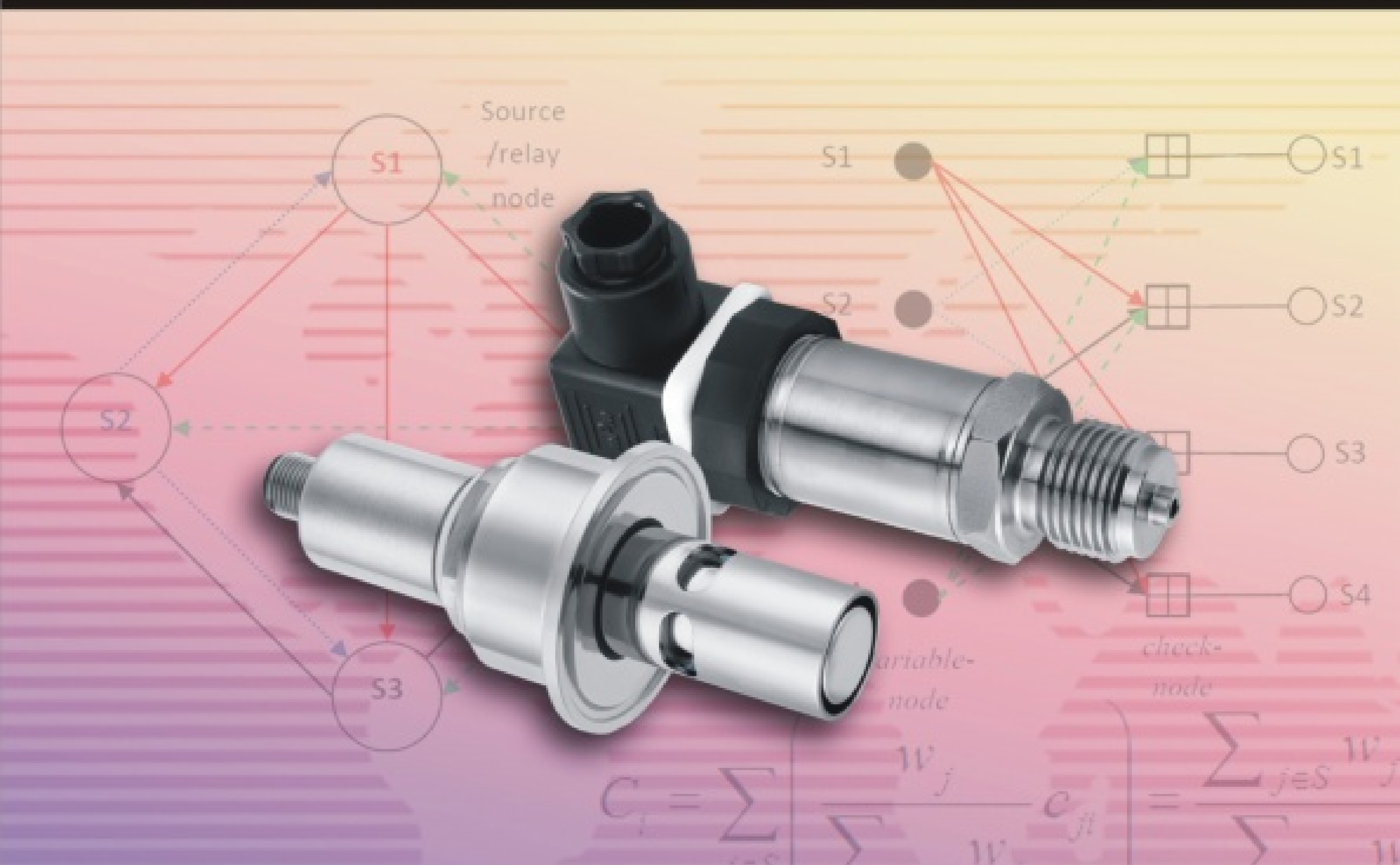
SENSORS & TRANSDUCERS

ISSN 1726-5479

vol. 20
Special
4/13



Sensors, Networks, Control Systems and Information Processing

International Frequency Sensor Association Publishing

Sensors & Transducers

International Official Journal of the International Frequency Sensor Association (IFSA) Devoted to Research and Development of Sensors and Transducers

Volume 20, Special Issue, April 2013

IFSA Publishing: Barcelona • Toronto

Printed in the USA.

