Sensors & Transducers, Vol. 24, Special Issue, August 2013, pp. 50-57

Sensors & Transducers

http://www.sensorsportal.com

Comparison Errors of the Image Conductivity Distribution Reconstructed by Direct and Indirect Algorithms in Electrical Tomography

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Received: 15 May 2013 /Accepted: 16 August 2013 /Published: 30 August 2013

Abstract: In this paper the two indirect conductivity distribution reconstruction algorithms in electrical tomography are presented. The first algorithm based on the previous resistivity reconstruction and the second one based on the usage of the inverse measured voltages. Efficiency of the presented algorithms is compared with the efficiency of the direct conductivity distribution reconstruction algorithm. Modeling results shows that due to better linearity the both indirect algorithms always provides better convergence of the iterative reconstruction process than direct algorithm. In indirect algorithms do not need to know exactly the level of initial approach of the conductivity distribution, because this level almost never affects the iterative process and initial approaching of the conductivity can always be uniform distribution of size 1 S·m. Therefore (at least in the first iterations) it is advisable to use indirect algorithms. *Copyright © 2013 IFSA.*

Keywords: Electrical tomography, Conductivity, Reconstruction algorithms, Error.

1. Introduction

The *electrical tomography* (ET) can be used to image reconstruction of a space distribution of electrical conductivity and other physical quantities depended of the conductivity in medicine and industry [1-3]. The data acquisition system (DASY) with the measuring electrodes and the computer are the base components of typical electrical tomography system (Fig. 1). Measuring electrodes are placed on the periphery of a research object and used for it current or voltage excitation. The object responses on this excitation in the form inter-electrode potential differences (output voltages) or as the electrode currents are measured and collected in DASY. The typical block scheme of the DASY is shown in Fig. 2 [4]. The main components of DASY are:

- Two multiplexer: MUXI (current excitation) and MUXV (measurement voltage);
- Current source (I0);
- Standard resistor (used for the corrections);
- Instrumental amplifier (IAMP) with reference voltage input Vref;
- Precision Analog-to-Digital and Digital-to-Analog (used for the corrections) converters, for example as data acquisition board (NI-DAQ) based on PCI bus with computer (PC) or PXI modular system.

Article number P_SI_431

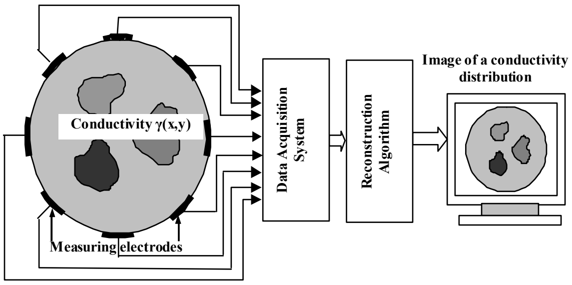
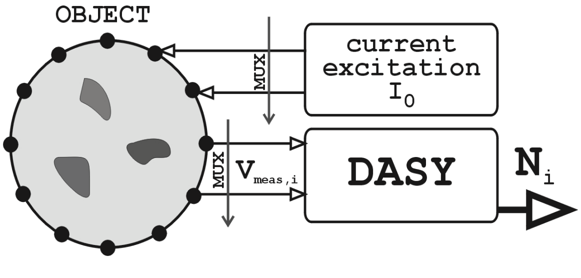
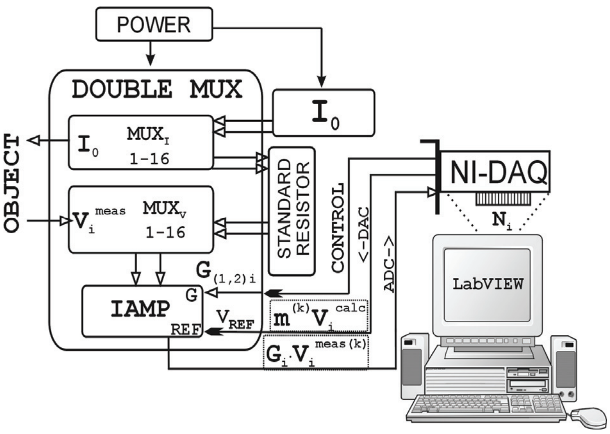


Fig. 1. The main components of the electrical tomography system.



