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Hardware Optimization of Compressed Sensing Based on FPGA

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Abstract: Realized the compressed sensing based on FPGA. The system sparseed the image date by using the 3-level lifting wavelet transform, measured the sparse data by using the hadamard matrix, and reconfigured the data by using the orthogonal matching pursuit. The system realized the compressed sensing based on NIOSII, optimized the hardware structure of stochastic measure matrix. It reduced the usage of resource and kept the speed. The system has the better reconfiguration effect, less distortion and uses less hardware resource.

Keywords: Compressed sensing, Sparse representation, Matching pursuit, Hardware realization, Hadamard matrix.

1. Foreword

Donoho D. L. points out the theory of compressed sensing (CS) [1~6] in 2004, the signal can be compressed by far lower than the Nyquist sampling theorem standard way to collect data, and it can still be accurately reconstructed back to the original signal. The theory is based on that signal is compressible characteristics, and it can complete the signal acquisition and compression simultaneously; the sampling rate depends entirely on the information structure and content in the signal, and is determined by the bandwidth of the signal, it makes a prominent advantages and broad application prospect in the field of signal processing.

CS has been extensively concerned, it is not only for many applied sciences such as statistics, information theory, coding theory, computer science and bring new inspiration, and has the corresponding application in many fields such as wireless communication [7], medical imaging [8], pattern recognition [9], image processing [10], optical imaging [11]. However, these studies are mainly based on the simulation and theoretical research, few studies are based on hardware implementation. With the continuous development of the VLSI (Very Large Scale Integrated Circuit) technology, FPGA (Field Programmable Gate Array) is the first choice of the hardware platform. FPGA is a hardware structure and style, has its irreplaceable advantages in the aspects of parallel computing and multi - channel data processing, so in many application fields of array signal processing, image processing and communication signal processing, FPGA is being used more and more. FPGA uses Harvard architecture; with a 32 instruction set based on the second generation programmable Nios embedded soft-core processor to provide more flexibility for complex algorithm on the FPGA platform by the hardware structure of the modular. This paper realizes the compressed sensing algorithm based on NiosII processor on the platform of the FPGA.

2. The Compressed Sensing Theory of Foundations

For a finite discrete time signal X, it can be seen as a column vector of a RN space N × 1 dimension, the elements are X[n], n =1, 2,... N. Any signal in RN space can be expressed by linear combination of the N × 1 dimension with the basis vectors {Ψ i}Ni=1. Assuming these basis vectors are orthonormal, the vector {Ψ i}Ni=1 as a column vector formed N × N matrix, Ψ=[Ψ1, Ψ2, ……, ΨN], so any signal X can be expressed as:

$$X=\Psi*\Theta \quad (1)$$

Among them, Θ is N × 1 column vector with the projection coefficient Θ=[Θi] =[<X, Ψi >]. If the non-zero number of is smaller than N, then the signal is compressible;

CS theory points out the setting the length of N signal in a group of orthogonal matrix or the transform coefficients in the tight frame Ψ is sparse, If we use a observation matrix Φ =M × N (M<<N), which is not related to transform matrix Ψ, to do the linear transformation for the coefficient vector, and get the observed set Y =M *1. Then you can use the optimal solution method to reconstruct the original signal collection from the observation precision.

Compressed sensing is a new theoretical framework which can do the sample and compression at the same time. The compression and sampling process is shown as Fig. 1. The three main parts in the process of compressed sensing are signal sparse representation, design of measurement matrix and reconstruction of signals.

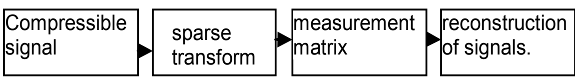


Fig. 1. Theoretical framework of CS.

