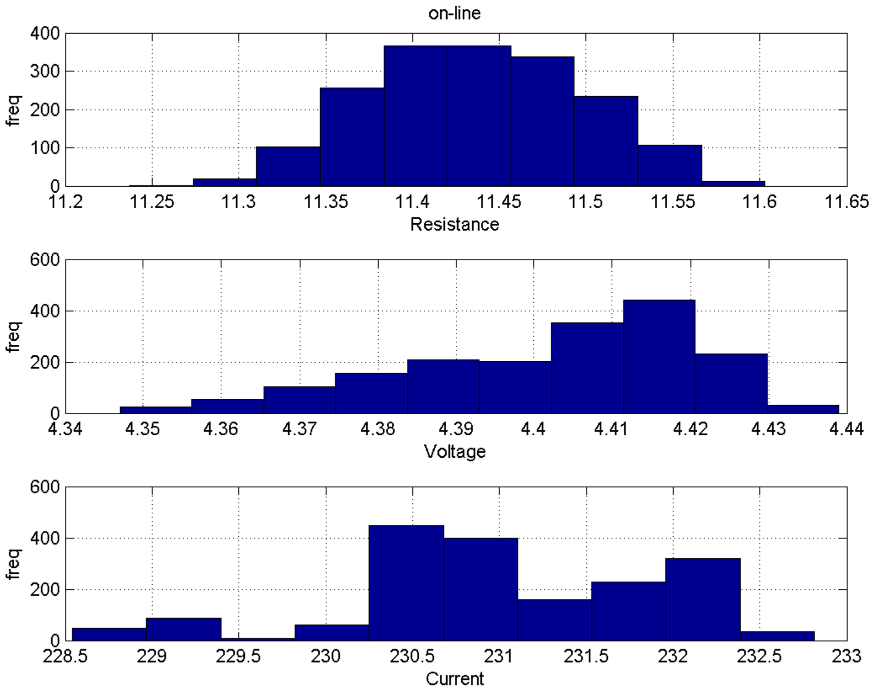


Fig. 4. Resistance, Voltage and Current.

Table 2. Covariance Matrices for Bandwidth Adjustment of Kalman Filter.

Q_n, R	**Covariance**			
	r_{11}	q_{11}	q_{22}	q_{33}
1	$2.0\text{x}10^{-2}$	$1\text{x}10^{-6}$	$1\text{x}10^{-6}$	$1\text{x}10^{-6}$
2	$2.0\text{x}10^{-2}$	$1\text{x}10^{-7}$	$1\text{x}10^{-7}$	$1\text{x}10^{-7}$
3	$2.0\text{x}10^{-2}$	$1\text{x}10^{-9}$	$1\text{x}10^{-9}$	$1\text{x}10^{-9}$
4	$2.0\text{x}10^{-2}$	$1\text{x}10^{-14}$	$1\text{x}10^{-14}$	$1\text{x}10^{-14}$
5	$2.0\text{x}10^{-2}$	$1\text{x}10^{-16}$	$1\text{x}10^{-16}$	$1\text{x}10^{-16}$
6	$2.0\text{x}10^{-2}$	$1\text{x}10^{-18}$	$1\text{x}10^{-18}$	$1\text{x}10^{-1}$
7	$2.0\text{x}10^{-2}$	$1\text{x}10^{-18}$	$1\text{x}10^{-18}$	$1\text{x}10^{-20}$
8	$2.0\text{x}10^{-2}$	$1\text{x}10^{-18}$	$1\text{x}10^{-18}$	$1\text{x}10^{-30}$
9	$2.0\text{x}10^{-2}$	$1\text{x}10^{-18}$	$1\text{x}10^{-10}$	$1\text{x}10^{-30}$
10	$2.0\text{x}10^{-2}$	$1\text{x}10^{-10}$	$1\text{x}10^{-10}$	$1\text{x}10^{-30}$
11	$2.0\text{x}10^{-2}$	$1\text{x}10^{-6}$	$1\text{x}10^{-10}$	$1\text{x}10^{-15}$
12	$2.0\text{x}10^{-2}$	$1\text{x}10^{-1}$	$1\text{x}10^{-10}$	$1\text{x}10^{-20}$

The covariances matrices variations and theirs associated gains are shown in Tables 2 and 3, respectively. The filter's gains are evaluated considering a mapping $f(Q;\ R) \rightarrow K_K$, the stochastic matrices spaces Q_{3x3} and R_{1x1} are mapped into K_3 *Kalman* gain.