(a)

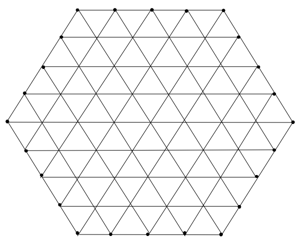


(b)

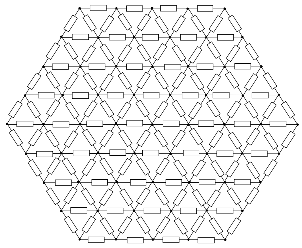
Fig. 2. Data Acquisition System (DASY) of ET [4]: the current excitation of object (a), the measurement process (b). (I_0 – excitation current, NI-DAQ – measurement board, IAMP – instrumental amplifier, G – gain of IAMP, DAC digital-to-analog converter, ADC analog-to-digital converter, N_i – output measurement digital data).

The quality of the conductivity image mainly depends on of the measurement errors in DASY, the reconstruction algorithm errors and the error of the approximation of the electrical field inside the investigated object. Each components of the DASY must be very precise because the uncertainties in measurement results are strongly amplified by used reconstructing algorithm. To increase the accuracy of the measurement results the difference method of measurement with additive and multiplicative correction techniques are used. To suppress the random effects additionally the averaging of measurement results is used.

Usually the inverse problem (image reconstruction of a conductivity space distribution) of electrical tomography is strongly non-linear, so iteration reconstruction algorithms are used to solve it. In general case to solve the electrical tomography inverse problem the continuous conductivity distribution must by approximated by discrete elements, for example, using the finite elements method [5-10]. From such approximation the researched area is covered for example by triangular elements (Fig. 3, a), which there corresponds an electrical network of resistors (Fig. 3, b) and (or) capacitors. If the rectangular finite elements are used then also the appropriate electrical network (with another configuration) can be constructed from resistors and (or) capacitors (Fig. 2, c) [6-10]. The network configuration and number of its resistors (capacitors) depend on the number of used finite elements and their arrangement.



(a)



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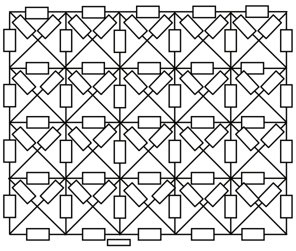

(c)

Fig. 3. Finite element mesh from triangle elements (a), equivalent network constructed from resistors (b), and electrical network approximated rectangular object (c).

The properties of these networks (Fig. 2, b, c) are used to calculate the Jacoby matrix as the most important component of reconstruction algorithm.

2. Direct and Indirect Reconstruction Algorithms

The goal of this paper is the investigation of the efficiency (convergence of the iterative reconstruction process) of the direct and two indirect (inverse conductivity (or resistivity) and inverse voltage) algorithms of the conductivity distribution image reconstruction [11-13]. Fig. 4 shows a numeric modeling block-scheme of the one iteration of the reconstruction process for these algorithms [13].

All three algorithms starts from calculation of the sensitivity (Jacobi) matrixes $\mathbf{J}$ depended on the *initial* conductivity $\sigma = \sigma_{init}$ (after discretization and FEM approximation of continuous distribution $\sigma_{init}(x, y)$). Then for the given domain model of the conductivity distribution $\sigma = \sigma_{\text{mod}}$ and given value for the injective current I the simulated measuring results $\mathbf{U}_{meas}$ are calculated. These calculations (calculation of the output electrode voltages) are the essence of the solving forward electrical tomography problem.

