

## Joint Features Extraction from Multiple Harmonic Sources Based on MUSIC Algorithm

<sup>1</sup> Hong-Ji Wang, <sup>1</sup> Ri-Jie Yang, <sup>1</sup> Yang Shen, <sup>2</sup> Li-Ye Tian

Naval Aeronautical Engineering Academy, 264001, China

Navla Submarine academy, 266001, China

Tel.: 008605356635813

E-mail: wanghongji-2008@163.com

*Received: 16 September 2013 / Accepted: 15 October 2013 / Published: 23 December 2013*

---

**Abstract:** Ship noise received by passive sonar is multiple harmonic sources, from which the recognition features of ship can be extracted. Based on the multiple signal classification estimation theory, a joint estimation method of harmonic sources number, harmonic orders and fundamental frequencies is developed in this paper. The observed signal vector is decomposed into multiple harmonics vector and noise vector, and the spectral factorization of spatial covariance matrix is evaluated to obtain the signal subspace and noise space. By exploiting the orthogonal property between subspaces, the spatial spectral representing cost function is presented. The number of harmonic sources, model orders and fundamental frequencies are obtained by maximizing the cost function. Simulation results verified the effectiveness of the proposed algorithm.

*Copyright © 2013 IFSA.*

**Keywords:** Fundamental frequency estimation, MULTiple Signals Classification (MUSIC) algorithm, Harmonic, Feature extraction

---

### 1. Introduction

Since the fundamental frequency is the essential feature of a harmonic signal, then the identification of the harmonic signal source is essentially to extract its fundamental frequency. At present, there are many scholars have studied single fundamental frequency extraction in-depth [1-3]. According to the processing domain, fundamental frequency extraction methods can be broadly divided time-domain method, frequency-domain methods and statistical methods [4]. In the time domain method, the early typical fundamental frequency extraction method is match filter by using the correlation function [5], which uses the maximum correlation coefficient of the

fundamental frequency as the extraction result. Such method can obviously get a "coarse" fundamental frequency estimation. In order to obtain more precise estimation of the fundamental frequency, there are many other methods such as based on subspace decomposition method [6, 7], based on the optimal filter method in the principle of minimum variance [8], comb-like filter array pattern method [9]. Since wavelet transform has a good localization quality in time-frequency domain, it has been widely used to extract fundamental frequency. As in literature [10-12], these methods have been compared to traditional methods in robustness and adaptability aspect. In addition, the method based on joint time and frequency analysis [13] not only combined the

advantages of the time-domain and frequency-domain method, but also achieved good fundamental frequency estimation results.

This paper studies the fundamental frequency extraction of the noise signals generated by ships, while received by passive sonar. Typically, the noise signals generated by ships and submarine mainly include machinery noise, propeller noise and hydrodynamic noise. When passive sonar uses these noise signals for recognition, it should firstly extract the fundamental frequency of the noise signal. However, noises often contain a plurality of harmonic signals. Therefore, compared to the fundamental frequency extraction in the single harmonic signal, it will be more complicated for multi-frequency harmonics signal extraction [14-16]. There is very important factor that the number of harmonics in the observed signal is unknown, and the number of the fundamental frequency need to estimate is unknown. Therefore, before the estimation of the fundamental frequency, we need to determine the number of harmonic in observed signals, and then study the fundamental frequency extraction method. When the order of harmonics is unknown, it is necessary to estimate the order of harmonic. This increases the difficulty of the fundamental frequency estimator design. For these problems, based on the estimation principle of MUSIC (multiple signal classification) algorithm [17], uses the "spatial spectrum" cost function in the MUSIC algorithm, search for maximization of the cost function, and propose a method, which can joint estimate of the harmonics number, each harmonic order and the fundamental frequency of harmonics. Finally, simulation of multi-harmonic signal was presented to evaluate the performance of the proposed method.

## 2. The Joint Extraction Method for Multi-harmonic Signal Characteristic Parameter

### 2.1. Estimation Principle of MUSIC Algorithm

The observed signal  $y(t)$  is the sum of harmonic signal and Gaussian white noise, i.e.

$$y(t) = \sum_{l=1}^L \alpha_l e^{j\theta_l} e^{jl\omega_0 t} + w(t), \quad (1)$$

where,  $t=1, \dots, N$  indicates the time index;  $L$  is the harmonic order of the harmonic signals;  $\alpha_l$  is the first  $l$  index constituent of the amplitude value of the harmonic signal,  $\theta_l$  is the index constituent for the corresponding phase, and in the uniform distribution

between  $[-\pi, \pi]$ ;  $\omega_0$  is fundamental frequency, which need to estimate;  $w(t)$  is complex symmetric white Gaussian noise.

