

## Transmit Waveform Selection for Polarimetric MIMO Radar Based on Mutual Information Criterion

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**Abstract:** Transmit waveform selection is essential to improve the performance of a polarimetric multiple-input multiple-output (MIMO) radar system. In this paper, an information theoretic criterion based method is proposed to select the polarization waveforms of the transmit array. The design criterion is to minimize the mutual information (MI) between the radar return signal at two adjacent coherent processing intervals. Based on this criterion, the waveform to be transmitted at next interval is selected according to the return signal at current interval. The criterion guarantees that the MIMO radar can catch more new information about the target at next time. A simple analytic form is attained when all these processes are proper Gaussian processes. An iterative optimization algorithm based on alternating projection is proposed to realize the procedure of waveform selection. The algorithm greatly reduces the amount of computation. Numerical examples are conducted to illustrate the effectiveness of the proposed method. *Copyright © 2013 IFSA.*

**Keywords:** MIMO radar, Waveform design, Polarization waveform, Mutual information criterion, Alternating projection.

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### 1. Introduction

Multiple-input multiple-output (MIMO) radar [1] has the advantage of flexible transmitting beam design, in which diverse forms of signal can be transmitted by multi-antennas. The system overcomes the performance declination of traditional phased array radar, brought by the target RCS scintillation. By introducing polarization sensitive arrays to the MIMO framework, the polarimetric MIMO radar has recently drawn considerable attention, in which the polarization information of signals can be fully utilized. The polarimetric MIMO radar system offers both spatial and polarization diversities that further improves the performance of radar target detection [2] and parameter estimation [3].

Commonly, a radar relies on waveform to realize its full potential. Thus, it is a crucial task to design waveform to unleash its advantages. The design method based on mutual information criterion (MIC) has been studied by researchers in recent years. Shannon mutual information (MI) is a measure of the information between a random variable (or vector) and another random variable (or vector). Early in 1993, M. R. Bell first proposed the method of waveform design aimed at improving the estimation and identification capability using mutual information between radar echoes and scattering targets, where the water-filling algorithm was employed [4].

For waveform design of MIMO radar, Yang Yang et al demonstrated the equality of the two methods of waveform design using maximum

conditional MI criterion and the minimum mean square error (MMSE) criterion in the conditions of deterministic the power spectrum density and that the power spectral density is limited [5][6], and gave a more practical and simplified method in which only finite sample values of power spectral density (PSD) of impulse response are needed, which has greatly simplified the algorithm. Based on these, a kind of waveform that meets the two criteria was designed via alternating projection algorithm [7]. Then under the condition that the power spectral density is unknown, the minimax robust waveform design was explored based on MI and MMSE estimation, and the conclusions were given demonstrating that the optimization solutions are different in the two situations [8]. It is shown that the MIC is an effective criterion for polarization type selection in synthetic aperture radar (SAR), and that the MI attains a simple analytic form for Gaussian-distributed scatterer and noise [9].

MIMO radar features waveform diversity. The waveform design of MIMO radar has been under intensive study in recent years, while the waveform design of polarimetric MIMO radar is rare to be investigated. The research in [9] provides a plausible solution which is used in single-input single-output (SISO) polarimetric radar for polarization states selection. In this article, we extend the model to the polarimetric MIMO radar. An information theoretic criterion based method is proposed to select the polarization waveforms of the transmit array. The design criterion is to minimize the mutual information between the radar return signal at two adjacent coherent processing intervals. Based on this criterion, the waveform to be transmitted at next interval is selected according to the return signal at current interval. The criterion guarantees that the MIMO radar can catch more new information about the target at next time. A simple analytic form is attained when all these processes are proper Gaussian processes. An iterative optimization algorithm based on alternating projection is proposed to realize the procedure of waveform selection. The algorithm greatly reduces the amount of computation.

Notation: Throughout this paper, we use bold upper case letters to denote matrices, and bold lower case letters to signify column vectors. Superscripts  $\{\cdot\}^H$  and  $\{\cdot\}^T$  denote complex conjugate transpose and transpose of a matrix respectively, and  $\{\cdot\}^{-1/2}$  denotes the inverse-square-root matrix operator.