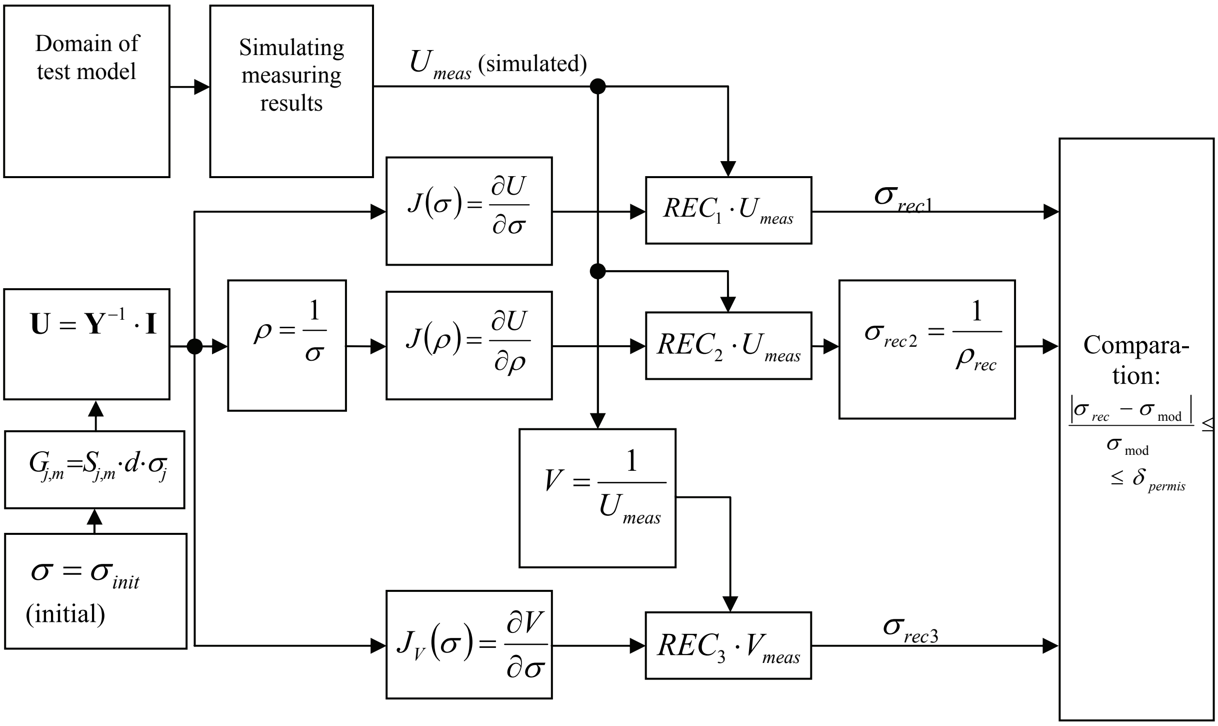

Fig. 4. Numeric modeling block-scheme of one iteration of three reconstruction algorithms (direct, resistivity (inverse conductivity), inverse voltage).

Using the Jacobi matrix and the simulated measuring results reconstructed conductivity values σ_{rec} are calculated (inverse tomography problem). Finally obtained values are compared with the conductivity values of the domain model. If the desired accuracy: maximum permissible error Δ_{permis} or relative δ_{permis}

$$\delta_\sigma = \left|\frac{\Delta_{rec}}{\sigma_{\text{mod}}}\right| = \left|\frac{\sigma_{rec} - \sigma_{\text{mod}}}{\sigma_{\text{mod}}}\right| \le \delta_{permis} \quad (1)$$

is achieved the reconstruction process stops. If not then new Jacobi matrix elements are calculated. Reconstructed conductivity values of the previous iteration $\sigma_{rec}^{(it-1)}$ are used instead the initial conductivity distribution σ_{init}.

Such comparison as described above can be implemented only for numeric modeling. In the real applications we don't know the real values of the domain conductivity. In that case we can compare calculated boundary voltages $U_{calc}(\sigma_{rec})$ with those that are measured U_{meas}:

$$\delta_U = \left|\frac{U_{calc}(\sigma_{rec}) - U_{meas}}{U_{meas}}\right| \le \delta_{U,permis}. \quad (2)$$

2.1. Direct Algorithm

Most of the algorithms implement a *direct* conductivity reconstruction with the prior sensitivity matrix – Jacobi matrix element $J_{U,i,j}(\sigma_j)$ calculation:

$$J_{U,i,j}(\sigma_j)=\frac{\partial U_{meas,i}}{\partial \sigma_j}, \quad (3)$$

where i is the number of the independent measurement result; j is the number of domain discretization element; $U_{meas,i}=F_i(\sigma(x,y))$ is the measured voltage value as the non-linear function $F_i(\cdot)$ of the unknown conductivity distribution $\sigma(x,y)$.