Using the signal model (1), the observation vector is defined as

$$\mathbf{y}(t) = [y(t), y(t-1), \dots, y(t-M+1)]^T, \quad (2)$$

where  $M$  is the dimension of the vector signal  $\mathbf{y}(n)$ , and  $t \geq M$ ,  $M \gg L$ ;  $\mathbf{T}$  is transpose symbols. Assuming  $\theta_l$  is independent of each other, then the covariance matrix of  $\mathbf{y}(n)$  can be expressed as

$$\mathbf{R} = E[\mathbf{y}(t)\mathbf{y}^H(t)] = \mathbf{A}\mathbf{P}\mathbf{A}^H + \sigma^2\mathbf{I}, \quad (3)$$

where  $E[\cdot]$  represents mathematical expectation;  $\mathbf{H}$  represents the conjugate transpose;  $\sigma^2$  is the noise variance;  $\mathbf{I}$  is a  $M \times M$  unit matrix.  $\mathbf{A}$  is Vander monde matrix [18], namely

$$\mathbf{A}_{M \times L} = [\mathbf{a}(\omega_0) \quad \mathbf{a}(2\omega_0) \quad \dots \quad \mathbf{a}(L\omega_0)], \quad (4)$$

where  $\mathbf{a}(\omega) = [1 \quad e^{-j\omega} \quad \dots \quad e^{-j(M-1)\omega}]^T$ .  $\mathbf{P}$  is the diagonal matrix, the  $l^{\text{th}}$  diagonal element of which is  $\alpha_l^2$ . Eigen value decomposition for  $\mathbf{R}$ , which can be expressed as

$$\mathbf{R} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H, \quad (5)$$

where  $\mathbf{U}_{M \times M} = [\mathbf{U}_1, \mathbf{U}_2, \dots, \mathbf{U}_M]$  is the matrix of  $\mathbf{R}$ , whose eigenvectors combination is mutually orthogonal;  $\mathbf{\Lambda}_{M \times M} = \text{diag}\{\lambda_1 \quad \lambda_2 \quad \dots \quad \lambda_M\}$  is a diagonal matrix which is consisted by the Eigen values of  $\mathbf{R}$ , and also  $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_M > 0$ . Because the rank of  $\mathbf{A}$  is  $L$ , and the rank of  $\mathbf{R}$  is  $M$ , and there is  $M \gg L$ . Therefore, by the relation (3) and (5), it shows that,  $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_L > \sigma^2$ ,  $\lambda_{L+1} = \dots = \lambda_M = \sigma^2$ .  $\mathbf{U}$  is divided into a signal feature vector  $\mathbf{U}_s = [\mathbf{U}_1, \mathbf{U}_2, \dots, \mathbf{U}_L]$  and a noise feature vector  $\mathbf{U}_n = [\mathbf{U}_{L+1}, \mathbf{U}_{L+2}, \dots, \mathbf{U}_M]$ , whose corresponding Eigen values are  $\{\lambda_1, \lambda_2, \dots, \lambda_L\}$  and  $\{\lambda_{L+1}, \lambda_{L+2}, \dots, \lambda_M\}$ , so  $\mathbf{R}$  can be rewritten as

$$\begin{aligned} \mathbf{R} &= \mathbf{U}_s \mathbf{\Lambda}_s \mathbf{U}_s^H + \mathbf{U}_n \mathbf{\Lambda}_n \mathbf{U}_n^H \\ &= \mathbf{U}_s \mathbf{\Lambda}_s \mathbf{U}_s^H + \sigma^2 \mathbf{U}_n \mathbf{U}_n^H \end{aligned} \quad (6)$$

Easy to know, and the subspace spanned by  $\mathbf{A}$  and  $\mathbf{U}_n$  is mutually orthogonal, i.e.

$$\mathbf{A}^H \mathbf{U}_n = \mathbf{0}, \mathbf{a}^H(\omega_0) \mathbf{U}_n = \mathbf{0} \quad (7)$$

MUSIC algorithm defines "spatial spectrum" as

$$J(\omega_0) = \frac{\|\mathbf{a}(\omega_0)\|_F^2 \|\mathbf{U}_n\|_F^2}{\|\mathbf{a}^H(\omega_0) \mathbf{U}_n\|_F^2}, \quad (8)$$

where,  $\|\cdot\|_F$  represents the Frobenius norm [19], and using the following equation estimates the parameter  $\omega_0$  of the  $J(\omega)$

$$\hat{\omega}_0 = \arg \max_{\omega_0 \in \Omega} J(\omega_0) \quad (9)$$

Equation (8) assuming the harmonics order  $L$  known, Christensen et al [6] extend the harmonic order to the unknown situation, use the principle of the MUSIC algorithm to study the joint estimation problem of the harmonic order  $L$  in harmonic signal  $y(t)$  and baseband  $\omega_0$ , presents a HMUSIC (Harmonic MUSIC) algorithm.

