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Hypernormal Form at Quintic of a Class of Four-dimensional Vector Fields with Linear Part has Two Pairs of Pure Imaginary Eigenvalues

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Abstract: Investigate of hypernormal form (the simplest normal form and the unique normal form) for high-dimensional vector fields which have applied in practical engineering can provide guidance for the principle of industry design and the theory of standardization in reality. In this paper, based on the method combined new grading function with multiple Lie brackets, we investigated the reduction hypernormal form problem for a class of four-dimensional vector fields with linear part has two pairs of pure imaginary eigenvalues. With the aid of Maple and new expressions of block matrices, we obtained the hypernormal form at quintic truncated and proved that was a unique one under certain condition. As an application, we studied the simply supported honeycomb sandwich plate dynamics model and obtained the hypernormal form at quintic truncated. *Copyright © 2013 IFSA.*

Keywords: High dimensional vector field, Honeycomb sandwich plate, Hypernormal form, New grading function, Multiple Lie brackets.

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1. Introduction

Research on complex dynamics behavior can not without further reduction of the dynamic system and normal forms are basis and powerful tool in bifurcation theory of vector fields for reduction. Through the development of the normal form theory, the international renowned scholars as Arnold [1], Baider, Sanders [2-3] and so on had made important contributions in the aspect of theory and calculation of normal form.

In recently years, people had done a lot of research on the normal forms problem for nonlinear dynamic system. Jana et al [4] used the normal form method and center manifold theorem to investigate the direction of the Hopf bifurcation and stability of the bifurcating limit cycle for a three dimensional parameter delay model. Guo et al [5] considered a hierarchically organized network and used normal form theory in order to investigate the codimension two bifurcations. After this reduction they found a great variety of equilibria, periodic and quasi-periodic oscillation patterns of maximal and submaximal symmetry which can be classified in a two-level pattern hierarchy. Yu et al [6] investigated the multi-pulse homoclinic orbits and chaotic dynamics for an axially moving viscoelastic beam are investigated in the case of 1:2 internal resonance. On the basis of the modulation equations derived by the method of multiple scales, the theory of normal form is utilized to find the explicit formulas of normal form associated with a double zero and a pair of pure imaginary eigenvalues. Kundu et al [7] dialed with dynamic behaviors on Hopfield type of ring neural network of four neurons having a pair of short-cut connections with multiple time delays. They used the normal form method and center manifold theory had determined the direction of the Hopf bifurcation, stability and the properties of Hopf-bifurcating periodic solutions. Ding et al [8] studied dynamics in a container crane model with delayed position feedback, with particular attention focused on non-resonant double Hopf bifurcation. By using multiple time scales and center manifold reduction methods, they obtained the equivalent normal forms near a double Hopf critical point in this neutral delayed differential system. Moreover, bifurcations are classified in a two-dimensional parameter space near the critical point. Zhang et al [9] investigated stability and local bifurcation behaviors for a simply supported functionally graded material (FGM) rectangular plate subjected to the transversal and in-plane excitations in the uniform thermal environment. With the aid of Maple and normal form theory, the explicit expressions of transition curves are obtained, which may lead to static bifurcation, Hopf bifurcation and 2-D torus bifurcation.

The classical normal form theory may not give the simplest form since only linear parts are used for simplifying the nonlinear terms, and hence one can not apply Poincare normal form theory to vector fields whose linear parts are identically zero. On the other hand, classical normal form is not unique in general. In order to get unique normal forms, further reduction of the classical normal form is necessary. Therefore, the hypernormal form (the simplest normal form and the unique normal form) is proposed to enrich and develop of the normal form theory. Study on hypernormal form problem is forward subject in high dimensional vector field of nonlinear dynamics.

In this paper, we applied the method combined new grading function with multiple Lie brackets to investigated the hypernormal form problem for a class of four-dimensional vector fields with linear part has two pairs of pure imaginary eigenvalues. With the aid of Maple and new expressions of block matrices, we obtained the hypernormal form at quintic truncated and proved that was a unique one under certain condition. As an application, we studied the simply supported honeycomb sandwich plate dynamics model and obtained the hypernormal form at quintic truncated.

2. Fundamental Theory

According to [10-13], this section gives the fundamental theory of new grading function and N^{th} order normal form, as well as the sufficient condition that N th order normal form is unique.

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2.1. New Grading Function

Let

$$D_n = \left\{ \prod_{i=1}^{n} x_i^{l_i} e_j \mid l_i \in Z^+,\, x_i \in R(orC),\, i,\, j = 1, \cdots, n \right\},$$

where e_j is the j th standard unit vector in R^n (or C^n). Consider the function $\delta : D_n \to Z$ defined by

$$\delta\left(\prod_{i=1}^{n} x_i^{l_i} \; e_j \right) = \sum_{i=1}^{n} a_i l_i - a_j ,$$

is a linear grading function, where $a_i, a_j \in N^+$, $i, j = 1, \cdots, n$.

2.2. N th Order Normal Form and Unique Normal Form

Let H_k be the linear space spanned by all monomials of degree k . Consider a formal vector field V defined by the following formal series

$$X = \sum_{k \in N} X_{\mu+k} \text{ , (2-1)}$$

where $X_k \in H_k$, $k \ge \mu$ and $X_\mu \ne 0$, we call (2-1) a zero order normal form and denote it as

$$V^{(0)} = \sum_{k \in N} V^{(0)}_{\mu+k} .$$

We may assume that X_μ is already in some simple form. Let $Y_k \in H_k$, and define an operator for any $k \in N$

$$L_k^{(1)} : H_k \to H_{\mu+k} ,\; Y_k \mapsto [Y_k , V_\mu^{(0)}] \text{ . (2-2)}$$

where $[u, v] = (Du)v - (Dv)u$, for any $u, v \in H_k$. It is obviously that $L_k^{(1)}$ is linear. Note that $L_k^{(1)}$ depends on $V_\mu^{(0)}$ and can be denoted by $L_k^{(1)} = L_k^{(1)}[V_\mu^{(0)}]$ .We count the kernel space and the range space of linear operator $L_m^{(1)}$ as $Ker\, L_m^{(1)}$ and $\operatorname{Im} L_m^{(1)}$ respectively.