2.1. Signal Sparse Representation

Assuming the signal F with a length of N, it can be expressed as a linear combination of the standard orthogonal matrix:

$$f=\sum_{i=1}^{N}\alpha_i\psi_i \text{或} f=\psi\alpha \quad (2)$$

A priori condition of CS algorithm is the sparse representation. If α in (2) is only non-zero number K (N>>K); or α approaches to zero after sorting according to the exponential decay, it can be considered that the signal is sparse. The common sparse methods are discrete cosine transform (DCT), fast Fourier transform (FFT), discrete wavelet transform (DWT). In recent years, the sparse signal decomposition in a redundant dictionary is another research focus. Substituted function with redundant functions over complete, called redundant dictionary. The dictionary should be selected as far as possible consistent with the approximated signal structure, which can be no restrictions. It is known as the redundant dictionary which is substituted base function with redundant functions completely; the dictionary should be selected as far as possible consistent with the approximated signal structure, which can be no restrictions.

2.2. The Measurement Matrix Construction

Using a measurement matrix M * N (M<<N) which is not related to transformation matrix to do the linear projection of the signal, then it can get the linear measurement matrix y:

$$y=\phi f \quad (3)$$

Because the measurement value of Y is M × 1 matrix, the measurement object is reduced from the N dimension to M dimension. The observation process is non-adaptive, it means that the selection for the measurement matrix Φ does not depend on the signal F. In the process of measurement, the measurement of K should not destroy the original signal information, ensures that the signal can be reconstructed exactly. Because the signal is sparse representation, formula 3 can be expressed as the following type:

$$y=\phi f=\phi\varphi\alpha=\theta\alpha \quad (4)$$

Among them, Θ is an M × N matrix, which is called the sensing matrix. The conversion process is shown in Fig. 2.

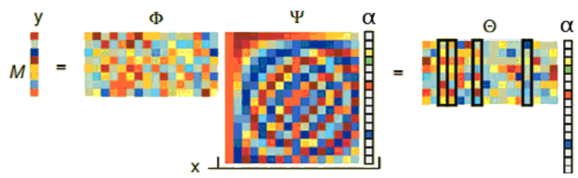


Fig. 2. Measurement of CS.

The random matrix is widely used in the measurement matrix, such as the Gauss random matrices, random Bernoulli matrix and so on; Gauss matrix is almost no related with any sparse signals, the required number of observations is minimum.

Many scholars have proposed a variety of deterministic measurement matrix, such as Chirp measurement matrix, the measurement matrix with Alltop sequence, half Fourier matrix and so on.

2.3. Signal Reconstruction

In the theory of compressed sensing, the number of M is much smaller than the observed signal length N, so the equation has no definite solution and cannot be reconstructed signal. However, because the signal is sparse K (K<<M), if matrix Θ in formula 4 satisfies the RIP (Restricted Isometry Property), for arbitrary K sparse signal f and constant $\delta_k \in (0,1)$, matrix Θ satisfies:

$$1-\delta_k \le \frac{\|\Theta f\|_2^2}{\|f\|_2^2} \le 1+\delta_k \tag{5}$$

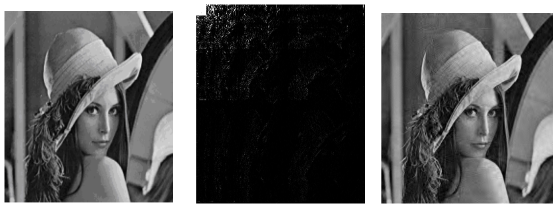
The K coefficient can be accurately reconstructed from M measurements. The equivalent condition of RIP is that the measurement matrix Θ and sparse matrix ψ are not related.

Now Compressed sensing reconstruction algorithms are mainly divided into two categories, one is the greedy algorithm, It is achieved signal vector approximated by selecting the appropriate atomic and using a series of progressive method, this algorithm consists of matching pursuit algorithm, orthogonal matching pursuit algorithm, regularized orthogonal matching pursuit algorithm. The other is a convex optimization algorithm; it is the 0 norm relaxed to 1 norm by linear programming, this algorithm mainly includes the gradient projection method, basis pursuit method, least angle regression method. The convex optimization algorithm is more accurate than the greedy algorithm for the solution, but it requires higher computational complexity.

