Sensors & Transducers, Vol. 25, Special Issue, December 2013, pp. 239-243

Sensors & Transducers

http://www.sensorsportal.com

Multi-scale Edge Detection of Digital Image Based on Improved Mallat Wavelet Decomposition Algorithm

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Received: 16 September 2013 /Accepted: 15 October 2013 /Published: 23 December 2013

Abstract: Multi-resolution analysis of digital image has attracted numerous studies since dyadic wavelet transform was introduced to this field. Aiming at achieving accurate and stationary edge detection, an improved Mallat wavelet decomposition algorithm was employed. This algorithm was defined by a low frequency component and three high frequency components. Two-dimensional signal can be reconstructed by dyadic wavelet transform. Maximum module of the wavelet transform was used for multi-scale edge detection. Results show high performance of the improved algorithm in local resolution of this improved algorithm. *Copyright © 2013 IFSA.*

Keywords: Multi-scale edge detection, Wavelet decomposition, Dyadic wavelet transform.

1. Introduction

Edge detection in the field of image processing is one of the most important areas of interest. Edges in images provide primary information about the objects presenting in a scene and the boundaries between them. It is often the crucial first step before sophisticated algorithms for object identification, feature extraction, and so on. The traditional methods mainly include edge detection method based on gradient [1, 2], second order derivation [3], Canny operator [4] and so on. However, these methods are limited by their resolution, especially in those images with objects in different depth of focus.

Nowadays, orthogonal wavelet and bi-orthogonal wavelet have been successfully applied in digital image processing for edge detection in multi-scale [5-10]. However, due to the lack of translation invariance, pseudo-Gibbs phenomena occur in the reconstruction signal. Accordingly, Mallat introduced dyadic wavelet with translation invariance for the first time, which was then used for image edge detection [11, 12]. Recently, wavelet lifting scheme was expanded to dyadic wavelet. Consequently, ordinary dyadic wavelet can be used to form high order vanishing moment [13-16]. Two-dimensional dyadic wavelet transform developed by Mallat has been widely used. The transform algorithm consists of one low frequency component and two high frequency components. For the sake of more details of the image, a novel wavelet transform algorithm was developed, which consists of one low frequency and two high frequency components.

2. Improved Two-dimensional Wavelet

2.1. Wavelet Construction

Two-dimensional dyadic wavelet introduced by Mallat consists of low frequency component and vertical and horizontal high frequency components. This transform can be used in multi-scale edge detection in digital image. The algorithm was improved by adding a high frequency component in the across corner direction. Supposing that $h, l, \tilde{h}, \tilde{l}$ are dyadic wavelet filter, then the corresponding scale functions and wavelets are $\phi, \psi, \tilde{\varphi}, \tilde{\psi}$. The two-dimensional separable dyadic wavelets are defined as:

$$\begin{cases} \psi^1(x,y) = \phi(x)\psi(y) \\ \psi^2(x,y) = \psi(x)\phi(y) \\ \psi^3(x,y) = \psi(x)\psi(y) \end{cases} \tag{1}$$

$$\begin{cases} \widetilde{\psi^1}(x,y) = \tilde{\phi}(x)\tilde{\psi}(y) \\ \widetilde{\psi^2}(x,y) = \tilde{\psi}(x)\tilde{\phi}(y) \\ \widetilde{\psi^3}(x,y) = \tilde{\psi}(x)\tilde{\psi}(y) \end{cases} \tag{2}$$

For any function$f(x,y) \in L^2(R^2)$, the two-dimensional dyadic wavelet transform of f at scale 2^j and is expressed as

$$W^k f(2^j, x, y) = \frac{1}{2^j} \iint_{R^2} f(u,v)\, \psi^k \left(\frac{u-x}{2^j}, \frac{v-x}{2^j}\right) dudv = 2^j \left(f * \overline{\psi_2^k j}\right)(x,y), k = 1,2,3 \tag{3}$$

Then

$$\sum_{k=1}^{3} \sum_{j=-\infty}^{+\infty} \widehat{\psi^k}\,(2^j\omega_x, 2^j\omega_x) \widehat{\widetilde{\psi^k}} * (2^j\omega_x, 2^j\omega_x) = 1 \tag{4}$$