Definition 2.1 $V = \sum_{k \in N} V_{\mu+k}$ is called a first order normal form, if

$$V_{\mu+k} \in N^{(1)}_{\mu+k} ,\; k = 1,2,\ldots ,$$

where $N^{(1)}_{\mu+k}$ is a complement subspace to $\mathrm{Im}\,L^{(1)}_k$ in $H_{\mu+k}$ and $L^{(1)}_k = L^{(1)}_k[V_\mu]$.

There is a sequence of formal transformations such that (2-1) is transformed into a first order normal form and can be denoted by

$$V^{(1)} = \sum_{k\in N} V^{(1)}_{\mu+k},$$

where $V^{(1)}_\mu = V^{(0)}_\mu$.

In order to make further reduction of a first order normal form, we define a sequence of linear operators $L^{(m)}_k$, $m, k \in N^+$. Let

$$V = \sum_{k\in N} V_{\mu+k},$$

be a formal series, where $V_m \in H_m$, $m \ge \mu$. Then we define $L^{(1)}_k = L^{(1)}_k[V_\mu]$ by (2-2) for any $k \in N$. If

$$L^{(m)}_k = L^{(m)}_k[V_\mu, V_{\mu+1}, \ldots, V_{\mu+m-1}],$$

is defined already for an $m \ge 1$ and any $k \in N$, then we define

$$L^{(m+1)}_k = L^{(m+1)}_k[V_\mu, V_{\mu+1}, \ldots, V_{\mu+m-1}, V_{\mu+m}],$$

by

$$L^{(m+1)}_k : KerL^{(m)}_k \times H_{m+k} \to H_{\mu+m+k}:$$

$$((Y_k, Y_{k+1}, \cdots, Y_{k+m-1}), Y_{k+m}) \mapsto \sum_{i=0}^{m} [Y_{k+i}, V_{\mu+m-i}].$$

Definition 2.2 $V = \sum_{k=0}^{N} V_{\mu+k} + \cdots$, where $V_m \in H_m$ for each $m \ge \mu$, is called an N th order normal form, if

$$V_{\mu+i} \in N^{(i)}_{\mu+i} \ (1 \le i \le N-1),$$

And

$$V_{\mu+j} \in N^{(N)}_{\mu+j} \ (j \ge N),$$

where $N^{(m)}_{\mu+k}$ is a complement subspace to $\mathrm{Im}\,L^{(m)}_{k-m+1}$ in $H_{\mu+k}$.

Lemma 2.3 For any $N \in \mathrm{N}$, every formal vector field can be changed by a sequence of near identity formal transformations to a N th order normal form.

Lemma 2.4 $V = \sum_{m\in N} V_{\mu+m}$ is an N th order normal form and also the unique normal form, if

(1) $KerL^{(N)}_k = \{0\} \times KerL^{(N-1)}_{k+1}$, $\forall 1 \le k \ne \mu$,

(2) $\mu \ge 1$ and for any $T_\mu \in KerL^{(1)}_\mu$, there exists an $\alpha \in R$, such that $\forall k \ge \mu$, $[T_\mu, V_k] = [\alpha V_\mu, V_k]$.

3. Hypernormal Form at Quintic of a Class of Four-Dimensional Vector Field

Considering the four dimensional vector field

$$\begin{aligned}
\dot{x}_1 &= x_2 + f_1(x_1, x_2, x_3, x_4) \\
\dot{x}_2 &= -x_1 + \alpha_1 x_1^3 + \alpha_2 x_1^2 x_3 + \alpha_3 x_1 x_3^2 + \alpha_4 x_3^3 \\
&\quad + f_2(x_1, x_2, x_3, x_4) \\
\dot{x}_3 &= x_4 + f_3(x_1, x_2, x_3, x_4) \\
\dot{x}_4 &= -x_3 + \beta_1 x_1^3 + \beta_2 x_1^2 x_3 + \beta_3 x_1 x_3^2 + \beta_4 x_3^3 \qquad (3\text{-}1) \\
&\quad + f_4(x_1, x_2, x_3, x_4).
\end{aligned}$$

Where $f_i(x_1, x_2, x_3, x_4)$, $i = 1,2,3,4$ are higher order terms defined by function δ as follows.

We define $\delta: D_4 \to Z$

$$\delta(x_1^m x_2^n x_3^p x_4^q \partial_{x_i}) = (m+p) + 2(n+q) - 1, \ i = 1,3,$$

$$\delta(x_1^m x_2^n x_3^p x_4^q \partial_{x_j}) = (m+p) + 2(n+q) - 2, \ j = 2,4.$$

The system (3-1) recorded under δ as

$$V^{(0)} = V^{(0)}_1 + V^{(0)}_2 + \cdots + V^{(0)}_m + \cdots,$$

where

$$\begin{aligned}
V^{(0)}_1 &= x_2 \partial_{x_1} \\
&\quad + (\alpha_1 x_1^3 + \alpha_2 x_1^2 x_3 + \alpha_3 x_1 x_3^2 + \alpha_4 x_3^3)\partial_{x_2} \\
&\quad + x_4 \partial_{x_3} \\
&\quad + (\beta_1 x_1^3 + \beta_2 x_1^2 x_3 + \beta_3 x_1 x_3^2 + \beta_4 x_3^3)\partial_{x_4}, \qquad (3\text{-}2)
\end{aligned}$$

The linear operators are defined as follows

$$L_k : H_k \to H_{k+1},\ Y_k \mapsto [Y_k,\ V^{(0)}_1],\ k = 1, 2, 3, 4,$$

and the basic vectors for spaces H_k, $k = 1, 2, 3, 4$ can be expressed as Table 1 shows.