## 2. Polarimetric MIMO Radar Model

Consider a polarimetric MIMO radar system with  $M$  transmitter antennas and  $N$  receiver antennas, each of which consists of horizontal and vertical components. A point target is located in far field. The transmitting and receiving signals are both in the form of full polarization. Then the received baseband signal can be described as

$$\mathbf{r} = \mathbf{H} \boldsymbol{\alpha}_t + \mathbf{w}, \quad (1)$$

where  $\boldsymbol{\alpha}_t$  denotes a  $4MN \times 1$  vector composed of  $MN$  complex-valued scattering coefficients in the ground region of interest, and each ground scatterer is characterized by the four elements of scattering matrix in the conventional horizontal (H) and vertical (V) axes at the scatterer location. The matrix  $\mathbf{H} \in \mathbb{C}^{2MN \times 4MN}$  denotes transmitting signal model matrix with  $M$  transmitting antennas. The white noise  $\mathbf{w}$  is a  $2MN \times 1$ , zero-mean, Gaussian-distributed complex-valued random vector that is uncorrelated over space and time and with covariance matrix  $\mathbf{R}_{\mathbf{w}\mathbf{w}} = E[\mathbf{w}\mathbf{w}^H]$ .

The received signal  $\mathbf{r}$  is a column vector which describes the echoes of  $N$  receiver antennas. It is defined as

$$\mathbf{r} = \left[ (\mathbf{r}^{11})^T \cdots (\mathbf{r}^{1N})^T \cdots (\mathbf{r}^{M1})^T \cdots (\mathbf{r}^{MN})^T \right]^T, \quad (2)$$

where  $\mathbf{r}^{mn}$  denotes an echo vector with respect to the signal from the  $m^{\text{th}}$  transmitter antenna to the  $n^{\text{th}}$  receiver antenna. The target scattering coefficient vector  $\boldsymbol{\alpha}_t$  has the form of

$$\boldsymbol{\alpha}_t = \left[ (\boldsymbol{\alpha}_t^{11})^T \cdots (\boldsymbol{\alpha}_t^{1N})^T \cdots (\boldsymbol{\alpha}_t^{M1})^T \cdots (\boldsymbol{\alpha}_t^{MN})^T \right]^T \quad (3)$$

Each item of  $\boldsymbol{\alpha}_t$  is also a column vector, i.e.

$$\boldsymbol{\alpha}_t^{mn} = \left[ \alpha_{iHH}^{mn} \quad \alpha_{iHV}^{mn} \quad \alpha_{iVH}^{mn} \quad \alpha_{iVV}^{mn} \right]^T \quad (4)$$

which is the scattering data between the  $m^{\text{th}}$  transmitter antenna and  $n^{\text{th}}$  receiver antenna. With respect to the elements of polarization scattering matrix in Eq. 4, the first polarization-specific subscript represents the transmit component, and the second polarization-specific subscript represents the receive component.  $\boldsymbol{\alpha}_t$  describes the features of target, which depends on scatterer scenario.

At the receiver, each polarimetric antenna can capture two parts of receiving signals from two orthogonal directions, namely,

$$\mathbf{r}^{mn} = \left[ r_H^{mn} \quad r_V^{mn} \right]^T \quad (5)$$

and

$$r_H^{mn} = \alpha_{iHH}^{mn} h_H^m + \alpha_{iVH}^{mn} h_V^m + w_H^{mn} \quad (6a)$$

$$r_V^{mn} = \alpha_{iHV}^{mn} h_H^m + \alpha_{iVV}^{mn} h_V^m + w_V^{mn}, \quad (6b)$$

where variables  $h_H^m$  and  $h_V^m$  represent the complex-value scalar modulation signal components for H and V, transmitted by the  $m^{\text{th}}$  antennas.