Jacobi matrix shows a sensitivity of measured quantity (in our case – measured voltage) to the conductivity value change. The main equation of the electrical conductivity tomography is the continuum version of Ohm's law:

$$j(x,y)=\sigma(x,y)E(x,y)=-\sigma(x,y)\nabla\varphi(x,y), \quad (4)$$

where $j(x,y)$ is the current density distribution; $E(x,y)$ is the electric field distribution; $\varphi(x,y)$ is the potential distribution.

In [7-10] was shown that if in (4) the electrical potential distribution $\varphi(x,y)$ inside research object is approximated by the triangle or rectangular elements then all Jacobi matrix elements $J_{U,i,j}(\sigma_j)$ can be calculated precisely using node potential differences (branch voltages) of the equivalent network constructed from resistors and (or) capacitors (Fig. 3, b, c) by formula:

$$J_{U,i,j}=\frac{1}{I_0}\sum_{m=1}^{M}S_{j,m}U_{j,m}(k)U_{j,m}(i). \quad (5)$$

where M = 3 (for triangle FEM) and M = 6 (for rectangular FEM) are the number of the branches (resistance elements) in the equivalent electrical network of each finite element; $S_{j,m}$ are the coefficients, their values depended only the shape parameters of the used FEM elements; $U_{j;m}(k)$, $U_{j;m}(i)$ are the voltages on the branch with number m of finite element with number j if the object is excited (by current I_0) from k and i branches (respectively) located in the object boundary.

The set of the all branch voltages $U_{j;m}(k)$, $U_{j;m}(i)$ (used in formula (5)) can be calculated for the given distribution of the finite element conductivities and excited boundary currents using for example the node potential method described by matrix equation:

$$\mathbf{U}=\mathbf{Y}^{-1}\cdot\mathbf{I}, \quad (6)$$

where U is the matrix of all nodal potentials, Y is the nodal admittance matrix of the equivalent electrical network; I is the matrix of excited current applied for the boundary nodes (electrodes).

Let value of conductivity $\sigma(x,y)$ increases to value $\sigma_{new}(x,y)$ by k times:

$$\sigma_{new}(x,y)=k\cdot\sigma(x,y). \quad (7)$$

In [11-12] we show that if the excited current is constant (I = I0 = const) then scale change in domain conductivity (7) will not cause changes in current distribution and causes inverse change in the potential distribution. So a new electrical potential distribution $\varphi_{new}(x,y)$ will be equal to:

$$\varphi_{new}(x,y)=\frac{\varphi(x,y)}{k}. \quad (8)$$

And therefore similarly dependency of the measured potential differences will be the same:

$$U_{meas,new}=\frac{U_{meas}}{k}, \quad (9)$$

where $U_{meas,new}=F(\sigma_{new}(x,y))$ is the new value of the measured voltage.

And for a new conductivity value $\sigma_{new}(x,y)$ the Jacobi matrix element $J_{U,i,j,new}(\sigma_{new,j})$ will be equal to [5]:

$$J_{U,i,j,new}(\sigma_{new,j})=\frac{\partial U_{meas,new,i}}{\partial \sigma_{new,j}}= =\frac{1}{k^2}\cdot\frac{\partial U_{meas,i}}{\partial \sigma_j}=\frac{1}{k^2}\cdot J_{U,i,j}(\sigma_j) \quad (10)$$

Equation (8) shows an inverse quadratic dependence of the Jacobi matrix elements to the conductivity value change that caused essential non-linearity of the *direct* reconstruction algorithm.