Let $(L^{(1)})_k$ is the coefficient matrix of transform for linear operator $L_k = L_k[V^{(0)}_1]$, $k = 1,2,3,4$. $\partial^{m,n}_{i,j}(M)$ ($0 < i \le m$, $0 < j \le n$) shows the block matrix of $m \times n$ and all its elements are also block matrix. For that mark, the (i, j) th element is matrix M and others are null matrices. For system (3-1), $(L^{(1)})_k$ can be regarded as $\sum_{i,j=1}^{4} \partial^{4,4}_{i,j}(L^{(1)}_{4(i-1)+j})_k$, $k = 1,2,3,4$. Where $L^{(1)}_5$, $L^{(1)}_{15}$ are negative unit matrices and $L^{(1)}_3$, $L^{(1)}_7$, $L^{(1)}_8$, $L^{(1)}_9$, $L^{(1)}_{13}$, $L^{(1)}_{14}$ are null matrices.

In order to further describe $(L^{(1)})_k$, we introduce the new marks of block matrices:

(1) $\eta_n(p) = p I_n$,

$$\eta^q_n(p) = diag(p, p-q, p-2q, \cdots, p-(n-1)q),$$

(2) ${}^{L}\eta^{q}_{n,l}(p)=\partial^{1,2}_{1,2}(\eta^{q}_{n}(p))$,

${}^{R}\eta^{q}_{n,l}(p)=\partial^{1,2}_{1,1}(\eta^{q}_{n}(p))$,

${}^{D}\eta^{q}_{n,l}(p)=\partial^{2,1}_{1,1}(\eta^{q}_{l}(p))$,

(3) ${}^{D}\eta^{q_1,q_2,\cdots q_j}_{n,l}(p_1,p_2,\cdots,p_j)={}^{D}\eta^{q_1}_{n,l}(p_1)$

$+\sum_{i=2}^{j}\partial^{2,1}_{2,1}({}^{D}\eta^{q_i}_{n-i+1,l}(p_i))$.

Table 1. The basic vectors for spaces H_k , $k=1, 2, 3, 4$.

k	The basic vectors for spaces H_k
1	$Span\{x_1^{l_1-\rho_3}x_2^{l_3-\rho_1}x_3^{\rho_3}x_4^{\rho_1}\partial_{x_4}$, $x_1^{l_2-\rho_2}x_2^{l_4-\rho_1}x_3^{\rho_2}x_4^{\rho_1}\partial_{x_3}$, $x_1^{l_1-\rho_3}x_2^{l_3-\rho_1}x_3^{\rho_3}x_4^{\rho_1}\partial_{x_2}$, $x_1^{l_2-\rho_2}x_2^{l_4-\rho_1}x_3^{\rho_2}x_4^{\rho_1}\partial_{x_1}$; $l_1+2l_3=3, l_2+2l_4=2\}$
2	$Span\{x_1^{l_1-\rho_4}x_2^{l_3-\rho_2}x_3^{\rho_4}x_4^{\rho_2}\partial_{x_4}$, $x_1^{l_2-\rho_3}x_2^{l_4-\rho_1}x_3^{\rho_3}x_4^{\rho_1}\partial_{x_3}$, $x_1^{l_1-\rho_4}x_2^{l_3-\rho_2}x_3^{\rho_4}x_4^{\rho_2}\partial_{x_2}$, $x_1^{l_2-\rho_3}x_2^{l_4-\rho_1}x_3^{\rho_3}x_4^{\rho_1}\partial_{x_1}$; $l_1+2l_3=4, l_2+2l_4=3\}$
3	$Span\{x_1^{l_1-\rho_5}x_2^{l_3-\rho_2}x_3^{\rho_5}x_4^{\rho_2}\partial_{x_4}$, $x_1^{l_2-\rho_4}x_2^{l_4-\rho_2}x_3^{\rho_4}x_4^{\rho_2}\partial_{x_3}$, $x_1^{l_1-\rho_5}x_2^{l_3-\rho_2}x_3^{\rho_5}x_4^{\rho_2}\partial_{x_2}$, $x_1^{l_2-\rho_4}x_2^{l_4-\rho_2}x_3^{\rho_4}x_4^{\rho_2}\partial_{x_1}$; $l_1+2l_3=5, l_2+2l_4=4\}$
4	$Span\{x_1^{l_1-\rho_6}x_2^{l_3-\rho_3}x_3^{\rho_6}x_4^{\rho_3}\partial_{x_4}$, $x_1^{l_2-\rho_5}x_2^{l_4-\rho_2}x_3^{\rho_5}x_4^{\rho_2}\partial_{x_3}$, $x_1^{l_1-\rho_6}x_2^{l_3-\rho_3}x_3^{\rho_6}x_4^{\rho_3}\partial_{x_2}$, $x_1^{l_2-\rho_5}x_2^{l_4-\rho_3}x_3^{\rho_5}x_4^{\rho_3}\partial_{x_1}$; $l_1+2l_3=6, l_2+2l_4=5\}$

where $\rho_i=0,1,\cdots,i$, $i=1,2,\cdots,7$,

$l_j\in N^{+}$, $j=1, 2, 3, 4$.

$(L^{(1)})_k$ can be expressed by these new marks as follows.

The situation for $k=1$.

$$(L^{(1)})_1=\sum_{l=1,3}\partial^{4,4}_{l,l}\{\partial^{3,2}_{1,1}\{[\sum_{i=1}^{2}\partial^{3,2}_{i+1,i}$$

$$({}^{R}\eta^{1}_{2(3-i)-1,2(3-i)}(2(3-i)-1))]$$

$$+\partial^{3,2}_{1,2}({}^{D}\eta^{0,0,0,0}_{5,2}(\alpha_1,\alpha_2,\alpha_3,\alpha_4))\}+\partial^{3,2}_{2,2}[\partial^{2,1}_{2,1}({}^{R}\eta^{1}_{1,2}(1))]$$

$$+\sum_{j=1}^{2}\partial^{3,2}_{j+1,j}[\sum_{i=1}^{3-j}\partial^{3-j,3-j}_{i,i}({}^{L}\eta^{-1}_{2(4-j-i)-1,2(4-j-i)}(1))]$$

$$+\partial^{3,2}_{1,2}[\partial^{3,1}_{1,1}({}^{D}\eta^{0,0,0,0}_{5,2}(\beta_1,\beta_2,\beta_3,\beta_4))]\}$$