## 2.2. Multi-harmonic Signal Characteristic Parameter Combination Extraction Method

In reality, the observed signal received may contain the signals from different sources. Harmonic order and fundamental frequency are emitted by those harmonic signal are different, and the number of the source, in some cases, is also unknown. In view of this situation, new proposed joint estimation method, which can estimate the number, harmonic order, the fundamental frequency of the multi-harmonic signal.

Let the observed signal  $y(t)$  includes  $K$  harmonic signals, i.e.

$$y(t) = \sum_{l=1}^{L_1} \alpha_l^1 e^{j\theta_l^1} e^{j l \omega_0^1 t} + \dots + \sum_{l=1}^{L_K} \alpha_l^K e^{j\theta_l^K} e^{j l \omega_0^K t} + w(t) \quad (10)$$

where, the parameters to be estimated are the harmonic number  $K$ , Harmonic order  $\{L_k, k=1, \dots, K\}$  and fundamental frequency  $\{\omega_0^k, k=1, \dots, K\}$ . Let

$$y_k(t) = \sum_{l=1}^{L_k} \alpha_l^k e^{j\theta_l^k} e^{j l \omega_0^k t},$$

and  $y(t)$  can be rewritten as

$$y(t) = y_1(t) + y_2(t) + \dots + y_K(t) + w(t) \quad (11)$$

Using equation (11) to build observation vector

$$\begin{aligned} \mathbf{y}(t) &= [y(t), y(t-1), \dots, y(t-M+1)]^T \\ &= \begin{bmatrix} y_1(t) + \dots + y_K(t) + w(t), \dots, \\ y_1(t-M+1) + \dots \\ + y_K(t-M+1) + w(t-M+1) \end{bmatrix}^T \\ &= \begin{bmatrix} y_1(t) + \dots + y_K(t) + w(t), \dots, \\ y_1(t-M+1) + \dots \\ + y_K(t-M+1) + w(t-M+1) \end{bmatrix}^T \\ &= \mathbf{y}_1(t) + \mathbf{y}_2(t) + \dots + \mathbf{y}_K(t) + \mathbf{w}(t), \end{aligned} \quad (12)$$

where  $\mathbf{w}(t) = [w_k(t), w_k(t-1), \dots, w_k(t-M+1)]^T$ . Assuming each harmonic signal in  $\mathbf{y}_k(t)$  are uncorrelated, and are independent with the noise signal, then according to the relationship (3), the covariance matrix can be expressed as

$$\begin{aligned} \mathbf{R} &= E[\mathbf{y}(t)\mathbf{y}^H(t)] \\ &= E\left\{[\mathbf{y}_1(t) + \mathbf{y}_2(t) + \dots + \mathbf{y}_K(t) + \mathbf{w}(t)][\cdot]^H\right\} \\ &= E[\mathbf{y}_1(t)\mathbf{y}_1^H(t)] + \dots \\ &\quad + E[\mathbf{y}_K(t)\mathbf{y}_K^H(t)] + E[\mathbf{w}(t)\mathbf{w}^H(t)] \\ &= \mathbf{A}_1 \mathbf{P}_1 \mathbf{A}_1^H + \dots + \mathbf{A}_K \mathbf{P}_K \mathbf{A}_K^H + \sigma^2 \mathbf{I}, \end{aligned} \quad (13)$$

where  $\mathbf{A}_k$  is the Vander monde matrix of the  $k^{\text{th}}$  harmonic signal, namely

$$\mathbf{A}_k = [\mathbf{a}(\omega_0^k) \quad \mathbf{a}(2\omega_0^k) \quad \dots \quad \mathbf{a}(L_k \omega_0^k)]$$

Similarly, do spectral decomposition to  $\mathbf{R}$  as shown in Equation (5), divide different signal subspace  $\mathbf{U}_S^k$  and noise subspace  $\mathbf{U}_n^k$  for different harmonic signal, and the Van der Monde matrix corresponding to each harmonic signal and the corresponding noise subspace are orthogonal, i.e.

$$\mathbf{A}_k^H \mathbf{U}_n^k = \mathbf{0}, \mathbf{U}_n^k = [\mathbf{U}_{L_k+1}^k, \mathbf{U}_{L_k+2}^k, \dots, \mathbf{U}_M^k] \quad (14)$$

where

$$\begin{aligned} \tilde{\mathbf{A}} &= \begin{bmatrix} \mathbf{A}_1 & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_2 & \dots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{A}_K \end{bmatrix}, \\ \tilde{\mathbf{U}}_n &= \begin{bmatrix} \mathbf{U}_n^1 & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{U}_n^2 & \dots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{U}_n^K \end{bmatrix} \end{aligned} \quad (15)$$