3. Simulation of Compressed Sensing System

According to the above theory, construction of compressed sensing system needs picking the hardware implementation and better performance algorithm easily from many algorithms. The design is based on Matlab, sparse transform of the signal is by using DCT transform, FFT transform and wavelet transform, measure the sparse image is by using random matrix and Hadamard matrix.

The reconstruction algorithms which are commonly used are based pursuit algorithm (BP), matching pursuit algorithm (MP) and orthogonal matching pursuit algorithm (OMP). BP uses a linear optimization model to obtain the accurate result, but it is to seek the optimization solution in different combinations of the entire measurement vector, so the computational complexity is very high. MP is low complexity and the asymptotic convergence, but it makes the projection which is selected on the set of the courtyard non orthogonal and the iteration results are suboptimal; so as to obtain convergence needs many times of iteration. In the condition of a sufficient linear measurement, OMP ensures the optimality of the iteration set by recursively on selected atoms orthogonal, thereby reducing the number of iterations to reach convergence. The accuracy and the speed of reconstruction of OMP algorithm are better than MP algorithm and BP algorithm [12]. This system reconstructs the image by using OMP algorithm. Using diagram A in the Fig. 3 as the original image to do the simulation, the images are shown as B and C diagram in Fig. 3 after wavelet transform and system reconstruction. In order to evaluate the effect of transformation, using PSNR (peak signal to noise) to measure the reconstruction effect, the value of PSNR is greater, the distortion is less, and the processed image quality is higher.



(a) (b) (c)

Fig. 3. Simulation of Compressed sensing.

The simulation results are shown in Table 1. In this image, wavelet transform of signal sparse effect is the best among the DCT transform, FFT transform and wavelet transform, and at the same time, the effect of using Hadamar matrix measurement is better than random matrix. Therefore, this system chooses the wavelet transform and the Hadamar matrix.

Table 1. Value of PSNR.

Sparse algorithm	DCT	FFT	WAVELET	DCT	FFT	WAVELET
Stochastic matrix	Random	Random	Random	Hadamar	Hadamar	Hadamar
Value of PSNR	26.5158	26.3285	30.7734	26.6121	26.4498	30.9298

4. Compressed Sensing System Hardware Implementation

Based on the system simulation results, this system sparse the signal by using the discrete wavelet transform, measures matrix by using the Hadamar matrix, and reconstructs the signal by using OMP algorithm.

4.1. The Lifting Wavelet Transforms

The sparse representation of signals uses discrete wavelet transform. Because the lifting wavelet transform only uses the simple addition operation, it can effectively improve the traditional wavelet high computation faults, save a large space for the processor, and is good for the FPGA platform realization.

Positive (negative) lifting wavelet transform structure generally includes three steps: Split, Predict, and update. According to the literature [13, 14], this system uses the three level lifting wavelet transform structure which has been optimized and can be hardware implemented; the system occupies the minimum resources, the optimizing three level lifting wavelet structure is shown in Fig. 4.

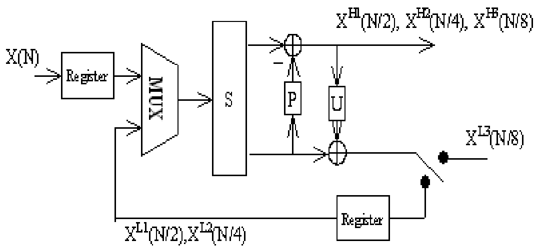


Fig. 4. Optimizing structure of three levels lifting wavelet.

4.2. Random Matrix Measurement

Random matrix measurement is the important step of compression perception, mainly is the matrix multiplication. For the matrix multiplication of C=A × B, which A, B and C are M × L, L × N and M × N matrix, It is calculated as

$$C_{i,j} = \sum_{k=1}^{L} A_{i,k} * B_{k,j}; 1 \le i \le M, 1 \le j \le N \tag{6}$$

The computational complexity of the above algorithm is 2 × M × L × N, namely O (N3), it needs the M × L × N multiplications and M × (L-1) × N addition operations. Matrix multiplication hardware implementation architecture is shown in Fig. 5, including A, B matrix storage module, register, buffer, and high performance computing point multiplication and adder calculation unit.