$$+\partial^{4,4}_{1,2}\{\sum_{j=1}^{2}\partial^{3,2}_{j,j}[\sum_{i=1}^{3-j}\partial^{4-j,3-j}_{i,i}$$

$$({}^{D}\eta^{0,0,0}_{2(4-j-i)+1,2(4-j-i)-1}(-\beta_2,-2\beta_3,-3\beta_4))]\}$$

$$+\partial^{4,4}_{1,4}\{\sum_{j=1}^{2}\partial^{3,2}_{j,j}[\sum_{i=1}^{3-j}\partial^{4-j,3-j}_{i,i}$$

$$({}^{D}\eta^{0,0,0}_{2(4-j-i)+1,2(4-j-i)-1}(-3\beta_1,-2\beta_2,-\beta_3))]\}$$

$$+\partial^{4,4}_{2,1}(\eta_8(-1))+\sum_{l=2,4}\partial^{4,4}_{l,l}\{\partial^{2,2}_{1,1}\{[\partial^{2,2}_{2,1}({}^{R}\eta^{1}_{2,3}(2))]$$

$$+[\partial^{2,2}_{1,2}({}^{D}\eta^{0,0,0,0}_{4,1}(\alpha_1,\alpha_2,\alpha_3,\alpha_4))]\}$$

$$+\partial^{2,2}_{2,1}[\partial^{1,2}_{1,1}({}^{L}\eta^{-1}_{2,3}(1))]$$

$$+\partial^{2,2}_{1,2}[\partial^{2,1}_{1,1}({}^{D}\eta^{0,0,0,0}_{4,1}(\beta_1,\beta_2,\beta_3,\beta_4))]\}$$

$$+\partial^{4,4}_{3,2}\{\sum_{j=1}^{2}\partial^{3,2}_{j,j}[\sum_{i=1}^{3-j}\partial^{4-j,3-j}_{i,i}$$

$$({}^{D}\eta^{0,0,0}_{2(4-j-i)+1,2(4-j-i)-1}(-\alpha_2,-2\alpha_3,-3\alpha_4))]\}$$

$$+\partial^{4,4}_{3,4}\{\sum_{j=1}^{2}\partial^{3,2}_{j,j}[\sum_{i=1}^{3-j}\partial^{4-j,3-j}_{i,i}$$

$$({}^{D}\eta^{0,0,0}_{2(4-j-i)+1,2(4-j-i)-1}(-3\beta_1,-2\beta_2,-\beta_3))]\}$$

$$+\partial^{4,4}_{4,3}(\eta_8(-1)).$$

The situation for $k=2$.

$$(L^{(1)})_2=\sum_{l=1,3}\partial^{4,4}_{l,l}\{\sum_{j=1}^{2}\partial^{3,3}_{j,j}\{[\sum_{i=1}^{3-j}\partial^{4-j,4-j}_{i+1,i}$$

$$({}^{R}\eta^{1}_{2(4-j-i),2(4-j-i)+1}(2(4-j-i)))]$$

$$+[\sum_{i=1}^{3-j}\partial^{4-j,4-j}_{i,i+1}({}^{D}\eta^{0,0,0,0}_{2(5-j-i),2(5-j-i)-3}$$

$$(i\alpha_1,i\alpha_2,i\alpha_3,i\alpha_4))]\}$$

$$+\sum_{j=1}^{2}\partial^{3,3}_{j+1,j}[\sum_{i=1}^{3-j}\partial^{3-j,4-j}_{i,i}({}^{L}\eta^{-1}_{2(4-j-i),2(4-j-i)+1}(1))]$$

$$+\sum_{j=1}^{2}\partial^{3,3}_{j,j+1}[\sum_{i=1}^{3-j}\partial^{4-j,3-j}_{i,i}({}^{D}\eta^{0,0,0,0}_{2(5-j-i),2(5-j-i)-3}$$

$$(j\beta_1, j\beta_2, j\beta_3, j\beta_4))]\}$$

$$+\partial^{4,4}_{1,2}\{\sum_{j=1}^{2}\partial^{3,2}_{j,j}[\sum_{i=1}^{3-j}\partial^{4-j,3-j}_{i,i}({}^{D}\eta^{0,0,0}_{2(5-j-i),2(5-j-i)-2}$$

$$(-\beta_2,-2\beta_3,-3\beta_4))]\}$$

$$+\partial^{4,4}_{1,4}\{\sum_{j=1}^{2}\partial^{3,2}_{j,j}[\sum_{i=1}^{3-j}\partial^{4-j,3-j}_{i,i}({}^{D}\eta^{0,0,0}_{2(5-j-i),2(5-j-i)-2}$$

$$(-3\beta_1,-2\beta_2,-\beta_3))]\}$$

$$+\partial^{4,4}_{2,1}(\eta_{14}(-1)) + \partial^{3,2}_{2,2}[\partial^{2,1}_{2,1}({}^{R}\eta^{1}_{1,2}(1))]$$

$$+\sum_{l=2,4}\partial^{4,4}_{l,l}\{\partial^{3,2}_{1,1}\{[\sum_{i=1}^{2}\partial^{3,2}_{i+1,i}({}^{R}\eta^{1}_{2(3-i)-1,2(3-i)}$$

$$(2(3-i)-1))]$$

$$+\partial^{3,2}_{1,2}({}^{D}\eta^{0,0,0,0}_{5,2}(\alpha_1,\alpha_2,\alpha_3,\alpha_4))\}$$

$$+\sum_{j=1}^{2}\partial^{3,2}_{j+1,j}[\sum_{i=1}^{3-j}\partial^{3-j,3-j}_{i,i}({}^{L}\eta^{-1}_{2(4-j-i)-1,2(4-j-i)}(1))]$$

$$+\partial^{3,2}_{1,2}[\partial^{3,1}_{1,1}({}^{D}\eta^{0,0,0,0}_{5,2}(\beta_1,\beta_2,\beta_3,\beta_4))]\}$$

$$+\partial^{4,4}_{3,2}\{\sum_{j=1}^{2}\partial^{3,2}_{j,j}[\sum_{i=1}^{3-j}\partial^{4-j,3-j}_{i,i}({}^{D}\eta^{0,0,0}_{2(5-j-i),2(5-j-i)-2}$$

$$(-\alpha_2,-2\alpha_3,-3\alpha_4))]\}$$

$$+\partial^{4,4}_{3,4}\{\sum_{j=1}^{2}\partial^{3,2}_{j,j}[\sum_{i=1}^{3-j}\partial^{4-j,3-j}_{i,i}({}^{D}\eta^{0,0,0}_{2(5-j-i),2(5-j-i)-2}$$

$$(-3\alpha_1,-2\alpha_2,-\alpha_3))]\}.$$

The situation for $k = 3$.