2.2. Resistivity (Inverse Conductivity) Algorithm

Let use instead of conductivity it inverse quantity – resistivity $\rho(x,y)$:

$$\rho(x,y)=\frac{1}{\sigma(x,y)}, \quad (11)$$

For a new conductivity value scaled by k times $\sigma_{new}(x,y)=k\cdot\sigma(x,y)$ a new resistivity value will be equal to:

$$\rho_{new}(x,y)=\frac{1}{k\sigma(x,y)}=\frac{\rho(x,y)}{k}. \quad (12)$$

Because for the resistivity $\rho(x, y)$ sensitivity matrix elements value is:

$$J_{U,i,j}(\rho_j) = \frac{\partial U_{meas,i}}{\partial \rho_j} = \frac{\partial U_{meas,i}}{\partial \sigma_j} \cdot \frac{\partial \sigma_j}{\partial \rho_j} = = -J_{U,i,j}(\sigma_j) \cdot \sigma_j^2 \quad (13)$$

then after (10) and (13) for a new resistivity value $\rho_{new}(x, y)$ the new sensitivity matrix elements value will not change:

$$J_{U,i,j,new}(\rho_{new,j}) = -J_{U,i,j,new}(\sigma_{new,j}) \cdot \sigma_{new,j}^2 = = -\frac{J_{U,i,j}(\sigma_j)}{k^2} \cdot k^2 \sigma_j^2 = J_{U,i,j}(\rho_j) \quad (14)$$

Equation (14) shows that the resistivity Jacobi matrix elements are not dependent on the scale conductivity (or resistivity) change. It gives an opportunity to set a prior conductivity approximation arbitrary, e.g. 1 *S·m*.

2.3. Inverse Voltage Algorithm

As we say, measured voltages are the non-linear function of conductivity distribution. Let denote a new function V_{meas} as inverse to the measured voltage U_{meas}:

$$V_{meas,i} = \frac{1}{U_{meas,i}}. \quad (15)$$

Then the Jacobi matrix element for inverse function will be equal to:

$$J_{V,i,j}(\sigma_j) = \frac{\partial V_{meas,i}(\sigma_j)}{\partial \sigma_j} = -\frac{J_{U,i,j}(\sigma_j)}{U_{meas,i}^2(\sigma_j)}. \quad (16)$$

If domain conductivity will change to the new value (equation (7)) the inverse function $V_{meas,i,new}$ value will change in same way. Accordingly to equations (9) and (16) for the new conductivity the new value of the Jacobi matrix element will not change:

$$J_{V,i,j,new}(\sigma_j) = -\frac{J_{U,i,j,new}(\sigma_{new,j})}{U_{meas,i}^2(\sigma_{new,j})} = = -\frac{k^2}{U_{meas,i}^2(\sigma_j)} \cdot \frac{J_{U,i,j}(\sigma_j)}{k^2} = = -\frac{J_{U,i,j}(\sigma_j)}{U_{meas,i}^2(\sigma_j)} = J_{V,i,j}(\sigma_j) \quad (17)$$

Equation (17) shows more linear dependency between inverse voltages and conductivity and that Jacobi matrix elements for the *inverse voltages* are not dependent on the scale conductivity change (by *k* times) same as in the *resistivity* algorithm.

3. Investigation of Algorithms Efficiency

For the given conductivity distribution $\sigma(x, y)$ and its FEM approximation by n_{el} discrete elements the electrical parameters (vector conductance $\mathbf{G}_{giv}$ of all branches) of equivalent network (Fig. 3) are calculated by formula [7-10]:

$$G_{j,m} = S_{j,m} \cdot d \cdot \sigma_j \quad (18)$$

where *d* is thickness of conducting medium.

If the regular discretization of the 96 triangular elements of the hexahedron domain (Fig. 3, a) is used then shape coefficient [10]:

$$S_{j,m} = S_0 = \cot(60^o) = 1/\sqrt{3}\,.$$

Conductances from formula (18) are used to calculate the nodal admittance matrix **Y** and next to calculate by formula (6) the matrix **U** of all nodal potentials of the equivalent network excited by current I_0 applied to the all boundary branches alternately. The vector of the measurement results $\mathbf{U}_{meas}$ is equal to the potential differences on boundary branches.

Measurement results generated for uniformly distributed domain with the prior conductivity value $\sigma_{m,j} = 0{,}001$ *S·m* which changes for 100 % (from 0,001 *S·m* to 0,002 *S·m*) for next test models:

1) 2 adjacent elements (with random number) in the center of domain;
2) 2 adjacent elements (with random number) on the boundary;
3) 6 adjacent elements in the center of the domain;
4) 6 adjacent elements on the domain's boundary;
5) 10 elements with random number;
6) random conductivity change (in 100 % range) of the all elements.