Since polarization is the locus of the electric field vector as a function of time, it describes the direction of wave oscillation in the plane perpendicular to the direction of propagation. Additionally, the locus is commonly an ellipse which can be described by its polarization tilt angle  $\tau \in [-\frac{\pi}{2}, \frac{\pi}{2}]$  and its polarization ellipticity angle  $\varepsilon \in [-\frac{\pi}{4}, \frac{\pi}{4}]$ . Assume that all signals have normalized amplitudes, thus, the electric field vector can be defined as Eq. 7 using the geometric descriptors,

$$\mathbf{E}^m = \begin{bmatrix} h_H^m \\ h_V^m \end{bmatrix} = \mathbf{Q}(\tau^m) \mathbf{h}(\varepsilon^m), \quad (7)$$

where  $\mathbf{Q}(\tau^m) = \begin{bmatrix} \cos \tau^m & \sin \tau^m \\ -\sin \tau^m & \cos \tau^m \end{bmatrix}$  is a rotation matrix and  $\mathbf{h}(\varepsilon^m) = \begin{bmatrix} \cos \varepsilon^m \\ j \sin \varepsilon^m \end{bmatrix}$  is an ellipticity vector.  $\mathbf{E}^m$  can be reformulated into

$$\mathbf{H}^m = \begin{bmatrix} h_H^m & \mathbf{0} & h_V^m & \mathbf{0} \\ \mathbf{0} & h_H^m & \mathbf{0} & h_V^m \end{bmatrix} \quad (8)$$

So Eq. 5 can be rewritten as

$$\mathbf{r}^{mn} = \mathbf{H}^m \mathbf{a}_t^{mn} + \mathbf{w}^{mn} \quad (9)$$

Therefore, the transmitting signal matrix  $\mathbf{H}$  in Eq. 1, consisting of the variable of polarization waveform we want to design, has a partitioned structure of the form

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}^1 & \dots & \mathbf{0} & \dots & \mathbf{0} & \dots & \mathbf{0} \\ \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \dots & \mathbf{H}^1 & \dots & \mathbf{0} & \dots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \mathbf{0} & \dots & \mathbf{0} & \dots & \mathbf{H}^M & \dots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \dots & \mathbf{0} & \dots & \mathbf{0} & \dots & \mathbf{H}^M \end{bmatrix}, \quad (10)$$

where each  $\mathbf{0}$  represents a  $2 \times 4$  matrix of zeros, and submatrix  $\mathbf{H}^m$  is repeated  $N$  times along the main block diagonal.

It is noted that the polarization waveform parameters  $\tau$  and  $\varepsilon$  is incorporated in  $\mathbf{H}$ , so we aim at selecting the optimal value for every  $\tau^m$  and  $\varepsilon^m$  to make the polarimetric MIMO radar system perform best.

### 3. Shannon MI and MI Criterion

#### 3.1. Shannon Mutual Information

As we know, entropy can be used to describe the average uncertainty of a random variable, and mutual

information is a measure of the statistical correlation between two random variables. Denote  $MI(\mathbf{r}_1, \mathbf{r}_2)$  as the mutual information between two random vectors  $\mathbf{r}_1$  and  $\mathbf{r}_2$ . The relationship between entropy and mutual information can be written as

$$MI(\mathbf{r}_1, \mathbf{r}_2) = H(\mathbf{r}_1) - H(\mathbf{r}_1 | \mathbf{r}_2) = H(\mathbf{r}_2) - H(\mathbf{r}_2 | \mathbf{r}_1) \quad (11)$$

$$MI(\mathbf{r}_1, \mathbf{r}_2) = H(\mathbf{r}_1) + H(\mathbf{r}_2) - H(\mathbf{r}_1, \mathbf{r}_2), \quad (12)$$

where  $H(\mathbf{r}_i)$  denotes the entropy of the random vector  $\mathbf{r}_i$ ,  $H(\mathbf{r}_i | \mathbf{r}_j)$  denotes conditional entropy of  $\mathbf{r}_i$  conditioned on  $\mathbf{r}_j$ , and  $H(\mathbf{r}_i, \mathbf{r}_j)$  denotes the joint entropy of the random vectors  $\mathbf{r}_i$  and  $\mathbf{r}_j$ . For a Gaussian continuous distribution, these measures are defined respectively as [10]