Table 3. Kalman Filter Gains.

K_n	**GAINS**		
Q_n, R	K_1	K_2	K_3
1	7.9390×10^{-2}	3.3170×10^{-3}	6.7846×10^{-5}
2	5.4792×10^{-2}	1.5545×10^{-3}	2.1739×10^{-5}
3	2.5815×10^{-2}	3.3867×10^{-4}	2.2070×10^{-6}
4	3.8314×10^{-3}	7.3575×10^{-6}	7.0575×10^{-9}
5	1.7835×10^{-3}	1.5906×10^{-6}	7.0756×10^{-10}
6	1.0302×10^{-3}	4.8365×10^{-7}	1.0014×10^{-10}
7	9.9985×10^{-4}	4.4470×10^{-7}	8.2456×10^{-11}
8	9.9953×10^{-4}	4.4429×10^{-7}	8.2272×10^{-11}
9	1.3716×10^{-3}	9.2155×10^{-7}	1.2745×10^{-10}
10	1.3716×10^{-3}	9.2155×10^{-7}	1.2745×10^{-10}
11	2.6494×10^{-3}	3.5128×10^{-6}	2.2332×10^{-9}
12	2.2552×10^{-2}	1.0026×10^{-5}	1.6729×10^{-9}

The SKF bandwidth variations and the time response are analyzed for the five *Kalman* gains adjustments are presented in Table 3 and are associated with Figs. 5, 6 and 7.

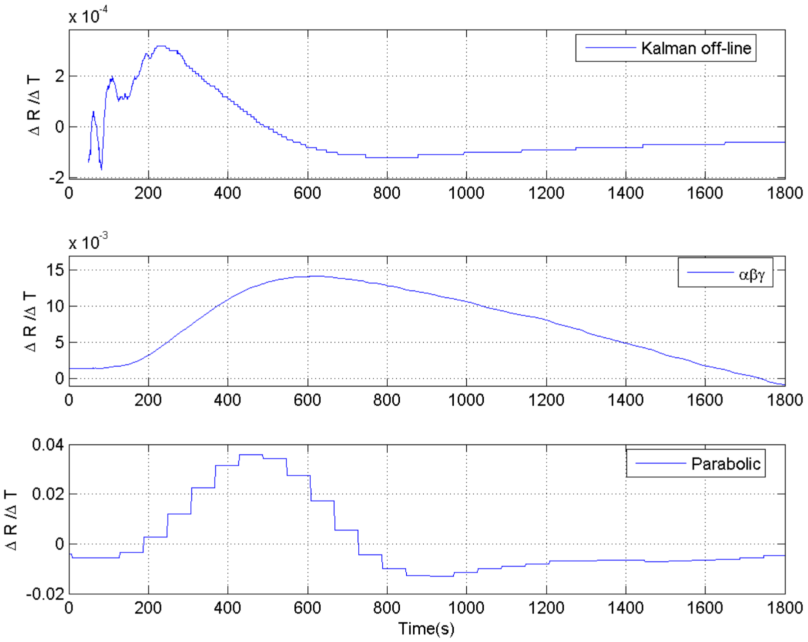


Fig. 5. Case 12 - Estimated Slope of the Reduction Cell.

Observing Figs. 5 and 6, we note that bandwidth is reduced as the Q matrix has its eigenvalues tending to zero, Table 2. Consequently, provokes the attenuation of the high frequencies behaviors. Comparing the SKF filter estimation with the parabolic filter in Figs. 5 and 6, we observe that SKF is able to reproduce. The same behavior of the comparison filters by the adjustment of matrices Q and R.