Then use the (14), we can build the following equivalence

$$\tilde{\mathbf{A}}^H \tilde{\mathbf{U}}_n = \mathbf{0} \quad (16)$$

Definition "spatial spectrum" objective function as

$$J(K, \mathbf{L}, \boldsymbol{\omega}_0) = \frac{\|\tilde{\mathbf{A}}^H\|_F^2 \|\tilde{\mathbf{U}}_n\|_F^2}{\|\tilde{\mathbf{A}}^H \tilde{\mathbf{U}}_n\|_F^2} \quad (17)$$

The objective function of the unknown parameters includes the number of harmonics  $K$ , harmonic order  $\mathbf{L}=\{L_k, k=1, \dots, K\}$ , and fundamental frequency  $\boldsymbol{\omega}_0=\{\omega_0^k, k=1, \dots, K\}$ . These parameters are given below through the joint estimation.

By the definition of Frobenius norm and  $\mathbf{A}_k$ , we have  $\|\mathbf{A}_k\|_F^2 = ML_k$ ,  $\|\mathbf{U}_n^k\|_F^2 = M - L_k$ , from equation (15), it can be obtained

$$\|\tilde{\mathbf{A}}^H\|_F^2 = ML_1 + \dots + ML_K = M(L_1 + \dots + L_K) \quad (18)$$

$$\begin{aligned} \|\tilde{\mathbf{U}}_n\|_F^2 &= (M - L_1) + \dots + (M - L_K) \\ &= KM - (L_1 + \dots + L_K) \end{aligned} \quad (19)$$

Therefore

$$\begin{aligned} J(K, \mathbf{L}, \boldsymbol{\omega}_0) &= \frac{\|\tilde{\mathbf{A}}^H\|_F^2 \|\tilde{\mathbf{U}}_n\|_F^2}{\|\tilde{\mathbf{A}}^H \tilde{\mathbf{U}}_n\|_F^2} \\ &= \frac{M(L_1 + \dots + L_K)[KM - (L_1 + \dots + L_K)]}{\|\tilde{\mathbf{A}}^H \tilde{\mathbf{U}}_n\|_F^2} \end{aligned} \quad (20)$$

Using the idea of MUSIC algorithm, when all the parameters  $K$ ,  $\mathbf{L}$  and  $\boldsymbol{\omega}_0$  in the multi-harmonic signal, make the cost function formula in (20) maxima, the value of those parameters are the best estimation in the cost function optimization criterion represents. These ideas can be expressed as a mathematical formula

$$\left( \hat{K}, \hat{\mathbf{L}}, \hat{\boldsymbol{\omega}}_0 \right) = \arg \max_{\omega_0 \in \Omega} \max_{\mathbf{L} \in \mathcal{L}} \max_{K \in \mathcal{K}} J(K, \mathbf{L}, \boldsymbol{\omega}_0) \quad (21)$$

where  $\Omega$ ,  $\mathcal{L}$ ,  $\mathcal{K}$  are sets of all possible values  $\boldsymbol{\omega}_0$ ,  $\mathbf{L}$ ,  $K$  respectively.  $\|\tilde{\mathbf{A}}^H \tilde{\mathbf{U}}_n\|_F^2$  can be calculated by Fast Fourier method given in Christensen [9]. The sample covariance matrix of  $\mathbf{y}(t)$  is estimated using the following formula

$$\hat{\mathbf{R}} = \frac{1}{N - M} \sum_{t=M}^N \mathbf{y}(t) \mathbf{y}^H(t) \quad (22)$$

Furthermore, when the observed signal  $y(t)$  contains only one harmonic signal, i.e.  $K=1$ , now the objective function (20) of the "spatial spectrum" is converted to

$$J(L, \omega_0) = \frac{ML(M - L)}{\|\tilde{\mathbf{A}}^H \tilde{\mathbf{U}}_n\|_F^2} \quad (23)$$

That is the objective function derived in [6] by Christensen et al. Therefore, the objective function shown in (23) can be seen as a special case of formula (20) deduced in this paper.

### 3. Simulation Analysis

In this section, multi-harmonic signal characteristic parameter extraction method given in the previous Section will be analyzed by simulation. First, the estimation accuracy of RMSE (Root Mean Squared Estimation Error) for the fundamental frequency, the harmonic order and the harmonic number estimation in the different SNR conditions were conducted. Then, the estimation accuracy trends of the fundamental frequency and harmonic order under observation vector dimension  $M$  in different values were analyzed. RMSE is calculated as follows

$$\text{RMSE} = \sqrt{\frac{1}{S} \sum_{s=1}^S (\hat{\omega}_0^s - \omega_0)^2} \quad (24)$$

where  $S$  is the Monte Carlo experiment number, which is 500;  $\hat{\omega}_0^s$  is the estimate of the fundamental frequency for the  $s$  th experiment;  $\omega_0$  is the true value of the fundamental frequency. Let the true signals contain two harmonics, the fundamental frequencies are  $\omega_0^1=0.15\text{rad/s}$  and  $\omega_0^2=0.12\text{rad/s}$  respectively, and harmonic orders are  $L_1 = 2$  and  $L_2=4$  respectively. They contaminated by white Gaussian noise of different intensities. 500 times Monte Carlo experiments under different SNR conditions were conducted. Fig. 1 shows the observed signal, whose SNR is 8 dB and length of time is 100 s.