$$(L^{(1)})_3 = \sum_{l=1,3}\partial^{4,4}_{l,l}\{\sum_{j=1}^{2}\partial^{4,3}_{j,j}\{[\sum_{i=1}^{4-j}\partial^{5-j,4-j}_{i+1,i}$$

$$({}^{R}\eta^{1}_{2(5-j-i)-1,2(5-j-i)}(2(5-j-i)-1))]$$

$$+[\sum_{i=1}^{3-j}\partial^{5-j,4-j}_{i,i+1}({}^{D}\eta^{0,0,0,0}_{2(5-j-i)+1,2(5-j-i)-2}$$

$$(i\alpha_1,i\alpha_2,i\alpha_3,i\alpha_4))]\}$$

$$+\sum_{j=1}^{3}\partial^{4,3}_{j+1,j}[\sum_{i=1}^{4-j}\partial^{4-j,4-j}_{i,i}({}^{L}\eta^{-1}_{2(5-j-i)-1,2(5-j-i)}(1))]$$

$$+\partial^{4,3}_{3,3}[\partial^{2,1}_{2,1}({}^{R}\eta^{1}_{1,2}(1))]$$

$$+\sum_{j=1}^{2}\partial^{4,3}_{j,j+1}[\sum_{i=1}^{3-j}\partial^{5-j,3-j}_{i,i}({}^{D}\eta^{0,0,0,0}_{2(5-j-i)+1,2(5-j-i)-2}$$

$$(j\beta_1, j\beta_2, j\beta_3, j\beta_4))]\}$$

$$+\partial^{4,4}_{1,2}\{\sum_{j=1}^{3}\partial^{4,3}_{j,j}[\sum_{i=1}^{4-j}\partial^{5-j,4-j}_{i,i}({}^{D}\eta^{0,0,0}_{2(5-j-i)+1,2(5-j-i)-1}$$

$$(-\beta_2,-2\beta_3,-3\beta_4))]\}$$

$$+\partial^{4,4}_{1,4}\{\sum_{j=1}^{3}\partial^{4,3}_{j,j}[\sum_{i=1}^{4-j}\partial^{5-j,4-j}_{i,i} + \partial^{4,4}_{2,1}(\eta_{20}(-1))$$

$$({}^{D}\eta^{0,0,0}_{2(5-j-i)+1,2(5-j-i)-1}(-3\beta_1,-2\beta_2,-\beta_3))]\}$$

$$+\sum_{l=2,4}\partial^{4,4}_{l,l}\{\sum_{j=1}^{2}\partial^{3,3}_{j,j}\{[\sum_{i=1}^{3-j}\partial^{4-j,4-j}_{i+1,i}$$

$$({}^{R}\eta^{1}_{2(4-j-i),2(4-j-i)+1}(2(4-j-i)))]$$

$$+[\sum_{i=1}^{3-j}\partial^{4-j,4-j}_{i,i+1}({}^{D}\eta^{0,0,0,0}_{2(5-j-i),2(5-j-i)-3}$$

$$(i\alpha_1,i\alpha_2,i\alpha_3,i\alpha_4))]\}$$

$$+\sum_{j=1}^{2}\partial^{3,3}_{j+1,j}[\sum_{i=1}^{3-j}\partial^{3-j,4-j}_{i,i}({}^{L}\eta^{-1}_{2(4-j-i),2(4-j-i)+1}(1))]$$

$$+\sum_{j=1}^{2}\partial^{3,3}_{j,j+1}[\sum_{i=1}^{3-j}\partial^{4-j,3-j}_{i,i}({}^{D}\eta^{0,0,0,0}_{2(5-j-i),2(5-j-i)-3}$$

$$(j\beta_1, j\beta_2, j\beta_3, j\beta_4))]\}$$

$$+\partial^{4,4}_{3,2}\{\sum_{j=1}^{3}\partial^{4,3}_{j,j}[\sum_{i=1}^{4-j}\partial^{5-j,4-j}_{i,i}$$

$$({}^{D}\eta^{0,0,0}_{2(5-j-i)+1,2(5-j-i)-1}(-\alpha_2,-2\alpha_3,-3\alpha_4))]\}$$

$$+\partial^{4,4}_{3,4}\{\sum_{j=1}^{3}\partial^{4,3}_{j,j}[\sum_{i=1}^{4-j}\partial^{5-j,4-j}_{i,i} + \partial^{4,4}_{4,3}(\eta_{20}(-1))$$

$$({}^{D}\eta^{0,0,0}_{2(5-j-i)+1,2(5-j-i)-1}(-3\alpha_1,-2\alpha_2,-\alpha_3))]\}.$$

The situation for $k = 4$.