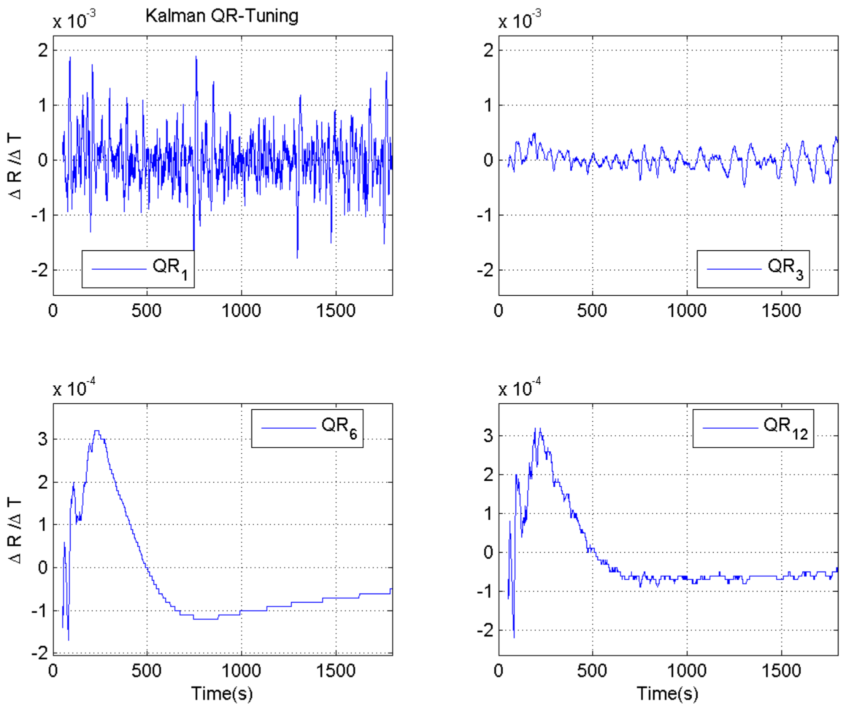


Fig. 6. Reduction Cell Estimated Slope for QR-Tuning.

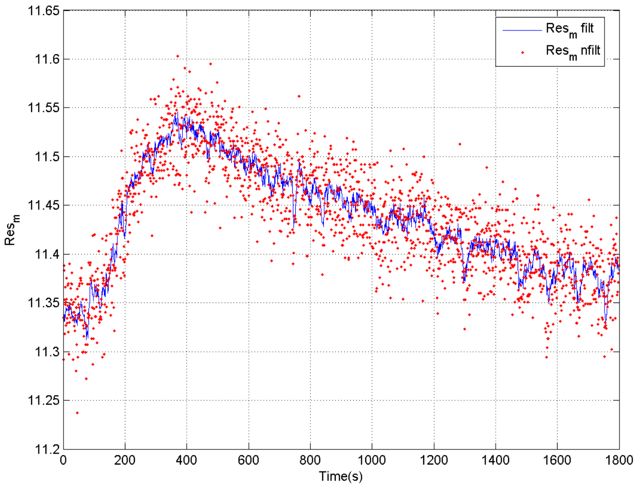


Fig. 7. Measured and Estimated Resistance.

The chosen heuristic to apply the QR duality principle is based on the relation. The R variance matrix is fixed in a value that is grater than Q values. Observing Figs. 5 and 6, is verified that the gains adjusted by QR duality principle, Table 3, imposes a band of operation in standard *Kalman* filter. This operation goes from a low to high bandpass operation.

A functional pattern for the gain adjustment can be proposed based on the statistics of the measurements. It is observed that Q matrix decreasing imposes a gain decreasing that has a $K_k \approx Q/R$ relation from 10^{-2} to 10^{-8} for the first Q and R variations of Table 2. Cases 3 and 4 had shown bandwidth enlargement as Q matrix coefficients had decreased.

Consequently, if the Q diagonal variance matrix has its coefficients reduced, means that the gain is reduced too. We can establish that if the gains are reduced the SKF works as low bandpass filter. If its gains are increased, the filter works as high bandpass filter. As can be seen in Fig. 6, the *Kalman* filter gains are designed to follow certain patterns that are imposed by the variation in the Q matrix.