Sensors & Transducers

Volume 20, Special Issue, April 2013 | **www.sensorsportal.com** | **ISSN 2306-8515 e-ISSN 1726-5479**

Sensors & Transducers Journal

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Volume 20
Special Issue
April 2013

www.sensorsportal.com

ISSN 1726-5479

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The Formation of the Local Gravitational Model Based on Point-Mass Method

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Received: 16 February 2013 /Accepted: 17 April 2013 /Published: 30 April 2013

Abstract: Virtual point-mass method has been widely used to approximating the local gravity field. However, the approximation area is usually too small. Thus point-mass model for large area is studied in this paper. And an experiment is conducted for the earth's gravity model in the region of Tibetan plateau, while the region is $74^{\circ}-104^{\circ}E$ and $25^{\circ}-40^{\circ}N$. The formatted point-mass model is a three-tier point-mass model which is on the base of the geopotential model with low degree and order. In addition, the resolution of the model is $2'\times 2'$. The observations of gravity anomaly are simulated from EGM2008 with degree and order 2159. The focus is on the formation of the point masses in the space domain and the solution of the formatted equation for virtual point masses. The results of the experiments show that the truncation error of gravity disturbance created by using the point-mass model is less than 1mGal and the geoid formatted by point-mass is applicable to engineering. Furthermore, this method is easy to deal with higher resolution model. *Copyright © 2013 IFSA.*

Keywords: Point-mass method, Local gravity field, Geoid, Toeplitz matrix.

1. Introduction

Geoid is the height datum system of the othomatric elevation and reflects a density distribution of the Earth's interior structure and the physical characteristics of the surface [1]. Researches in this area have become increasingly active. With the development of CHAMP (CHAllenging Minisatellite Payload) program, GRACE (Gravity Recovery And Climate Experiment) project, GOCE (Gravity field and steady state Ocean Circulation Explorer) satellite technology plans, it is possible to making the precise determination of Earth's external gravity field model and the corresponding geoid [2, 3]. It is noteworthy that there are many difficulties to establishment of centimeter geoid, but the establishment of centimeter-level regional geoid is no longer a problem [4]. Furthermore, with study progress of the regional geoid, the establishment of a wider regional geoid has become a focus research project. In spite of the fact that the geopotential models for the global gravity field have become accuracy increasingly, these models are still not available for the local gravity field. Actually, the local gravity field theory is an important topic for practical application in boundary value problems [5]. There are some ways to approximate the local gravity fields, in the 1960s, Bjerhanmmar [6, 7] proposed a theory which equivalently converts the Molodesky boundary value problem to a simple virtual spherical boundary value problem. And Paul[8] and Sunkel [9] gave a comprehensive study of point mass model. This theory is first studied by Xiaoping Wu [10] in China and he gave the construction progress of point-mass model. after that, the finite element method adopted by Hanjiang Wen [11] is to approximate the gravity field model which is based on virtual theory.

Now point mass model theory has been widely used in spacecraft gravitational perturbation calculation [12, 13], because it has three advantages, which are, (1) the kernel function of the model is simple and can be calculated fast, (2) the calculation of the singularity will not occur in low altitude areas, and (3) it is better to get the disturbing gravity with different frequencies by the quality of team points which can be simply added [10]. In this paper, we introduced the theory of point-mass model to approximate the local gravity field. And the characteristics of the elements in the coefficient matrix for the model construction are analyzed by numerical calculation. The observations are calculated by using EGM2008 [14] which is internally recognized as the best currently. The point-mass model on the base of geopotential model with degree and order 36 from low frequency to high frequency is applied to approximate the gravity field. Point-mass model provides the different frequencies truncation error which can be the reference for the selection of optimum point groups. The results of the simulation have shown that the point-mass groups effects on disturbing gravity are quite difference in the outer space of the earth. Furthermore, the point masses model is a good tool to approximated local gravity field. However, the approximation area is usually very small, for instance, $2^\circ \times 2^\circ$, this disadvantage will limit its usage. The altitude of Tibet Plateau is very high, and the terrain is complexity, therefore, this region can be used as a typical area as the experimental target. In this paper, we extend the approximation range by split temple and give an experiment on the region of $74^\circ - 104^\circ E$ and $25^\circ - 40^\circ N$ which is cover the Tibetan plateau. We format a geoid model on the basis of the virtual sphere theory.

2. Extending Method

The virtual sphere theory is valid for the Keldysh-Lavrentiev theorem [4, 15] which is: If the Earth's surface Σ is sufficiently regular (e.g. continuously differentiable), then any function f, harmonic and regular outside Σ and continuous outside and on Σ, may be uniformly approximated by functions y, harmonic and regular outside an arbitrarily given sphere σ inside the Earth, in the sense that for any given ε >0, the relation |f-y|< ε holds everywhere outside and on Σ. We assume that there is a virtual sphere σ plotted in Fig. 1 with the radius, R_B and sphere density, μ. In this figure, Δg is the gravity unit, in this paper, we set it is the gravity anomaly which is the character of gravity anomaly vector by magnitude.