Accordingly, signal $f(x,y)$ can be reconstructed by its dyadic wavelet transform, namely

$$f(x,y) = \sum_{j=-\infty}^{+\infty} \sum_{k=1}^{3} \frac{1}{2^j} \left(W^k f(2^j, \cdot, \cdot) * \widetilde{\psi_2^k} j\right)(x,y), \tag{5}$$

where $f * \widetilde{\psi_2^k} j$ represents convolution of f and $\widetilde{\psi_2^k} j$; $\widehat{\widetilde{\psi^k}} *$ represents complex conjugate of $\widehat{\psi^k}$.The wavelet transform defined by was two-dimensional stationary wavelet transform of dyadic wavelet. $\widetilde{\psi^1}, \widetilde{\psi^2}, \widetilde{\psi^3}$ are called reconstructed wavelet of ψ^1, ψ^2, ψ^3.

2.2. Discrete Fast Algorithm

Supposing that the input signal is an image $a^0 = \{a_{n,m}^0\}_{n,m \in Z}$ with sampling interval is 1. Supposing that $\varphi(x,y) = \phi(x)\phi(y)$ Accordingly, $\varphi_{2^j}(x,y) = \frac{1}{2^j}\phi(\frac{x}{2^j})\phi(\frac{y}{2^j})$. Then there is a two-dimensional function $f(x,y) \in L^2(R^2)$, making $(f * \bar{\varphi})(n,m) = a_{n,m}^0$. For any $j \geq 0$:

$$a_{n,m}^j = 2^j (f * \widetilde{\varphi_2} j)(n,m) \tag{6}$$

For $j > 0$, at grid *(n.m)*, there is

$$d_{n,m}^{j,k} = W^k f(2^j, n, m) = 2^j \left(f * \overline{\psi_2^k j}\right)(n,m), k = 1,2,3 \tag{7}$$

$d_{n,m}^{j,k}$ is wavelet coefficient at grid of two-dimensional dyadic stationary wavelet. With regard to any scale $2^j > 1$, discrete signal sequence $\{d^{1,1}, d^{1,2}, d^{1,3}\}$, $\{d^{2,1}, d^{2,2}, d^{2,3}\}$,⋯, $\{d^{J,1}, d^{J,2}, d^{J,3}\}$ and a^J are called two-dimensional dyadic stationary wavelet transform of a^0. The two-dimensional discrete dyadic stationary wavelet decomposition can be expressed as:

$$\begin{cases} a_{n,m}^{j+1} = \sum_{k,p} \overline{h_k}\, \overline{h_p} a_{n-2^j k, m-2^j p}^j \\ d_{n,m}^{j+1,1} = \sum_{k,p} \overline{h_k}\, \overline{l_p} a_{n-2^j k, m-2^j p}^j \\ d_{n,m}^{j+1,2} = \sum_{k,p} \overline{l_k}\, \overline{h_p} a_{n-2^j k, m-2^j p}^j \\ d_{n,m}^{j+1,3} = \sum_{k,p} \overline{l_k}\, \overline{l_p} a_{n-2^j k, m-2^j p}^j \end{cases} \tag{8}$$

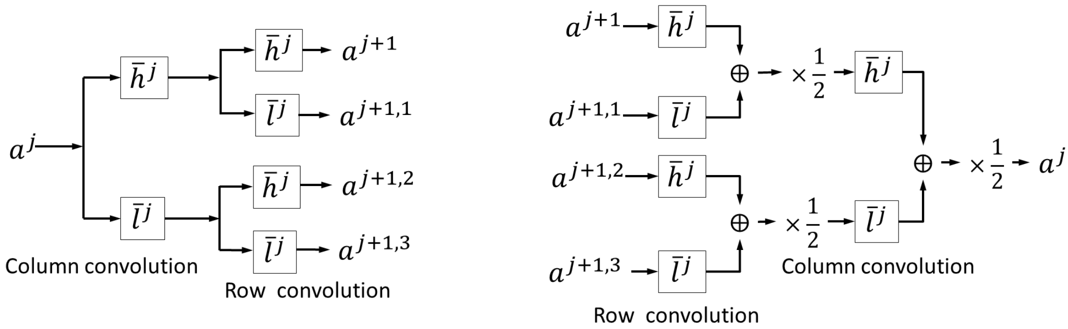
The two-dimensional discrete dyadic stationary wavelet reconstructed can be expressed as:

$$a^j = \frac{1}{4}[a^{j+1} * (\tilde{h}^j, \tilde{h}^j) + d^{j+1,1} * (\tilde{h}^j, \tilde{l}^j) + d^{j+1,2} * (\tilde{l}^j, \tilde{h}^j) + d^{j+1,3} * (\tilde{l}^j, \tilde{l}^j)] \tag{9}$$

The filter bank of improved two-dimensional dyadic wavelet transform is shown in Fig. 1. In the figure, a^{j+1}, $d^{j+1,1}$, $d^{j+1,2}$, $d^{j+1,3}$ represent the profile image and detail image in the horizontal, vertical, across corner direction. Through the improved two-dimensional dyadic wavelet transform, detail information in the corner direction of the image can be obtained. Moreover, it provides dyadic filter to replace orthogonal wavelet or bi-orthogonal wavelet in the Mallat double channel filter bank.

3. Multiscale Edge Detection

After dyadic wavelet transform of two-dimensional signal of the image, edges can be detected. Supposing that image D has a dimension $N \times N$ pixel, then the edge detection is processed in the following steps:



(a) Decomposition process (b) Reconstruction process

Fig. 1. Filter bank of improved two-dimensional dyadic wavelet transform.

1) On scale2^j, calculate two-dimensional wavelet transform of image *D*:

$W^1f(2^j,n,m)$, $W^2f(2^j,n,m)$,*n,m=0,1,···,N-1*.

The decomposition scale can be determined as needed.

2) Calculate module value and tangent value of phase.

For every pixel point *(n, m)*, calculate

$$Mf(2^j,n,m) = \sqrt{|W^1f(2^j,n,m)|^2 + |W^2f(2^j,n,m)|^2}$$
$$tanAf(2^j,n,m) = \frac{W^2f(2^j,n,m)}{W^1f(2^j,n,m)} \quad (10)$$

If wavelet $W^1f(2^j,n,m)$ and $W^2f(2^j,n,m)$ are not equal to zero at point *(n, m)*, the point is not edge point.

3) Determine threshold $T_j > 0$, for *n,m=0,1,···,N-1*, if

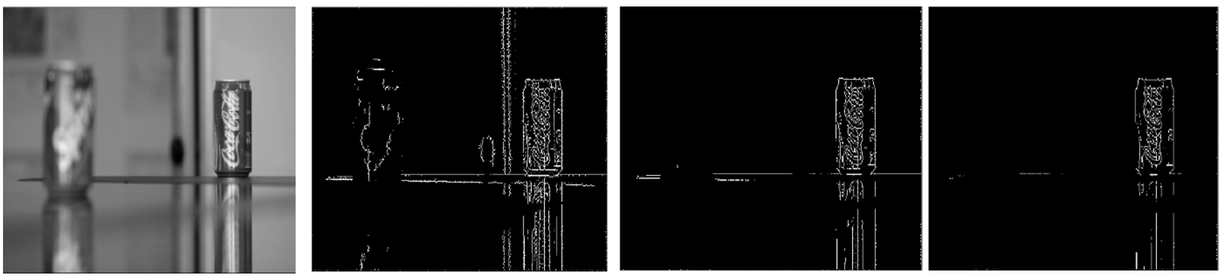
$$Mf(2^j,n,m) \geq T_j \quad (11)$$

and there is local maximum at point *(n, m)*, *(n, m)* is a border point.

4) Connect border points on scale 2^j, then modulus evaluation curve along the border at this scale can be formed.