The initial conductivity distribution approach assumed as uniform ($\sigma_{j.init} = \sigma_0 = 1\,S \cdot m$). Therefore the initial vector of conductance is constant: $\mathbf{G} = \mathbf{G}_0$. Then at first the nodal admittance matrix **Y** is calculated and next the matrix **U** of all nodal potentials of the equivalent network excited by current $\mathbf{I} = \mathbf{I}_0$ applied to the all boundary branches is determined by formula (6). The elements of the base Jacobi matrix $\mathbf{J}(\sigma)$ are calculated after formula (5).

Using reconstruction (with regularization) Levenberg-Marquardt algorithm [14] the vector of search conductance $\mathbf{G}_{REC}^{(it)}$ in iteration with number *it* in direct algorithm was determined by formula:

$$\mathbf{G}_{REC}^{(it)} = \mathbf{REC}_1^{(it)} \cdot \mathbf{U}_{meas} = \mathbf{REC}_\sigma^{(it)} \cdot \mathbf{U}_{meas}, \quad (19)$$

where $\mathbf{REC}_1^{(it)} = \mathbf{REC}_\sigma^{(it)}$ is direct conductivity reconstruction matrix that depended on number (n_{el}) and shape (triangular, rectangular, etc.) of FEM approximation and the current value of conductivity distribution ($\sigma^{(it-1)}$):

$$\mathbf{REC}_\sigma^{(it)} = \left(\mathbf{J}(\sigma)^{(it)T}\mathbf{J}(\sigma)^{(it)} + \mu^{(it)} \cdot \mathbf{1}\right)^{-1} \cdot \mathbf{J}(\sigma)^{(it)T}, \quad (20)$$

where: $\mathbf{J}(\sigma)^{(it)}$ is current Jacobi matrix, $\mu^{(it)} = r^{(it)} \cdot \sqrt{n_{el} \cdot m_{eps}} \cdot norm2\left(\mathbf{J}(\sigma)^{(it)T}\mathbf{J}(\sigma)^{(it)}\right)$ is regularization term; m_{eps} is so called machine epsilon, $norm2\left(\mathbf{J}(\sigma)^{(it)T}\mathbf{J}(\sigma)^{(it)}\right)$ is the norm of matrix $\mathbf{H}(\sigma)^{(it)} = \mathbf{J}(\sigma)^{(it)T}\mathbf{J}(\sigma)^{(it)}$; $r^{(it)} = f\left(U_{meas} / \mathrm{U}_{calc}^{(it)}\right)$ is the coefficient that take into account the degree of inequality of measured $\mathbf{U}_{meas}$ and calculated $\mathbf{U}_{calc}^{(it)} = \mathbf{J}(\sigma)^{(it-1)} \cdot \mathbf{G}_{REC}^{(it)}$ voltages; **1** is a unitary diagonal matrix.

The vector of search conductivity $\sigma_{REC}^{(it)}$ is determined from inverse to (18) formula:

$$\sigma_{REC,j}^{(it)} = G_{REC,j}^{(it)} \Big/ S_0 \cdot d. \quad (21)$$

Condition for completion of the iteration process is maximum error to be less than 1 %. Errors calculated according to the equation:

$$\delta_{j,\max} = \left| \frac{\sigma_{REC,j}^{(it)} - \sigma_{m,j}}{\sigma_{m,j}} \cdot 100\% \right| \le 1\%, \quad (22)$$

where $\sigma_{m,i}$ is the model conductivity value of *j* – th discrete element of the domain.