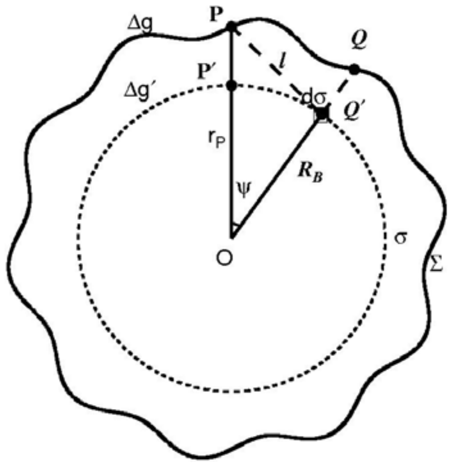

Fig. 1. Bjerhammar sphere.

The distance l in Fig. 1 between two points of which one is the point P and the other is the moving point $d\sigma$ at the sphere σ is given by

$$l^2 = R_B^2 + r_P^2 - 2R_B r_P \cos\psi , \tag{1}$$

In Eq. (1), r_P is the radius of the computation point P, ψ is the angle between the radius of P and $d\sigma$, λ and φ are longitude and geocentric latitude respectively. For convenience of reference, we give the formula to obtain the angle φ, as follows [16].

$$\begin{aligned}\cos\psi &= \sin\varphi_P \sin\varphi_\sigma \\ &+ \cos\varphi_P \cos\varphi_\sigma \cos(\lambda_P - \lambda_\sigma)\end{aligned} , \tag{2}$$

The range of the angle ψ is from 0° to 180°, so we can obtain $\sin\psi = \sqrt{1-\cos^2\psi}$. We always use the triangle functions of the angle ψ and the functions can be calculated through recurrence calculation of the triangle functions.

According to Newton's law of gravitation, the potential of gravitation at an arbitrary point P in outer space can be written as follows:

$$V_P = G\int_\sigma \frac{\mu}{l} d\sigma , \tag{3}$$

where G is Newton's gravitational constant, $d\sigma$ is the element of virtual sphere area.

To simplify the mathematics, one decomposes the Earth's gravity field into the sum of the normal gravity field and the anomalous gravity field. The normal gravity field, a first approximation of the actual gravity field, is generated by an ellipsoid of revolution with its centre at the geo-centre, called the referenced ellipsoid which of which the surface is an equi-potential surface. Since the normal gravity field can be directly evaluated from simple closed formulas, the problems are converted to the determination of the disturbing potential. We assume that the anomalous potential T' caused by virtual sphere is consistent with the one T caused by real earth. The feasibility that the outside reconcile disturbance field approximates the real field is supported by the Keldysh-Lavrentiev theorem. By assuming that the set of point masses are $\{m_i\}(i=1,2,\cdots,n)$, we can get

$$T_{\mathrm{P}}^{'} = f\sum_{i=1}^{n} m_i l_{\mathrm{Pi}}^{-1} = \sum_{i=1}^{n} M_i l_{\mathrm{Pi}}^{-1}, \qquad (4)$$

where the virtual masses $M_i = Gm_i$, l_{Pi} is the distance between the computation point and the moving point which is on the virtual sphere. We get the gravity anomaly Δg which is usually as the observation data in geodesy and geometric, and the boundary condition is given as follows [4]

$$\Delta g_j = -\frac{\partial T_j^{'}}{\partial r_j} - \frac{2T_j^{'}}{r_j}, \qquad (5)$$

Inserting Eq. (4) into Eq. (5), we get

$$\Delta g_j = \sum_{i=1}^{n}[(r_j - R_B\cos\psi_{ji})l_{ji}^{-3} - 2r_j^{-1}l_{ji}^{-1}]M_i, \qquad (6)$$

This formula is the observation equation which is a linear model. Thus we can get the unknown parameters M_i according to Eq. (6). In this paper, we use least squares method to deal with the solution of observation equation. Then the anomalous potential T' can be obtained by Eq. (4). Therefore, the geoid unbulation can be written as follows according to the relation between the potential and the gravity disturbance vectors[5]:

$$N = \frac{T}{\gamma} = \frac{1}{\gamma}\sum_{i=1}^{n} M_i l_{\mathrm{Qi}}^{-1}, \qquad (7)$$

where γ is the normal gravity which can be calculated through a simple function.