5. SKF Real Time Performance

In this section, the indirect measurement system device is evaluated for on-line implementations. The results and practical aspects are discussed in context of hardware-software devices to evaluate the Al_2O_3 concentration in the electrolytic bath. The main goal is to evaluate the *Kalman* filter tuning algorithm based on the QR duality of its gains.

5.1. On line Estimation

The on-line filtering of resistance by the *Kalman* filter is presented in Fig. 7 and it is associated operational condition state estimation is shown in Fig. 3. This figure is used as reference for gains adjustment, in this manner, the Q and R matrices elements deviations effects are observed in the filtered resistance and non realist matrices are discarded as solutions.

5.2. Comparison

The on-line results estimated slope by Kalman filter, Fig. 8, is compared with the αβγ and parabolic filters. The main purpose is to evaluate the effectiveness of the proposed adjustment as a pattern tracking by QR duality gains adjustments.

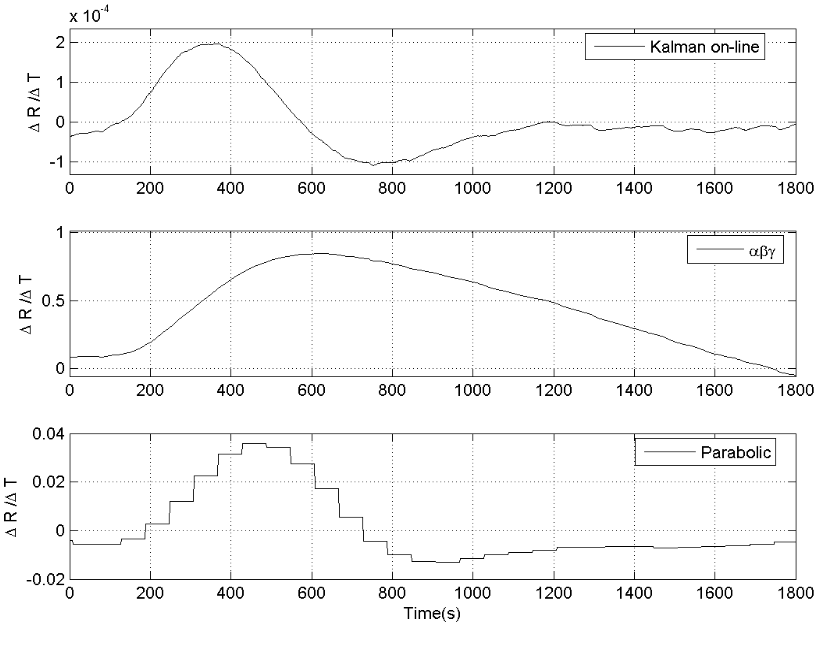


Fig. 8. Real Time Slopes of Kalman, αβγ and Parabolic Filters.

The first association of Figs. 3 and 8, the *Kalman* filter has a sensitivity for alumina concentration variation by the reduction cell resistance deviations, as can be seen in overfeed transition to under-feed phase. As a second association in Fig. 7 as the level of noise that is not filtered out depends on Q matrix selection.

6. Concluding Remarks

In this article it was presented a *Kalman* filtering bandwidth tuning procedure, based on QR matrices duality principle, for indirect measurement of Al_2O_3 concentration in the electrolytic bath. Specifically, the main goal was the indirect measurement of the Al_2O_3 concentration in reduction cell by deviations on the reduction cell resistance.

This research results had been applied to a real reduction cells. It focuses the use of the QR duality principle as a design framework for *Kalman* filter gain adjustment. The framework succeeds in the determination of trial and error heuristic. The improvement of the framework performance can be done by the inclusion of genetic algorithms for the Q and R searches and a Bayesian inference engine. Likewise, the development of neural networks to solve the algebraic *Riccati* equation for multivariable stochastic state space observers, as an alternative way to by pass the inverse problem computation due to multiple outputs. Also, a metaheuristic and evolutionary algorithms can be ~~s~~ consider to help the tuning of the *Kalman* filter gains.