Under different SNR conditions, results of the RMSE index are shown in Fig. 2 (a) and (b) respectively, whose corresponding fundamental frequency were 0.15 rad/s and 0.12 rad/s. As can be seen from Fig. 2(a), with the improvement of SNR, the RMSE estimation result of the fundamental frequency of 0.15 rad/s show in oscillation downward trend, which means that the estimation accuracy is improved with the improvement of the SNR, while

estimation error of the fundamental frequency 0.12 rad/s SNR is oscillation decrease before SNR arrival 9 dB, and RMSE tends to flatten after 13 dB. This gradual trend also occurred in the RMSE curve of the fundamental frequency 0.15 rad/s, all of which indicate that it has little effect in the aspect of increasing the SNR to enhance the role of the estimation accuracy, under the conditions of this experiment.

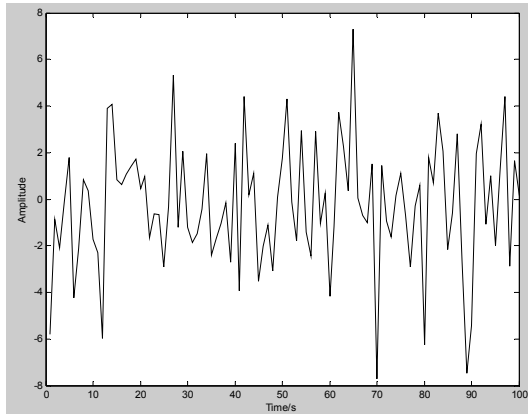
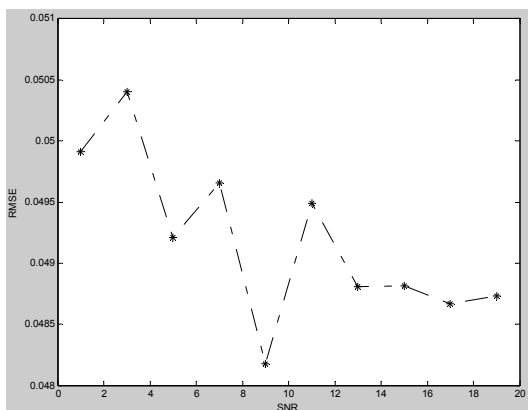
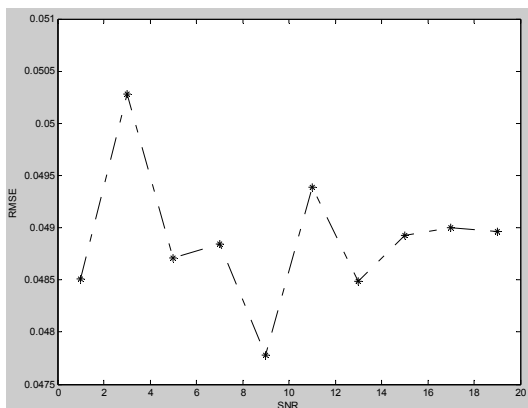


Fig. 1 . Simulated multiple harmonic signals.



(a) RMSE corresponding to  $\omega_0^1 = 0.15$  rad/s.



(b) RMSE corresponding to  $\omega_0^2 = 0.12$  rad/s.

Fig. 2. RMSE of Fundamental frequencies estimation under different SNRs.

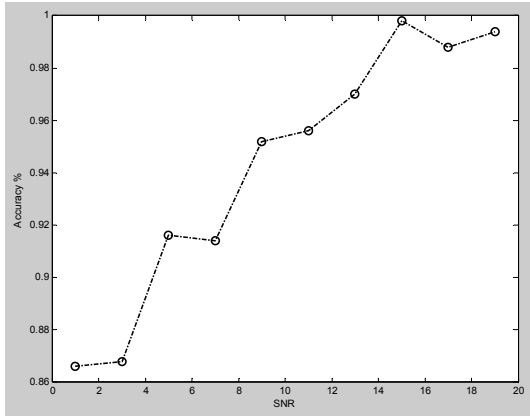
The reason is that the characteristic of the algorithm determines a search space and a search step of the fundamental frequency must be set in advance. Since the existence of step length, estimated fundamental frequency generally only can be closer to the real value of the fundamental frequency, but cannot be exactly equal with them. Therefore, when the SNR is gradually increased so that the fundamental frequency estimation value reaches the vicinity of the true value, and then increasing the SNR cannot obtain that the estimated fundamental frequency is equal to the value of the true fundamental frequency, but only in the vicinity of the oscillation, so that both RMSE indicators of the fundamental frequency does not change much, when SNR is as much high as a certain value. Overall, estimation errors of two fundamental frequencies are below 5% through the proposed algorithm, the deviation of estimation results may be due to the fundamental frequency of the search step too wide, making the obtained fundamental frequency not completely fit the real value of the fundamental frequency. The solution is to narrow the search for the fundamental frequency step size, or use gradient descent method so that the estimated value of the fundamental frequency gradually become the real value of the fundamental frequency. But this time the amount of computation will be increased, the reality is that requires precise estimates of the fundamental frequency, or just need an approximate value, which can be determined according to the demand.