3. Computational Experiments

Before we format the model, we calculate the formation matrix units. If the resolution is $1^\circ \times 1^\circ$ and the area is $9^\circ \times 9^\circ$, then we get the matrix is 100×100. The parameter R_B is set to approximately equal to the resolution of observation data in the experiment, so we set the depth is 100 km. Thus we get the matrix is shown in Fig. 2. In fact the unit value is too small, so we multiply the matrix units by a constant number which is 10^6. We can see that the matrix is a split toeplitz matrix. In fact we can get the same conclusion according to the Eq. (6). For $a=(r_j - R_B\cos\psi_{ji})l_{ji}^{-3} - 2r_j^{-1}l_{ji}^{-1}$ is the unit of the formation matrix, we can see that a is the functions of r, R and ψ. If we set the r and R as constant values, we will get the results that for any two units, as long as $|i-j|$ is equal, the two units have the same value. Therefore, the coefficient matrix of formation model is a split toeplitz matrix.

In order to deal with the toeplitz matrix of formation model faster, we should format the robust matrix. It should be noted that when the virtual point of quality and surface observations have the same spherical coordinates, when the coefficient matrix of the stability of the structure is best [17]. Thus, two numerical simulations have been done to know the coefficient matrix units. Fig. 2 is in the case that the distance between the virtual point and the earth surface is 15 km. In face, the units' value is very small, so in order to facilitate matrix operation, the units is multiplied by a constant number 10^{-5}. From Fig. 2, we can see that the matrix is a split toeplitz matrix, and most of the units are close to 0.

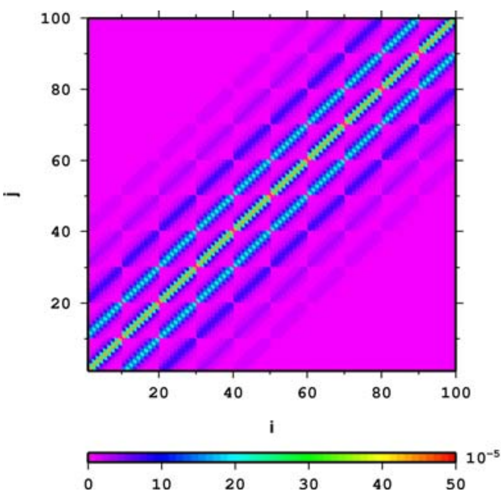

Fig. 2. Variation of the matrix unit for the depth of virtual points is 15 km.

The other experiment is in the case that the distance is 50 km, and the results are shown in Fig. 3. Of course, the values of the units are also small and we set the constant number is 10^{-5} as we did in the former simulation. Comparing Fig. 2 and Fig. 3, we can see that the maximum number of the former matrix is larger then the latter one. In addition, only a little variation of the matrix in Fig. 3 is close to 0.

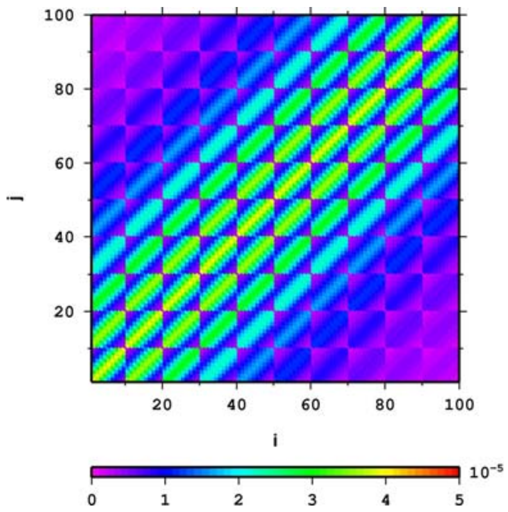

Fig. 3. Variation of the matrix unit for the depth of virtual points is 50 km.

We can also see that the main diagonal matrix elements are the maximum value of every split toeplitz matrix. This offers us a good condition to solve the formation equation.

Now we set the region is $74^{\circ}-104^{\circ}E$ and $25^{\circ}-40^{\circ}N$, and the point masses model is composed by three groups which are from low frequency to high frequency of the geoid. We know that this area is the Tibet Plateau which is the youngest and largest part of the Qinghai-Tibet PlateauIt. It stretches approximately 1000 km north to south and 2500 km east to west. The average elevation is over 4,500 m, and all 14 of the world's 8,000 m and higher peaks are found in the region. Tibet Plateau can offer a complex geoid which is a typical simulation target. Let us take back to the simulation. The resolution of three groups are $1^{\circ}\times 1^{\circ}$, $20'\times 20'$ and $3'\times 3'$ correspondingly. However, the third group's resolution is so dense that the formation matrix is huge. This will lead to the problem that it is difficult to solve the formation formula. Hence we split the approximation area many little regions of which the area is $2^{\circ}\times 2^{\circ}$. The observation data are calculated by EGM2008 and the simulated geoid is shown in Fig. 4. The geoid undulation is about - 70 m ~ - 10 m. Therefore, the information of the gravity is plentiful. That's to say, this place is a typical region for our experiments.