The proposed method had proven to be good alternative to make the gain adjustment of *Kalman* when it is used for the indirect measurement of Al_2O_3 concentration. Especially good is its flexibility on patterns tracking, based on the covariances matrices estimation for the bandwidth adjustment and to improve transient response. In addition to this, the robustness of the filter had been kept along various stages of the normal pot operation, in counterpoint to others filtering methods that have been used widely in the aluminium industry.

References

[1]. P. C. A. B. J. W. Fiona, Stevens McFaden, Geoffrey P. Bearne, Application of Advance Process Control to Aluminium Reduction Cell – A Review, 2001, p. 1233.

[2]. H.-G. Yeh, Real-Time Implementation of a Narrow-Band Kalman Filter With a Floating- Point Processor DSP-32, *IEEE Transaction on Industrial Electronics*, 31, 9, 1990, pp. 13–18.

[3]. E. A. S. P. Borg, Dynamic Model and Estimator for Online Supervision of the Alumina Reduction Cell, 08, 1988.

[4]. S. M. Bozic. *Digital and Kalman Filtering*, Edward Arnold, London, 1st edition, 1984.

[5]. G. N. S. Pasupathy, Adaptative Estimation of Noise Covariance Matrices in Real-Time Preprocessing of Geophysical Data, IEEE 05, IEEE, 1997.

[6]. S J Kwon, Robust Kalman Filtering with perturbation Estimation Process doe uncertain Systems. *IEE Proceedings*, 2006.

[7]. J. de Jesus Barros, State Space Estimation of the Sonda Rockets Impact Points., Estimação no Espaço de Estado do Ponto de Impacto de Foguetes, *Master thesis in Portuguese- Dissertação de mestrado*, Universidade Federal do Maranhão, São Luís-MA-Brasil, 2005.

[8]. G. C. C. K. Chui, A Modified Adaptative Kalman Filter for Real-Time Applications, IEEE 06, IEEE, 1991.

[9]. João Viana DA Fonseca Neto, Carlos. Braga. Nilton F. Nagen, Jorge A. Farid and Fábio Nogueira da Silva, Bandwidth Adjustment of Digital Kalman Filter for Indirect Measurement of Electrolytic Bath State Variables, *2nd International Conference on Sensing Technology 2007*, November 26-28, 2007 at Palmerston North, New Zealand.

Guide for Contributors

Aims and Scope

Sensors & Transducers Journal (ISSN 1726-5479) provides an advanced forum for the science and technology of physical, chemical sensors and biosensors. It publishes state-of-the-art reviews, regular research and application specific papers, short notes, letters to Editor and sensors related books reviews as well as academic, practical and commercial information of interest to its readership. Because it is an open access, peer review international journal, papers rapidly published in *Sensors & Transducers Journal* will receive a very high publicity. The journal is published monthly as twelve issues per annual by International Frequency Association (IFSA). In additional, some special sponsored and conference issues published annually.

Topics Covered

Contributions are invited on all aspects of research, development and application of the science and technology of sensors, transducers and sensor instrumentations. Topics include, but are not restricted to:

- Physical, chemical and biosensors;
- Digital, frequency, period, duty-cycle, time interval, PWM, pulse number output sensors and transducers;
- Theory, principles, effects, design, standardization and modeling;
- Smart sensors and systems;
- Sensor instrumentation;
- Virtual instruments;
- Sensors interfaces, buses and networks;
- Signal processing;
- Frequency (period, duty-cycle)-to-digital converters, ADC;
- Technologies and materials;
- Nanosensors;
- Microsystems;
- Applications.

Submission of papers

Articles should be written in English. Authors are invited to submit by e-mail editor@sensorsportal.com 6-14 pages article (including abstract, illustrations (color or grayscale), photos and references) in both: MS Word (doc) and Acrobat (pdf) formats. Detailed preparation instructions, paper example and template of manuscript are available from the journal's webpage: http://www.sensorsportal.com/HTML/DIGEST/Submition.htm Authors must follow the instructions strictly when submitting their manuscripts.

Advertising Information

Advertising orders and enquires may be sent to sales@sensorsportal.com Please download also our media kit: http://www.sensorsportal.com/DOWNLOADS/Media_Kit_2008.pdf