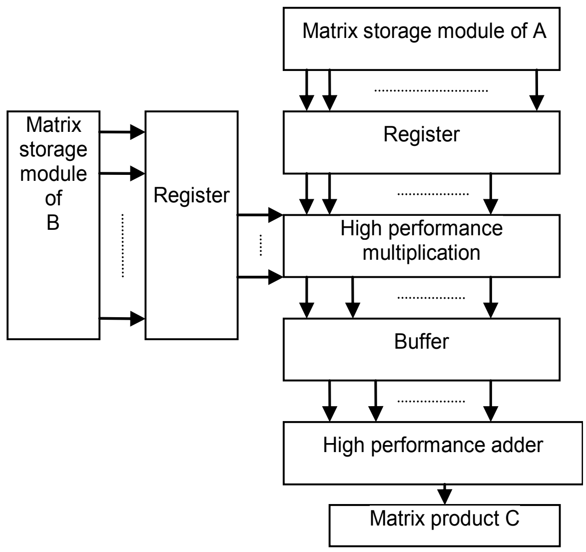


Fig. 5. Hardware implementation architecture of matrix multiplication.

So matrix multiplication hardware implementation requires a large number of multipliers and adders, and occupies a lot of the hardware resources. Therefore it needs to be optimized to reduce resource in the hardware structure. In order to reduce the number of the use of multipliers and adders, multipliers and adders can be shared. The multipliers and adders sharing will cause the speed slowly in the same clock; PLL can be used for frequency doubling. [15, 16] Reducing the use of resources does not make the speed slow down. The optimized structure is shown as Fig. 6, it use PLL to generate eight times the frequency of the clock signal, matrix B can be divided into eight parts sharing multiplier. This structure can maintain the same speed multiplier based on the number of the original use for number 1/8, greatly reduces the use of resources.

The random matrix is widely used in the measurement matrix, although the observed number of Gauss matrix is minimum, it belongs to the non adaptability measurement, it is high complexity in the actual implementation and difficult to apply in large scale problems; And from the above simulation result, using Hadamar matrix measurement result is more effective than normal random matrix. At the same time, matrix multiplication is equivalent to adding or subtracting the input vector of each element when it does the product of elements by using the Hadamar matrix measurement, and the complex multiplication unifies for calculation of addition and subtraction. Therefore, it can be further simplified hardware structure. The specific implementation framework is shown in Fig. 7. It stores image data sparse in ROM, and then produces the Hadamar matrix by a 32 bit parallel linear feedback shift register (LFSR). Using this structure greatly reduces the difficulty of hardware realization, and it saves the hardware resources.