$$H(\mathbf{r}_i) = - \int_{-\infty}^{\infty} \dots \int \ln[p(\mathbf{r}_i)] p(\mathbf{r}_i) d\mathbf{r}_i \quad (13)$$

$$H(\mathbf{r}_i, \mathbf{r}_j) = - \int_{-\infty}^{\infty} \dots \int \ln[p(\mathbf{r}_i | \mathbf{r}_j)] p(\mathbf{r}_i, \mathbf{r}_j) d\mathbf{r}_i d\mathbf{r}_j \quad (14)$$

$$H(\mathbf{r}_1, \mathbf{r}_2) = - \int_{-\infty}^{\infty} \dots \int \ln[p(\mathbf{r}_1, \mathbf{r}_2)] p(\mathbf{r}_1, \mathbf{r}_2) d\mathbf{r}_1 d\mathbf{r}_2 \quad (15)$$

In these definitions, natural logarithms are utilized, which is convenient when Gaussian distribution is involved.

#### 3.2. Mutual Information Criterion

A complex random vector  $\mathbf{s}$  is called (strictly) proper if  $C_{ss} = E[\mathbf{s}\mathbf{s}^T]$  [11]. Assume  $\mathbf{a}_t$  is also a zero-mean, Gaussian-distributed complex-valued vector with covariance matrix  $\mathbf{R}_u = E[\mathbf{a}_t \mathbf{a}_t^H]$ . The random vectors  $\mathbf{a}_t$  and  $\mathbf{w}$  are independent of each other, and each is assumed to be an analytic signal. Thus each is a proper random vector and admits a complex Gaussian probability density function (PDF), denoted as  $p[\cdot]$ , with the form of

$$p(\mathbf{a}_t) = \pi^{-M} |\mathbf{R}_u|^{-1} \exp[-\mathbf{a}_t^H \mathbf{R}_u^{-1} \mathbf{a}_t] \quad (16)$$

Based on above assumptions, it can be inferred from Eq. 1 that the vector  $\mathbf{r}$  is also a proper process and admits a complex Gaussian PDF. Accordingly the MI between  $\mathbf{r}_1$  and  $\mathbf{r}_2$  could attain a simple form, which is proved in [9].

Let  $\mathbf{r}_i$  denote the received signal vector for the waveform  $\{\mathbf{H}_i(\boldsymbol{\tau}_i, \boldsymbol{\varepsilon}_i)\}$  at this coherent processing interval, and  $\mathbf{r}_j$  denote the received signal vector for the waveform  $\{\mathbf{H}_j(\boldsymbol{\tau}_j, \boldsymbol{\varepsilon}_j)\}$  to be transmitted at the

next internal. The space-time whitening transformations of vectors  $\mathbf{r}_1$  and  $\mathbf{r}_i$  are defined as

$$\mathbf{X} = \mathbf{R}_{11}^{-1/2} \mathbf{r}_1 \quad (17)$$

$$\mathbf{Z} = \mathbf{R}_{ii}^{-1/2} \mathbf{r}_i \quad (18)$$

So the cross-covariance matrix of these two random vectors is

$$E[\mathbf{Z}\mathbf{X}] = \mathbf{R}_{ZX} = \mathbf{R}_{ii}^{-1/2} \mathbf{R}_{i1} \mathbf{R}_{11}^{-1/2} \quad (19)$$

with

$$\mathbf{R}_{11} = E[\mathbf{r}_1 \mathbf{r}_1^H] = \mathbf{H}_1 \mathbf{R}_u \mathbf{H}_1^H + \mathbf{R}_{ww} \quad (20)$$

$$\mathbf{R}_{i1} = E[\mathbf{r}_i \mathbf{r}_1^H] = \mathbf{H}_i \mathbf{R}_u \mathbf{H}_1^H \quad (21)$$

$$\mathbf{R}_{ii} = E[\mathbf{r}_i \mathbf{r}_i^H] = \mathbf{H}_i \mathbf{R}_u \mathbf{H}_i^H + \mathbf{R}_{ww}, \quad (22)$$

where  $\mathbf{R}_{11}$ ,  $\mathbf{R}_{ii}$  and  $\mathbf{R}_{i1}$  are the auto-covariance and cross-covariance matrices of echoes, respectively.