Fig. 3(a) and (b) are the harmonic order estimation accuracy under different SNR conditions, which means that the estimated accuracy equals correct estimate of the number / total number of experiments. In general, as higher as the SNR, the estimation accuracy of the harmonic order is also higher. At SNR = 17 dB, the estimated accuracy is 1, which indicates that estimation results of each experiment are correct under the SNR. Fig. 4 (a) and (b) are the harmonic order estimation result of 500 experiments, corresponding to the harmonic order of 2 and 4, respectively, under the conditions SNR=15 dB. From Fig. 4 (a), it can be seen that the estimated results=2 of the experiments make majority (correct estimate of the number is 476, and estimated accuracy is 95.2 %), Followed by the estimation results=3 of the experiments. In Fig 4 (b), a correct estimate of the number is 468, and the estimation accuracy is 93.6 %. In addition, the estimation accuracy of the harmonic number reaches 100 % under every SNR conditions, which is shown in Fig. 5.

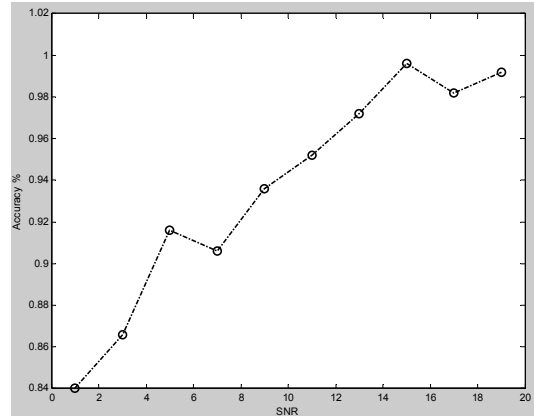
In the simulation, the dimension  $M$  of the observation has a relatively large impact on the estimation accuracy. In the experiment, the observed length of the signal is  $N=100$ , we take eight different values, the range is  $\lfloor (1/10)N \rfloor \sim \lfloor (8/10)N \rfloor$ , and the step is  $\lfloor (1/10)N \rfloor$ . Fig. 6 (a) and (b) show the RMSE respectively corresponding to  $\omega_0^1=0.15$  and  $\omega_0^2=0.12$

under the conditions of different values. In general, as  $M$  is much larger, the estimation accuracy of the fundamental frequency is much higher. Because, when the value of  $M$  is larger, the vector Observation signal  $y(t)$  contains more information that make covariance matrix  $\mathbf{R}$  contain more

abundant information, which is good for the estimate of the fundamental frequency. Therefore, the value  $M$  can be taken larger in practical application, and be ensured that  $M \gg L$ . Reference [9] generally recommended  $M = \lfloor (4/5)N \rfloor$ .

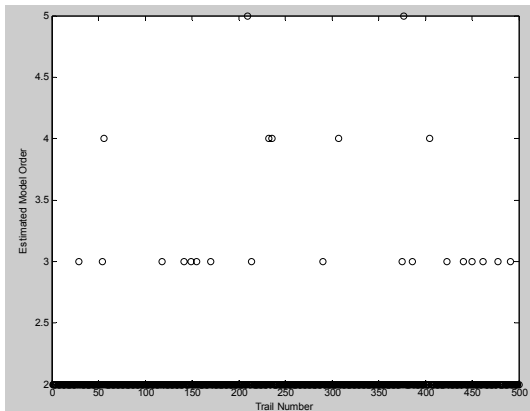


(a) Estimation accuracy corresponding to  $L_1 = 2$ .