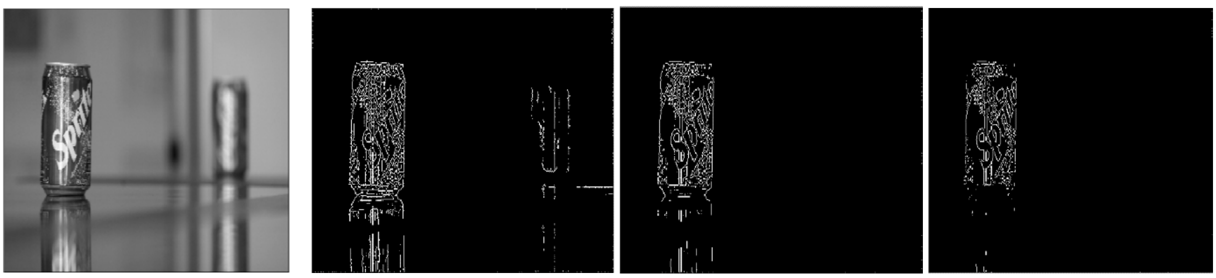
4. Results

For the purpose of validating the performance of the improved Mallat wavelet decomposition algorithm, can1 and can 2 were used for edge detection, as indicated in Fig. 2 and Fig. 3.



(a)Original Image (b)Threshold 0.1001 (c) Threshold 0.2001 (d) Threshold 0.3001

Fig. 2. Multi-scale edge detection result of Can 1.



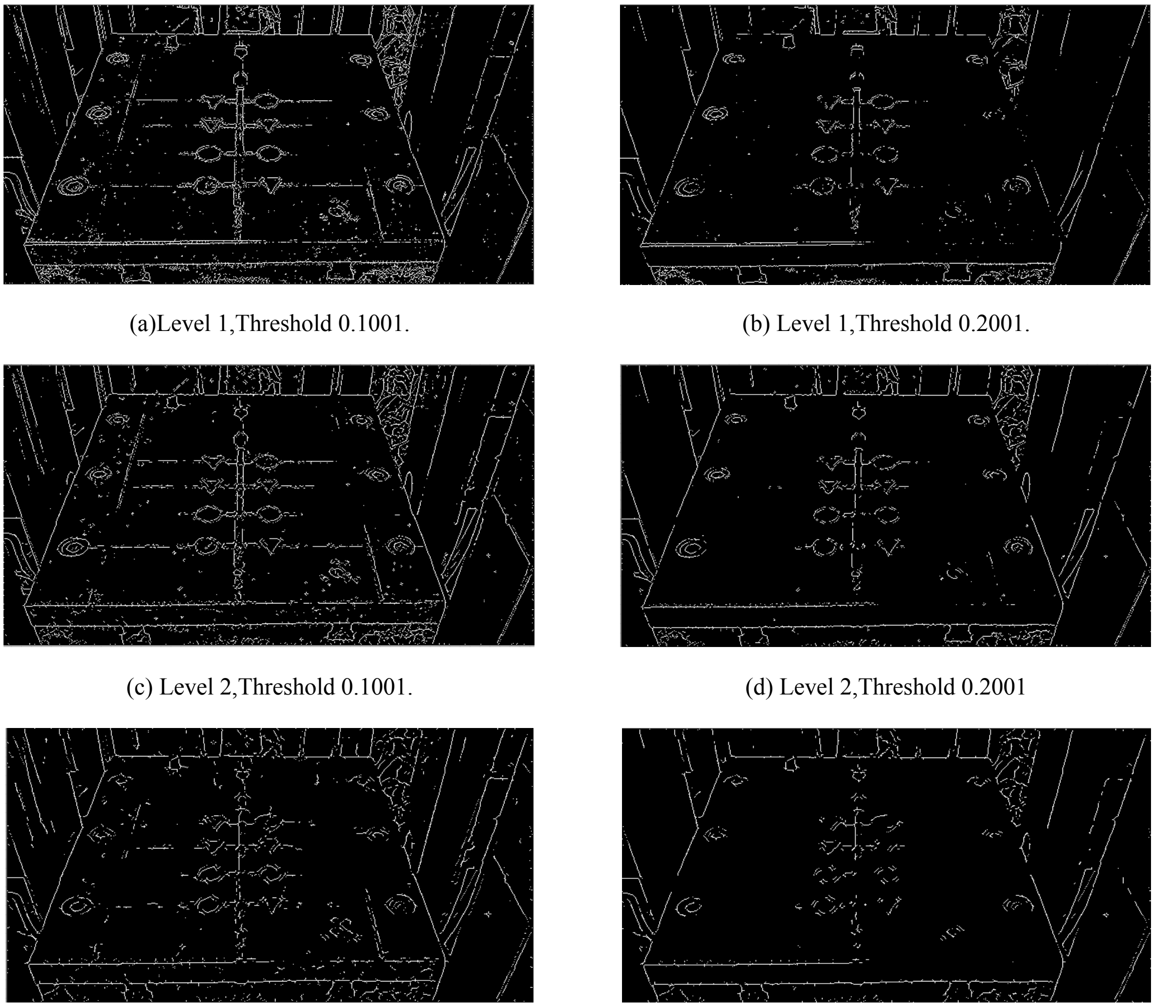
(a)Original Image (b)Threshold 0.1001 (c) Threshold 0.2001 (d) Threshold 0.3001