In *resistivity algorithm* the current resistivity Jacobi matrix $\mathbf{J}(\rho)^{(it)}$ is calculated by formula (13) using conductivity Jacobi matrix $\mathbf{J}(\sigma)^{(it)}$ and reconstruction matrix was calculated by similar to (20) formula

$$\begin{aligned} &\boldsymbol{REC}_2^{(it)} = \boldsymbol{REC}_\rho^{(it)} = \\ &= \left(\boldsymbol{J}(\rho)^{(it)T}\boldsymbol{J}(\rho)^{(it)} + \mu^{(it)} \cdot \boldsymbol{1}\right)^{-1} \cdot \boldsymbol{J}(\rho)^{(it)T} \end{aligned} \quad (23)$$

Then vector of search resistances $\mathbf{R}_{REC}^{(it)}$ in iteration with number *it* was calculated by formula:

$$\mathbf{R}_{REC}^{(it)} = \mathbf{REC}_1^{(it)} \cdot \mathbf{U}_{meas} = \mathbf{REC}_\rho^{(it)} \cdot \mathbf{U}_{meas}, \quad (24)$$

and the search conductivity $\sigma_{REC,j}^{(it)}$ is determined from formula

$$\sigma_{REC,j}^{(it)} = 1 \Big/ R_{REC,j}^{(it)} \cdot S_0 \cdot d. \quad (25)$$

In *inverse voltage algorithm* the current Jacobi matrix $\mathbf{J}_V(\sigma)^{(it)}$ was calculated by formula (14) using conductivity Jacobi matrix $\mathbf{J}(\sigma)^{(it)}$ and reconstruction matrix is calculated by formula

$$\begin{aligned} &\boldsymbol{REC}_3^{(it)} = \boldsymbol{REC}_V^{(it)} = \\ &= \left(\boldsymbol{J}_V(\sigma)^{(it)T}\boldsymbol{J}_V(\sigma)^{(it)} + \mu^{(it)} \cdot \boldsymbol{1}\right)^{-1} \cdot \boldsymbol{J}_V(\sigma)^{(it)T} \end{aligned} \quad (26)$$

Then vector of search conductance $\mathbf{G}_{REC}^{(it)}$ in iteration with number it was determined by formula

$$\mathbf{G}_{REC}^{(it)} = \mathbf{REC}_3^{(it)} \cdot \mathbf{V}_{meas} = \mathbf{REC}_V^{(it)} \cdot \mathbf{V}_{meas}, \quad (27)$$

and the search conductivity $\sigma_{REC,j}^{(it)}$ is determined from formula (21).

Fig. 5 – 7 shows maximum errors of the all elements on the each of the first 4 – 6 iteration for the three reconstruction algorithms and described above test models. Because on the first iteration *direct* reconstructed algorithm provide not real conductivity values and the maximum error is more than 100 % on Fig. 5 – 7 these errors are not presented.

4. Conclusions

Modeling results shows that the scale differences between the domain conductivity distribution and the prior approach of the conductivity distribution takes into account automatically by inverse conductivity (resistivity) and inverse voltage algorithms; direct algorithm didn't works without additional means. That's why value of the maximum error for direct algorithm on the first iteration is so large (Fig. 5 – 7). Using of initial distribution scaling after first iteration provides convergence of the process for the direct algorithm.

Similar results of the reconstruction process investigation for other test conductivity distributions with the different regularization parameters also obtained. These test distributions are: 100 % conductivity change in the single element, located in different parts of the space (from the centre to the boundary of object); 100 %

conductivity changes in the several adjacent elements also in different parts of the space; 100 % conductivity changes of the half elements, located randomly.

Due to better linearity the both indirect algorithms always provides better convergence of the iterative reconstruction process than direct algorithm. And, in addition, indirect algorithms do not need to know exactly the level of the initial approach of the conductivity distribution, because this level almost never affects the iterative process. Initial approaching of the conductivity can always be uniform distribution for example of size 1 S·m. Therefore (at least in the first iterations) it is advisable to use indirect reconstructed algorithms.

Modeling implemented in MatLab 7.10.0.