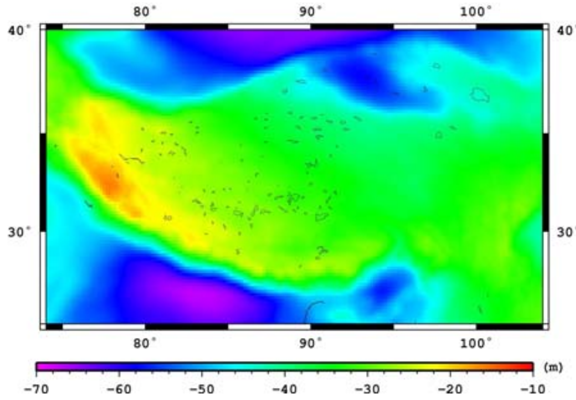

Fig. 4. The geoid of the local area.

The statistics of the solutions of the observation equation are shown in Table 1. Table 1 makes it clear that the maximum error is less than $1mGal$. It has to note that no matter in any group, the formatted geoid is close to the theoretical curve. Hence, the precision of the equation's solution is good, that to say, the formation matrix is not an ill-posed condition. In fact, the number of point groups could be larger than three.

Table 1. The results of the simulation.

Error type	Group A	Group B	Group C
Minimum error(Gal)	5.42101086242752E-013	5.42101086242752E-012	0.0000000000000
Maximum error(Gal)	2.86229373536173E-010	1.55431223447522E-008	2.22261445359528E-011
RMS(Gal)	6.05360942663373E-011	2.28746041692529E-009	5.00773615478669E-013

4. Discussions

Geoid is an element of Earth's gravity field, and it can be calculated by the gravity field model. Tibet Plateau is a large region and the geoid fluctuates widely. If we format the geoid model through traditional methods, it is hard to meet the needs of high resolution. However, the virtual point method can be relatively easy to achieve the requirements of the model. Another very important advantage is that the speed of the calculation of the geoid's unit is fast. We should take the note that Point mass model may not be the best approximation of the gravity field model in this region, but the region can best approximate the geoid. Such approximating polynomials of virtual sphere always exist for arbitrary accuracy requirements. In geodesy we usually speak of Runge's theorem. In essence, the geoid we formatted is a harmonic geoid because the geoid is an equi-potential surface of an analytical downward continuation. And besides, the point masses are below the topographic surface.

The fundamental theorems on the asymptotic behavior of Eigen values and inverses of banded Toeplitz matrices and Toeplitz matrices with absolutely summable elements are widely used in engineer. Typically, we need to get the inverse of

Toeplitz matrices quickly. Therefore how to format a good Toeplitz matrix is a very important issue. In the construction of the point mass model, a good approach is that the virtual point mass and the observation have the same geo-coordination according to the simulations. Mathematics concept is simple, versatile and elegant. Covariance matrix of the linear model applicable to discrete-time stochastic processes is as a result of an application and their factors.

5. Conclusions

According to the analysis and simulation above, the conclusions can be drawn: (1) of the quality of construction of the regional geoid model, can be very high resolution geoid approaching the region; (2) the point mass model building, although is not the best approximation regional gravity field of the real situation, but can best approximate the geoid in this region, therefore, this method provides a reference for the establishment of the global geoid; (3) in the process of regional geoid model construction by point mass method, the key issue is to build a stable factor structure matrix; (4) the complexity of the physical characteristics of the Tibetan plateau, the point mass approximation of the geoid in the region is up to centimeter level accuracy. However, there are still some problems which are not resolved, especially the fast calculation of the inverse of the formatted matrix. In addition, approximation region of this method is limited, so this issue should be in-depth studied.

Acknowledgements

The author wishes to thank Dr. Xing Cheng for his constructive suggestions and valuable comments on the original manuscript. This work was supported in part by National Natural Science Foundation of China (grant No. 41161069, and grant No. 41204003) , open research fund program of the key laboratory of Geospace Environment and geodesy, Ministry of Education, China (grant No. 11-01-01), Jiangxi Province Key Lab for Digital Land (grant No. DLLJ201307) and Foundation of Jiangxi Educational Committee (grant No. GJJ12394).

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