$$(L^{(1)})_4 = \sum_{l=1,3}\partial^{4,4}_{l,l}\{\sum_{j=1}^{3}\partial^{4,4}_{j,j}\{[\sum_{i=1}^{4-j}\partial^{5-j,5-j}_{i+1,i}$$

$$({}^{R}\eta^{1}_{2(5-j-i),2(5-j-i)+1}(2(5-j-i)))]$$

$$+[\sum_{i=1}^{4-j}\partial^{5-j,5-j}_{i,i+1}({}^{D}\eta^{0,0,0,0}_{2(6-j-i),2(6-j-i)-3}$$

$$(i\alpha_1,i\alpha_2,i\alpha_3,i\alpha_4))]\}$$

$$+\sum_{j=1}^{3}\partial_{j+1,j}^{4,4}[\sum_{i=1}^{4-j}\partial_{i,i}^{4-j,5-j}({}^{L}\eta_{2(5-j-i),2(5-j-i)+1}^{-1}(1))]$$

$$+\sum_{j=1}^{3}\partial_{j,j+1}^{4,4}[\sum_{i=1}^{4-j}\partial_{i,i}^{5-j,4-j}({}^{D}\eta_{2(6-j-i),2(6-j-i)-3}^{0,0,0,0}$$

$$(j\beta_1, j\beta_2, j\beta_3, j\beta_4))]\}$$

$$+\partial_{1,2}^{4,4}\{\sum_{j=1}^{3}\partial_{j,j}^{4,3}[\sum_{i=1}^{4-j}\partial_{i,i}^{5-j,4-j}({}^{D}\eta_{2(6-j-i),2(6-j-i)-2}^{0,0,0}$$

$$(-\beta_2,-2\beta_3,-3\beta_4))]\}$$

$$+\partial_{1,4}^{4,4}\{\sum_{j=1}^{3}\partial_{j,j}^{4,3}[\sum_{i=1}^{4-j}\partial_{i,i}^{5-j,4-j}({}^{D}\eta_{2(6-j-i),2(6-j-i)-2}^{0,0,0}$$

$$(-3\beta_1,-2\beta_2,-\beta_3))]\}$$

$$+\partial_{2,1}^{4,4}(\eta_{30}(-1))$$

$$+\sum_{l=2,4}\partial_{l,l}^{4,4}\{\sum_{j=1}^{2}\partial_{j,j}^{4,3}\{[\sum_{i=1}^{4-j}\partial_{i+1,i}^{5-j,4-j}$$

$$({}^{R}\eta_{2(5-j-i)-1,2(5-j-i)}^{1}(2(5-j-i)-1))]$$

$$+[\sum_{i=1}^{3-j}\partial_{i,i+1}^{5-j,4-j}({}^{D}\eta_{2(5-j-i)+1,2(5-j-i)-2}^{0,0,0,0}$$

$$(i\alpha_1, i\alpha_2, i\alpha_3, i\alpha_4))]\}$$

$$+\sum_{j=1}^{3}\partial_{j+1,j}^{4,3}[\sum_{i=1}^{4-j}\partial_{i,i}^{4-j,4-j}({}^{L}\eta_{2(5-j-i)-1,2(5-j-i)}^{-1}(1))]$$

$$+\partial_{3,3}^{4,3}[\partial_{2,1}^{2,1}({}^{R}\eta_{1,2}^{1}(1))]$$

$$+\sum_{j=1}^{2}\partial_{j,j+1}^{4,3}[\sum_{i=1}^{3-j}\partial_{i,i}^{5-j,3-j}({}^{D}\eta_{2(5-j-i)+1,2(5-j-i)-2}^{0,0,0,0}$$

$$(j\beta_1, j\beta_2, j\beta_3, j\beta_4))]\}$$

$$+\partial_{3,2}^{4,4}\{\sum_{j=1}^{3}\partial_{j,j}^{4,3}[\sum_{i=1}^{4-j}\partial_{i,i}^{5-j,4-j}({}^{D}\eta_{2(6-j-i),2(6-j-i)-2}^{0,0,0}$$

$$(-\alpha_2,-2\alpha_3,-3\alpha_4))]\}$$

$$+\partial_{3,4}^{4,4}\{\sum_{j=1}^{3}\partial_{j,j}^{4,3}[\sum_{i=1}^{4-j}\partial_{i,i}^{5-j,4-j}({}^{D}\eta_{2(6-j-i),2(6-j-i)-2}^{0,0,0}$$

$$(-3\alpha_1,-2\alpha_2,-\alpha_3))]\}$$

$$+\partial_{4,3}^{4,4}(\eta_{30}(-1)).$$

We change $(L^{(1)})_k$ by elementary row transformations and make $(L_6^{(1)})_k$ and $(L_{16}^{(1)})_k$ into null matrices. Under transformations, the matrix can be showed that

$$(\widetilde{L}^{(1)})_k = \partial_{1,1}^{2,2}[\partial_{1,1}^{2,2}(L_1^{(1)})_k + \partial_{1,2}^{2,2}(\widetilde{L}_2^{(1)})_k$$

$$+\partial_{2,1}^{2,2}(-I)] + \partial_{1,2}^{2,2}({}^{L}\eta_{2,2}^{0}(L_4^{(1)})_k)$$

$$+\partial_{2,2}^{2,2}[\partial_{1,1}^{2,2}(L_{11}^{(1)})_k + \partial_{1,2}^{2,2}(\widetilde{L}_{12}^{(1)})_k + \partial_{2,1}^{2,2}(-I)$$

$$+\partial_{2,1}^{2,2}({}^{L}\eta_{2,2}^{0}(L_{10}^{(1)})_k), \quad (3\text{-}3)$$

make further efforts，we show (3-3) as

$$(\widetilde{L}^{(1)})_k \underline{\underline{\Delta}}\, {}^{D}\eta_{4,1}^{0,0,0,0}(M_1, M_2, M_3, M_4) \quad (3\text{-}4)$$

where $M_1 = (\widetilde{L}_A{}^{(1)})_k$, $M_2 = (\widetilde{L}_B{}^{(1)})_k$,

$$M_3 = (\widetilde{L}_C{}^{(1)})_k,\ M_4 = (\widetilde{L}_D{}^{(1)})_k,\ k = 1,2,3,4.$$