Let  $d_i \in [0,1]$  denote the 1<sup>st</sup> singular value of the cross-covariance matrix  $\mathbf{R}_{ZX}$ , then mutual information between proper random variables  $\mathbf{r}_1$  and  $\mathbf{r}_i$  has the form of

$$MI(\mathbf{r}_1, \mathbf{r}_i) = - \sum_{l=1}^{2MN} \ln[1 - d_l^2] \quad (23)$$

We hope to select desirable waveform to be transmitted at next time from the alternative waveform candidates in order to make the radar capture more new information about the target. It is to say that the waveform to be selected guarantees the return signals different from the moment returns as possible, namely, to minimize the mutual information between  $\mathbf{r}_1$  and  $\mathbf{r}_i$ . Hence the mutual information criterion can be expressed as

$$MIC = \min_{\mathbf{H}_i(\boldsymbol{\tau}_i, \boldsymbol{\varepsilon}_i)} \{MI(\mathbf{r}_1, \mathbf{r}_i)\}, \quad (24)$$

where  $\boldsymbol{\tau}_i$  and  $\boldsymbol{\varepsilon}_i$  are the parameter vectors of waveform  $\mathbf{H}_i$  to be transmitted from  $M$  antennas at next time.

#### 4. Waveform Selection Process via Alternating Projection

To select waveform effectively, we present an iterative optimization algorithm in this section that features an alternating projection approach within  $M$  subspaces. For the convergence properties of

alternating projection, we can refer to literature [12]. Our algorithm can be stated as Table 1.

**Table 1.** Alternating projection algorithm.

<ul style="list-style-type: none"> <li>• Initial <math>\boldsymbol{\tau}_i</math> and <math>\boldsymbol{\varepsilon}_i</math>, let <math>\tau_0^m = \tau_0</math>, <math>\varepsilon_0^m = \varepsilon_0</math>, <math>m = 1, \dots, M</math>.</li> <li>• <math>i = 0</math>;</li> <li>• <b>while</b> end condition is not satisfied <b>do</b></li> <li>• <b>for</b> <math>m=1:M</math></li> <li>• Let <math>\tau_i^l = \tau_i^l</math>, <math>\varepsilon_i^l = \varepsilon_i^l</math>, <math>l = 1, \dots, m-1</math>, and <math>\tau_i^k = \tau_{i-1}^k</math>, <math>\varepsilon_i^k = \varepsilon_{i-1}^k</math>, <math>k = m+1, \dots, M</math>. Find <math>\tau_i^m \in [-\frac{\pi}{2}, \frac{\pi}{2}]</math>, <math>\varepsilon_i^m \in [-\frac{\pi}{4}, \frac{\pi}{4}]</math> to minimize MI;</li> <li>• <b>end for</b></li> <li>• <math>i = i + 1</math>;</li> <li>• <b>end while</b></li> </ul>
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The main idea of this iterative algorithm is to find the optimal parameters ( $\tau^m, \varepsilon^m$ ) for the  $m$ th antenna in the finite candidate set, while parameters of other  $M-1$  antennas maintain their latest values unchanged. Once the parameters of one antenna have changed, the optimal values of parameters of the others may change as well. Thus, we optimize parameters of each dimension of antenna alternately until the end condition is satisfied.

#### 5. Simulation and Analysis

In these section, we use the example of the model with  $M = 2$  transmitter antennas and  $N = 2$  receiver antennas to illustrate the waveform design solution derived.