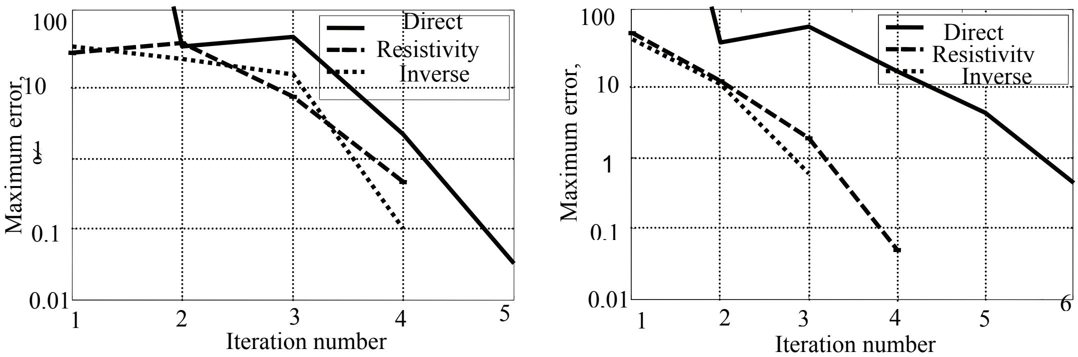

Fig. 5. The maximum reconstruction error for the 100 % conductivity changes of 2 adjacent elements in the center of the domain (a) and on the domain's boundary (b).

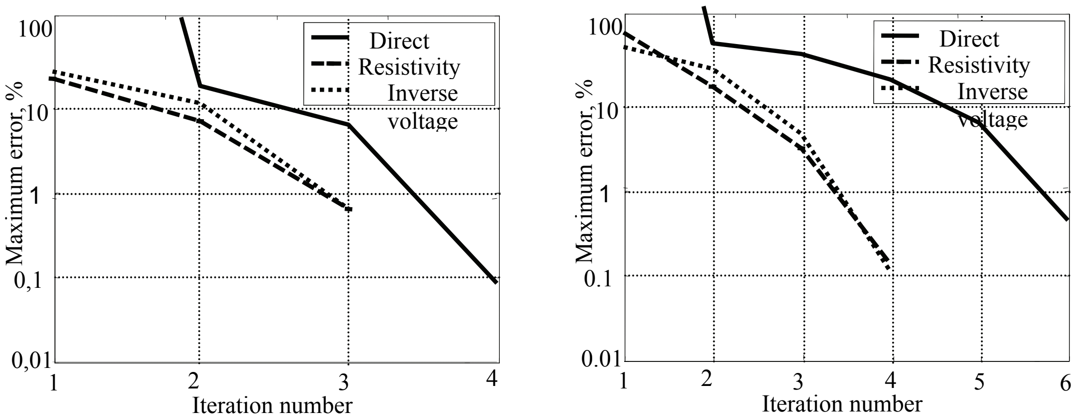

Fig. 6. The maximum reconstruction error for the 100 % conductivity changes of 6 adjacent elements in the center of the domain (a) and on the domain's boundary (b).

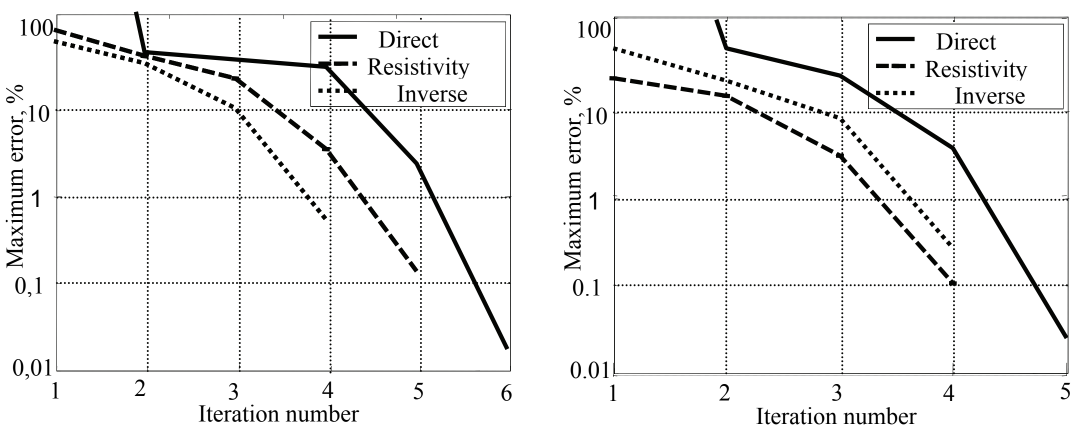

Fig. 7. The maximum reconstruction error for the 100 % conductivity changes of 10 random located elements (a) and random 100 % conductivity changes of the all elements (b).

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