We have to delete some rows on $(\widetilde{L}_A{}^{(1)})_k$ and $(\widetilde{L}_C{}^{(1)})_k$, such that (3-4) is square matrix with full rank as follow

$$(\widetilde{L}^{(1)^*})_k = \partial_{1,1}^{2,2}[\partial_{1,1}^{2,2}(L_1{}^{(1)^*})_k + \partial_{1,2}^{2,2}(\widetilde{L}_2{}^{(1)^*})_k$$

$$+\partial_{2,1}^{2,2}(-I)] + \partial_{1,2}^{2,2}({}^{L}\eta_{2,2}^{0}(L_4{}^{(1)^*})_k)$$

$$+\partial_{2,2}^{2,2}[\partial_{1,1}^{2,2}(L_{11}{}^{(1)^*})_k + \partial_{1,2}^{2,2}(\widetilde{L}_{12}{}^{(1)^*})_k$$

$$+\partial_{2,1}^{2,2}(-I)] + \partial_{2,1}^{2,2}({}^{L}\eta_{2,2}^{0}(L_{10}{}^{(1)^*})_k), \quad (3\text{-}5)$$

in accordance with the expansion of negative unit matrices for (3-5)，we obtain the result as follows

$$(\widetilde{L}^{(1)^{**}})_k = \partial_{1,1}^{2,2}[\partial_{1,1}^{2,2}(\widetilde{L}_2{}^{(1)^*})_k + \partial_{1,2}^{2,2}(L_4{}^{(1)^*})_k$$

$$+\partial_{2,1}^{2,2}(L_{10}{}^{(1)^*})_k + \partial_{2,2}^{2,2}(\widetilde{L}_{12}{}^{(1)^*})_k], \quad (3\text{-}6)$$

Remark 3.1 With the aid of Maple, when $\alpha_4 \neq 0$， we calculate that

$$KerL_2 = KerL_3 = KerL_4 = \{\mathbf{0}\},$$

and $KerL_1$ is

$$span\{x_2\partial_{x_1} + (b_1x_1^3 + b_2x_1^2x_3 + b_3x_1x_3^2 + b_4x_3^3)\partial_{x_2}$$

$$+x_4\partial_{x_3} + (d_1x_1^3 + d_2x_1^2x_3 + d_3x_1x_3^2 + d_4x_3^3)\partial_{x_4}\}.$$

Theorem 3.2 When $\alpha_4 \neq 0$， the first order normal form at Quintic truncated for system (3-1) is

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \alpha_1x_1^3 + \alpha_2x_1^2x_3 + \alpha_3x_1x_3^2 + \alpha_4x_3^3$$

$$+\sum_{\substack{p+2q+s+2t=4\\ qt=0,p+s=3,p\geq s}} d_{p,q,s,t}x_1^px_2^qx_3^sx_4^t + \sum_{\substack{p+2q+s+2t=5\\ p=1,2,3,q=0,1\\ s=0,1,t=0,1,2}} d_{p,q,s,t}x_1^px_2^qx_3^sx_4^t$$

$$+\sum_{\substack{p+2q+s+2t=6\\ q=0,1,s=0,1\\ t=0,2,p=1,2,3,4}} d_{p,q,s,t}x_1^px_2^qx_3^sx_4^t + \sum_{\substack{p+2q+s+2t=7\\ p=3,5,q=0,1\\ s=5,7,t=0,1}} d_{p,q,s,t}x_1^px_2^qx_3^sx_4^t$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = \beta_1x_1^3 + \beta_2x_1^2x_3 + \beta_3x_1x_3^2 + \beta_4x_3^3$$

$$+\sum_{p+2q+s+2t=4} d_{p,q,s,t}x_1^px_2^qx_3^sx_4^t + \sum_{\substack{p+2q+s+2t=5\\ p=t=1,s\neq 2\\ s=3,t\neq 1}} d_{p,q,s,t}x_1^px_2^qx_3^sx_4^t$$

$$+\sum_{\substack{p+2q+s+2t=6\\ p=s=1,t\neq 2\\ s=2,t\neq 2}} d_{p,q,s,t}x_1^p x_2^q x_3^s x_4^t+\sum_{\substack{p+2q+s+2t=7\\ q=t=1,p\neq 0,1\\ q=2t=2,p\neq 0,1\\ t=2,s\geq 2,p\neq 0,1}} d_{p,q,s,t}x_1^p x_2^q x_3^s x_4^t\,,$$

all the coefficients here are determined uniquely by system (3-1).

Theorem 3.3 By Lemma 2.4, equation (3-2) and Remark 3.2, the result in **Theorem 3.3** is unique normal form.

4. Hypernormal Form at Quintic Truncated for Honeycomb Sandwich Plate Dynamics Model

Honeycomb sandwich plate as **Fig. 1** showed has the advantages of light weight, high stiffness, good fatigue resistance and small thermal conductivity. It has good performance on the composite insulation and bearing force. It is widely used in spacecraft structures, the basic structure of large attachments as well as the design of thermal protection structure.

This section research the hypernormal form of honeycomb sandwich plate dynamics model by methods showed at section 2 and section 3.

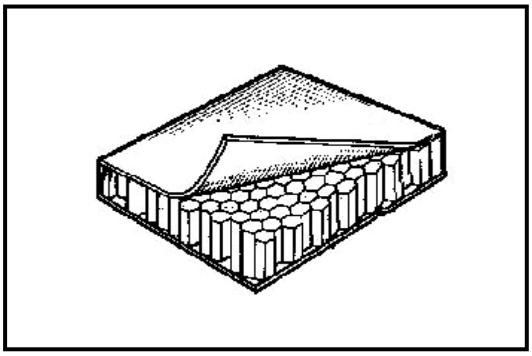

Fig 1. Structure of honeycomb sandwich plate.