Assume that  $\boldsymbol{\alpha}_i$  is space-time uncorrelated, namely, the target scattering coefficients between different antennas in different times is independent and identically-distributed. Thus, the covariance matrix  $\mathbf{R}_u$  is block diagonal and in our example, it can be described as

$$\mathbf{R}_u = \begin{bmatrix} \mathbf{R}^{11} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}^{22} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{R}^{33} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{R}^{44} \end{bmatrix}, \quad (25)$$

where  $\mathbf{R}^{ii} = [\boldsymbol{\alpha}^{ii}(\boldsymbol{\alpha}^{ii})^H]$  is the covariance matrix between the  $i$ th transmitting antenna and the  $i$ th receiving antenna. In realistic scene,  $\mathbf{R}_u$  can be measured, however, we can not obtain the value of  $\boldsymbol{\alpha}^{ii}$  since it is a random statistical variable. Here we directly give the value of  $\boldsymbol{\alpha}^{ii}$  to obtain  $\mathbf{R}_u$  for notational simplicity, they are given as

$$\begin{aligned} \mathbf{\alpha}^{11} &= [0.1 + j0.2 \quad 0.3 + j0.7 \quad 0.6 + j0.4 \quad 0.3 + j0.1]^T \\ \mathbf{\alpha}^{22} &= [0.01 + j0.02 \quad 0.3 + j0.7 \quad 0.6 + j0.4 \quad 0.3 + j0.1]^T \\ \mathbf{\alpha}^{33} &= [0.5 - j0.1 \quad 0.4 + j0.3 \quad 0.6 + j0.4 \quad 0.2 + j0.4]^T \\ \mathbf{\alpha}^{44} &= [0.1 + j0.7 \quad 0.4 - j0.5 \quad 0.2 + j0.4 \quad 0.7 + j0.1]^T \end{aligned}$$

Assume that the transmitting signals are completely polarized, so they are the functions of polarization tilt angle  $\tau$  and polarization ellipticity angle  $\varepsilon$ . From Eq. 24, we know that there are 4 variables to be selected in our simulation, namely,  $\tau_i^1$ ,  $\varepsilon_i^1$ ,  $\tau_i^2$ ,  $\varepsilon_i^2$ . They are the parameters of transmitting signals from the first and the second transmitter antennas, respectively. Here we assume that  $\tau_i^i$  and  $\varepsilon_i^i$  are respectively chosen from  $\left\{ \tau_i^i \left| -\frac{\pi}{2}, \left(-\frac{\pi}{2} + 1 \times \frac{\pi}{8}\right), \dots, \left(-\frac{\pi}{2} + 8 \times \frac{\pi}{8}\right) \right\}$  and  $\left\{ \varepsilon_i^i \left| -\frac{\pi}{4}, \left(-\frac{\pi}{4} + 1 \times \frac{\pi}{16}\right), \dots, \left(-\frac{\pi}{4} + 8 \times \frac{\pi}{16}\right) \right\}$ .

It is clear that there are totally  $9^4$  kinds of waveform in our example. If the number of transmitter antennas is more than 2, it will be much greater. How to select the waveform effectively becomes a problem. Here an iterative algorithm via alternating projection is employed to solve it.

Assume the polarization parameters of the transmitted waveform at current interval are  $\tau_i^1 = \tau_i^2 = -\frac{\pi}{2}$  and  $\varepsilon_i^1 = \varepsilon_i^2 = -\frac{\pi}{4}$ . We use alternating projection approach in the two subspaces  $(\tau_i^1, \varepsilon_i^1)$  and  $(\tau_i^2, \varepsilon_i^2)$  to select the optimal parameters satisfying MIC. So the problem of waveform design can be break into two sub-problems. First, choose  $\tau_i^1$  and  $\varepsilon_i^1$  in the subspace  $(\tau_i^1, \varepsilon_i^1)$  to minimize  $MI(\mathbf{r}_1, \mathbf{r}_i)$  with the latest  $\tau_i^2$  and  $\varepsilon_i^2$  involved. Next, using the new values of  $\tau_i^1$  and  $\varepsilon_i^1$ , renew  $\tau_i^2$  and  $\varepsilon_i^2$  in subspace  $(\varepsilon_i^2, \tau_i^2)$  to minimize  $MI(\mathbf{r}_1, \mathbf{r}_i)$ . Alternately do the two steps above until the values of  $\tau_i^1, \varepsilon_i^1, \tau_i^2, \varepsilon_i^2$  are all unchanged in two adjacent iterations. In this case we obtain the optimal waveform. Additionally, the initial values can be arbitrary, since the results of this iterative algorithm are irrelevant to these initial data.