Fig. 3. Multi-scale edge detection result of Can 2.

The used wavelet package was Haar wavelet. For can 1, the focus is on the right can, while focus is on the left can in can 2. Both cans have the same profile. However, they are in different depth of focus in the two images. It is difficult to detect the edge using traditional method due to the different detail scale of the two cans. Using the multi-scale method, it becomes possible to identity the can in different depth of focus. In this example, the level of discrete wavelet analysis was used to analysis was specified as one. Different thresholds were used as shown in Fig. 2 and Fig. 3. In Fig. 1, when the threshold was 0.1001, the edge of the right can was clear, so as its ground and left border. Even the inverted reflection of the can in the ground was legible. However, few edges of the left can were detected too. As the threshold increases to 0.2001, the left can was gone. In this condition, the inverted reflection of the can was still visible. The same result was also founded in Fig. 4. Based on this result, it can be concluded that the improved Mallat wavelet decomposition is useful in multi-scale edge detection in for digital image.

The final purpose of the multi-scale edge detection is to be used in practice such as contour extraction, defect detection and so on. Fig. 4 shows an injection mould. In the forming process of plastic production, position and defect of injection mould are important factors. To obtain these information, the first step is detect the edge of mould. In Fig. 5, edge detection results under different discrete levels and thresholds are shown. First of all, the discrete level was set to be one, results of edge detection using two different threshold are shown in Fig. 5(a) and Fig. 5(b).

Fig. 4. Original image of injection mould.



(a)Level 1,Threshold 0.1001.

(b) Level 1,Threshold 0.2001.

(c) Level 2,Threshold 0.1001.

(d) Level 2,Threshold 0.2001

(e) Level 3, Threshold 0.1001.

(f) Level 3, Threshold 0.2001

Fig. 5. Multi-scale edge detection result of injection mould.

When the threshold was 0.1001, the edge of the mould was clear, so as the cavity of the mould. As the threshold increased to 0.2001, the background noise decreased remarkablely. The cavity of the mould was still identifiable. However, the right edge of the mould was not as clear as previous one. And then the discrete level was set to be two, results of edge detection using two different threshold are shown in Fig. 5(c) and Fig. 5(d). In this condition, the background noise is less than Fig. 5(a) and Fig. 5(b). The image detail was more clear and identifiable. The edges were also clear to identify. However, as the discrete level increased to three, the edge became fuzzy and not identifiable. There for, the most appropriate discrete level of wavelet transform is one or two. To decrease the noise level, threshold can be increased.

5. Conclusion

Aiming at achieving accurate and stationary edge detection, an improved Mallat wavelet decomposition algorithm was developed. The transform algorithm consists of one low frequency component and two high frequency components. Two-dimensional signal can be reconstructed by dyadic wavelet transform. Maximum module of the wavelet transform was for multi-scale edge detection. Based on the detection result of selected images, it can be concluded that the improved Mallat wavelet decomposition is powerful in multi-scale edge detection in for digital image. Results show high performance of the improved algorithm in translation invariance and local resolution of this improved algorithm.

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