4.1. Averaged Equations In Cartesian Form

The four-dimensional average equations in Cartesian coordinate form for honeycomb sandwich plate dynamics model as follows

$$\begin{aligned}
\dot{x}_1 =& -\sigma_1 x_2+2e_{15}x_1x_3x_4-e_{15}x_2\left(x_3^2-x_4^2\right)\\
&+2e_{16}x_1x_2x_3-e_{16}x_4\left(x_1^2-x_2^2\right)\\
&+2e_{15}x_2\left(x_3^2+x_4^2\right)+2e_{16}x_4\left(x_1^2+x_2^2\right)\\
&+3e_{14}x_2\left(x_1^2+x_2^2\right)+3e_{17}x_4\left(x_3^2+x_4^2\right)\\
\dot{x}_2 =& \sigma_1 x_1-2e_{15}x_2x_3x_4-e_{15}x_1\left(x_3^2-x_4^2\right)\\
&-2e_{16}x_1x_2x_4-e_{16}x_3\left(x_1^2-x_2^2\right)\\
&-2e_{15}x_1\left(x_3^2+x_4^2\right)-2e_{16}x_3\left(x_1^2+x_2^2\right)\\
&-3e_{14}x_1\left(x_1^2+x_2^2\right)-3e_{17}x_3\left(x_3^2+x_4^2\right)\\
\dot{x}_3 =& -\sigma_2 x_4+2g_{15}x_1x_2x_3-g_{15}x_4\left(x_1^2-x_2^2\right)\\
&+2g_{16}x_1x_3x_4-g_{16}x_2\left(x_3^2-x_4^2\right)\\
&+2g_{15}x_4\left(x_1^2+x_2^2\right)+2g_{16}x_2\left(x_3^2+x_4^2\right)\\
&+3g_{14}x_4\left(x_3^2+x_4^2\right)+3g_{17}x_2\left(x_1^2+x_2^2\right)\\
\dot{x}_4 =& \sigma_2 x_3-2g_{15}x_1x_2x_4-g_{15}x_3\left(x_1^2-x_2^2\right)\\
&-2g_{16}x_2x_3x_4-g_{16}x_1\left(x_3^2-x_4^2\right)\\
&-2g_{15}x_3\left(x_1^2+x_2^2\right)-2g_{16}x_1\left(x_3^2+x_4^2\right)\\
&-3g_{14}x_3\left(x_3^2+x_4^2\right)-3g_{17}x_1\left(x_1^2+x_2^2\right) \quad (4\text{-}1)
\end{aligned}$$

coefficient matrix of linear part for system (3-3) is

$$\begin{aligned}A=&\partial_{1,1}^{2,2}[\partial_{1,2}^{2,2}(-\sigma_1)+\partial_{2,1}^{2,2}(\sigma_1)]\\
&+\partial_{2,2}^{2,2}[\partial_{1,2}^{2,2}(-\sigma_2)+\partial_{2,1}^{2,2}(\sigma_2)]\,,\end{aligned}$$

it has two pairs of pure imaginary eigenvalues obviously, and system (4-1) accord with the form of (3-1).

4.2. Hypernormal Form at Quintic Truncated

Through the function δ , $V_1^{(0)}$ of system (4-1) can be showed as

$$\begin{aligned}V_1^{(0)} =& x_2\partial_{x_1}\\
&-(3e_{14}x_1^3+3e_{16}x_1^2x_3+3e_{15}x_1x_3^2+3e_{17}x_3^3)\partial_{x_2}\\
&+x_4\partial_{x_3}\\
&-(3g_{17}x_1^3+3g_{15}x_1^2x_3+3g_{16}x_1x_3^2+3g_{14}x_3^3)\partial_{x_4}\,,\end{aligned}$$

We define linear operator

$$L_m^{(1)}:H_m\to H_{m+1},\ Y_m\mapsto[Y_m,V_1^{(0)}]\,,$$

$m\in N^+$ .According to Theorem 3.3 and Theorem 3.4, we have the result as below

Theorem 4 The first order normal form for honeycomb sandwich plate dynamics model is unique and the Hypernormal form at Quintic truncated is

$$\begin{aligned}
\dot{x}_1 =& x_2\\
\dot{x}_2 =& 3e_{14}x_1^3+3e_{16}x_1^2x_3+3e_{15}x_1x_3^2+3e_{17}x_3^3\\
&+\sum_{\substack{p+2q+s+2t=4\\ qt=0,p+s=3,p\geq s}} d_{p,q,s,t}x_1^p x_2^q x_3^s x_4^t+\sum_{\substack{p+2q+s+2t=5\\ p=1,2,3,q=0,1\\ s=0,1,t=0,1,2}} d_{p,q,s,t}x_1^p x_2^q x_3^s x_4^t\\
&+\sum_{\substack{p+2q+s+2t=6\\ q=0,1,s=0,1\\ t=0,2,p=1,2,3,4}} d_{p,q,s,t}x_1^p x_2^q x_3^s x_4^t+\sum_{\substack{p+2q+s+2t=7\\ p=3,5,q=0,1\\ s=5,7,t=0,1}} d_{p,q,s,t}x_1^p x_2^q x_3^s x_4^t\\
\dot{x}_3 =& x_4\\
\dot{x}_4 =& 3g_{17}x_1^3+3g_{15}x_1^2x_3+3g_{16}x_1x_3^2+3g_{14}x_3^3\\
&+\sum_{p+2q+s+2t=4} d_{p,q,s,t}x_1^p x_2^q x_3^s x_4^t+\sum_{\substack{p+2q+s+2t=5\\ p=t=1,s\neq 2\\ s=3,t\neq 1}} d_{p,q,s,t}x_1^p x_2^q x_3^s x_4^t
\end{aligned}$$

$$+ \sum_{\substack{p+2q+s+2t=6 \\ p=s=1,t\neq 2 \\ s=2,t\neq 2}} d_{p,q,s,t} x_1^p x_2^q x_3^s x_4^t + \sum_{\substack{p+2q+s+2t=7 \\ q=t=1,p\neq 0,1 \\ q=2t=2,p\neq 0,1 \\ t=2,s\geq 2,p\neq 0,1}} d_{p,q,s,t} x_1^p x_2^q x_3^s x_4^t ,$$

all the coefficients here are determined uniquely by system (4-1).

5. Conclusion

This paper used the method combined new grading function with multiple Lie brackets, the hypernormal form at quintic truncation of a class of four-dimensional vector field was obtained by the new mark of block matrices. This paper also from the practical point of view, studied the reduction problem of honeycomb sandwich plate dynamics model, and gets the hypernormal form at quintic truncated for that model.

In recent years, the investigation of the normal form theory has obtained great progress, especially in the development of natural science and engineering application [14-16]. Therefore, the research of hypernormal form for high dimension nonlinear vector field will focus on new way to obtain relevant form, and development from the general theory of the system to the practical engineering application.

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