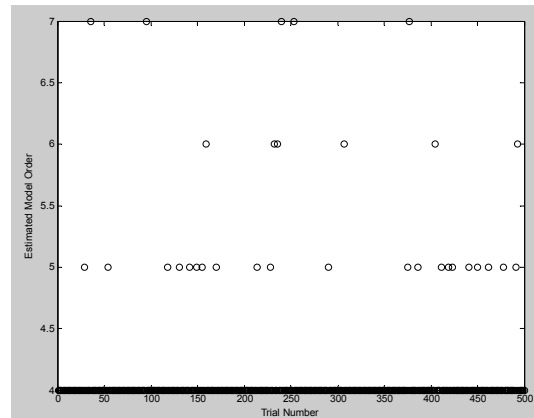


(b) Estimation accuracy corresponding to  $L_2 = 4$ .

Fig. 3. Estimation accuracy of harmonic orders under different SNRs.



(a) Estimation results corresponding to  $L_1 = 2$ .



(b) Estimation results corresponding to  $L_2 = 4$ .

Fig. 4. Estimation results of harmonic orders in each experiment with SNR=15 dB.

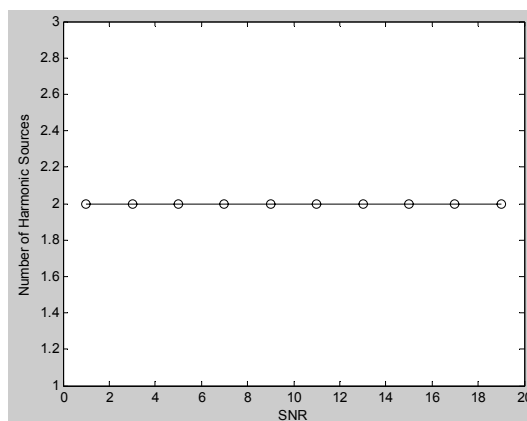
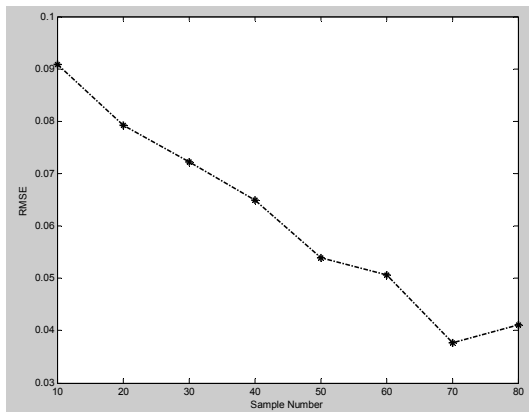
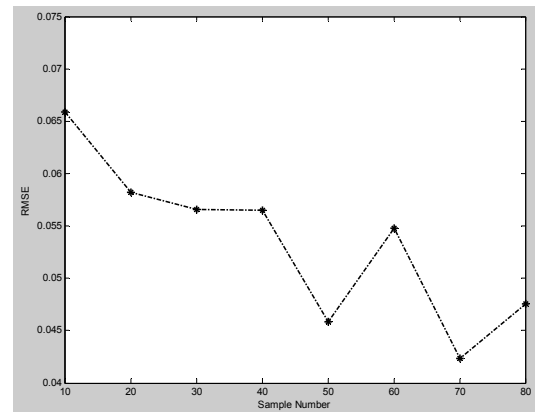


Fig. 5. Estimation results of number of harmonic sources under different SNRs.

(a) RMSE corresponding to  $\omega_0^1 = 0.15$  rad/s.(b) RMSE corresponding to  $\omega_0^2 = 0.12$  rad/s.Fig. 6. RMSE of fundamental frequencies estimation under different  $M$ .

## 4. Conclusions

Based on MUSIC algorithm, this paper presented a joint estimation method to number of harmonics, harmonic order, and harmonic fundamental frequency. Compared to some other frequency estimation methods, such as the correlation coefficient method and nonlinear least-squares fitting etc, which need to know the harmonic order of the fundamental in advance, the advantage of this algorithm can estimate the above characteristic parameters, without the need for any prior information and assumptions of the above parameters. However, the algorithm also faces a computational problem, since it needs to search max value in three dimensions. As can be seen from the simulation results and the estimation results of the Fundamental frequency are satisfactory on the whole, the estimation accuracy of the harmonic order and harmonics number is relatively high. It shows that the proposed algorithm is effective. On the other hand, considering that it cannot be quantitative validation, this paper did not use the measured data for analysis in the experiments, which is our next step work.