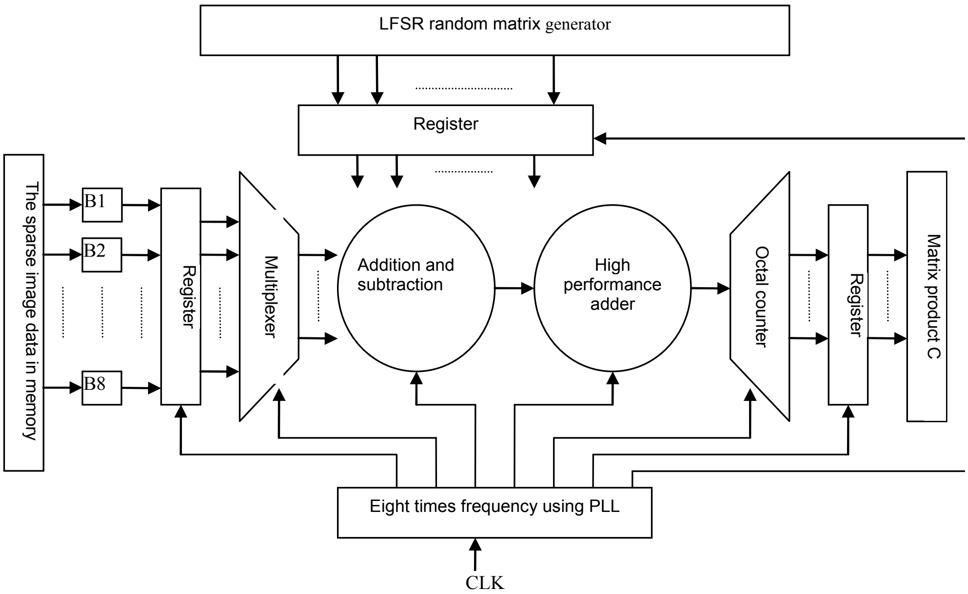


Fig. 6. Optimized structure of matrix multiplication.

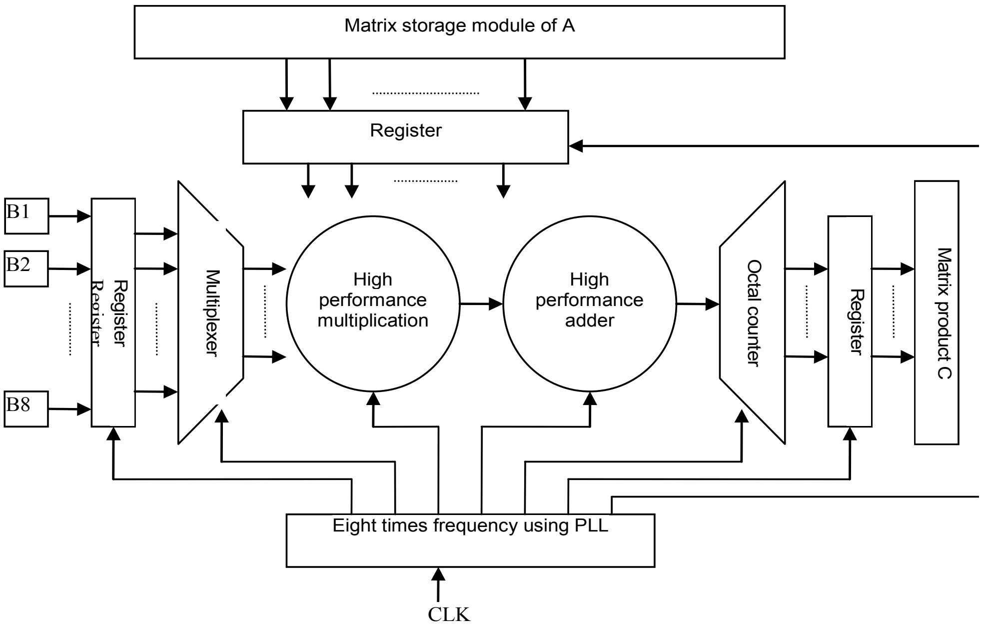


Fig. 7. Optimized hardware implementation using Hadamar matrix.

4.3. The Hardware Construction of CS System

This system used OMP algorithm as the signal reconstruction algorithm, because the reconstruction accuracy and speed of OMP is better than MP and BP algorithm. This system is based on the DE2 platform of Altera, it uses NIOSII processor to realize the hardware construction. It uses VHDL language to construct wavelet transform process and Hadamar matrix and matrix multiplication, uses C language in NIOSII to complete OMP algorithm, and finally completes the system hardware implementation; System resources' information is shown in Table 2.

Table 2. System resources' information.

Name of resources	Amount
Total Logic Elements	20947
Total combinational functions	16,550
Dedicated logic registers	14,358
Total registers	14410
Total memory bits	308,864
Added Multiplier 9-bit elements	44

5. Conclusions

The design is based on FPGA platform to realize the compressed sensing system. Three level lifting wavelet transform is used to optimize the sparse signal, it uses Hadamar matrix measurement, and it is easy for the hardware implementation and matrix multiplication into addition and subtraction, reduces the hardware resource utilization greatly, finally uses the OMP algorithm for image reconstruction. The system has better reconstruction effect, the distortion is smaller, and uses less hardware resources.

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