## References

- [1]. LUO Xiao-dong, REN Wei-Jun. A New Fundamental Frequency Estimation Algorithm in the Frequency Domain, *Computer Engineering & Science*, 25, 5, 2003, pp. 93-96.
- [2]. Ma Xiao-Min, Zheng Wen-Si, Chen Qi, Autocorrelation fundamental frequency extraction algorithm in MATLAB, *Journal of Northwest University for Nationalities (Natural Science)*, 31, 80, 2010, pp. 54-59.
- [3]. Yang Bai-Ding, Ta DE-An, Wang Wei-Qi. The Research of mean trabecular spacing by Fundamental frequency estimation algorithm, *Journal of Acoustics*, 36, 2, 2011, pp. 172-178.
- [4]. Zhang Jie, Long Zi-Ye, Zhang Bo, etc., The Summary of fundamental frequency extraction Algorithms on Speech Signal Processing, *Journal of Electronic Technology University*, 39, 04, 2010, pp. 99-102.
- [5]. B. Yegnanarayana, K. S. R. Murty, Event-based instantaneous fundamental frequency estimation from speech signals, *IEEE Trans. Audio, Speech Lang. Process.*, 17, 4, 2009, pp. 614-624.
- [6]. A. Nehorai, B. Porat. Adaptive comb filtering for harmonic signal enhancement, *IEEE Trans. Acoust., Speech, Signal Process.*, 34, 5, 1986, pp. 1124-1138.
- [7]. M. G. Christensen, A. Jakobsson, S. H. Jensen, Joint high-resolution fundamental frequency and order estimation, *IEEE Trans. Audio, Speech Lang. Process.*, 15, 5, 2007, pp. 1635-1644.
- [8]. J. X. Zhang, M. G. Christensen, S. H. Jensen, et al., A robust and computationally efficient subspace-based fundamental frequency estimator, *IEEE Trans. Audio, Speech Lang. Process.*, 18, 3, 2010, pp. 487-497.
- [9]. M. G. Christensen, J. H. Jensen, A. Jakobsson, et al., On optimal filter designs for fundamental frequency estimation, *IEEE Signal Proces. Letters*, 15, 2008, pp. 745-748.
- [10]. Zhu Li-min, He En-Hua, Ding Han, The Map Array of the Comb filter and its Application, *Chinese Journal of Mechanical Engineering*, 42, 5, 2006, pp. 1-5.
- [11]. Zhong Jin-Hong, Yang Shan-Lin, Lin Yi-Rong, Lu Kui, A New Algorithm for Voice Fundamental Frequency Extract Based on Wavelet Transform, *Journal of Electronics and Information Technology*, 24, 9, 2002, pp. 1218-1224.
- [12]. Dong Lu, Xin Wei-Ran, Ye Dun-Fan, Audio Signal Pitch Extraction Based on Wavelet Transform, *Journal of Electro-acoustic Technology*, 6, 2005, pp. 50-55.
- [13]. Wu Jin, Liu Zheng-Xin, Audio Fundamental Frequency Detection Based on Wavelet Transform, *J. of Wuhan Univ. of Sci & Tech. (Natural Science Edition)*, 23, 2, 2000, pp. 190-192.
- [14]. D.-J. Liu, C. T. Lin, Fundamental frequency estimation based on the joint time-frequency analysis of harmonic spectral structure, *IEEE Trans. Speech and Audio Proces.*, 9, 6, 2001, pp. 609-621.
- [15]. C. Yeh, A. Roebel, X. Rodet, Multiple fundamental frequency estimation and polyphony inference of polyphonic music signals, *IEEE Trans. Audio, Speech Lang. Process.*, 18, 6, 2010, pp. 1116-1126.
- [16]. A. P. Klapuri, Multiple fundamental frequency estimation based on harmonicity and spectral

- smoothness, *IEEE Trans. Speech and Audio Proces.*, 11, 6, 2003, pp. 804-816.
- [17]. Z. Y. Duan, B. Pardo, C. S. Zhang, Multiple fundamental frequency estimation by modeling spectral peaks and non-peak regions, *IEEE Trans. Audio, Speech Lang. Process.*, 18, 8, 2010, pp. 2121-2133.
- [18]. H. Krim, M. Viberg, Two decades of array signal processing research: the parametric approach, *IEEE Signal Proces. Magazine*, 13, 4, 1996, p. 67-94.
- [19]. Luo Jia-Hong, Matrix Analysis Introduction, *Press of South China Technology University*, Gong Zhou, 2005.

---

2013 Copyright ©, International Frequency Sensor Association (IFSA). All rights reserved.  
(<http://www.sensorsportal.com>)



**Universal Sensors and Transducers Interface (USTI-EXT) for extended temperature range**

**-55 °C ... +150 °C**

26 measuring modes for all frequency-time parameters, rotational speed, capacitance Cx, resistance Rx, resistive bridges  
Frequency range, 0.05 Hz ... 7.5 MHz (120 MHz);  
Programmable relative error, % 1 .... 0.0005 %  
Conversion speeds 6.25 us ... 12.5 ms  
SPI, I2C, RS232 (master and slave, up to 76 800 baud rate)  
Packages: 32-lead, 7x7 mm TQFP and 32-pad, 5x5 mm (QFN/MLF)

**Applications: automotive industry, avionics, military, etc.**

<http://www.techassist2010.com/>    [info@techassist2010.com](mailto:info@techassist2010.com)