Based on the data given above, the simulation results indicate that the minimal MI value is 7.8624 with  $\tau_i^1 = 0.7854$ ,  $\varepsilon_i^1 = -0.3927$ ,  $\tau_i^2 = 0.7854$ ,  $\varepsilon_i^2 = 0.3927$ . However, the value of MI may fluctuate a little each time for the random noise, even under the same waveform parameters. Our findings are illustrated in Fig. 1 and Fig. 2. Note that Fig. 1 presents the fact when  $\varepsilon_i^1 = 0.7854$ ,  $\tau_i^1 = -0.3927$ , the minimal MI value is located at  $\varepsilon_i^2 = 0.7854$ ,  $\tau_i^2 = 0.3927$ . Furthermore, from Fig. 2 it is also

verified that the best results of the four parameters still have the same values, with  $\varepsilon_i^2 = 0.7854$ ,  $\tau_i^2 = 0.3927$  unchanged. Using alternating projection approach, the amount of calculation is reduced to  $n \times 9^2$ , where  $n$  is the number of alternating iterations. Obviously, it is much less than  $9^4$ .

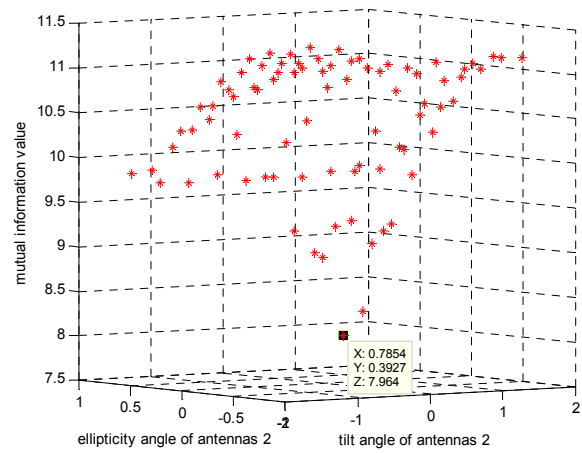


Fig. 1. The selection of  $\tau_i^2$  and  $\varepsilon_i^2$  when  $\tau_i^1$  and  $\varepsilon_i^1$  is finally optimized.

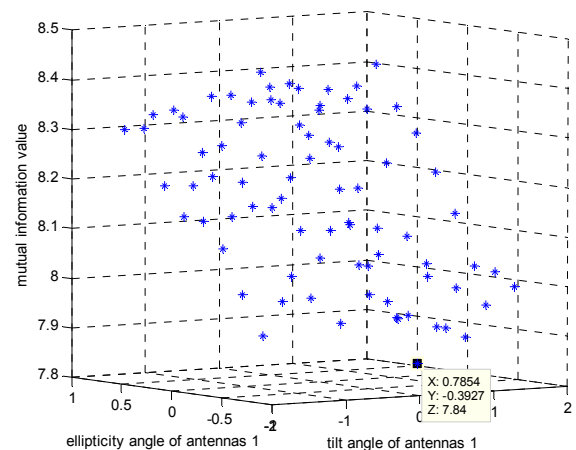


Fig. 2. The selection of  $\tau_i^1$  and  $\varepsilon_i^1$  when  $\tau_i^2$  and  $\varepsilon_i^2$  is finally optimized.

## 6. Conclusions

In this paper, a solution to polarization waveform selection for polarimetric MIMO radar is presented. The criterion is based on the mutual information between the radar return signals at two adjacent instants of time. It can attain a simple form for the proper Gaussian case. The optimal transmit polarization waveform is effectively designed according to the second-order statistics of the target. Further, an effective waveform optimization method based on alternating projection is proposed. It is an iterative algorithm which has greatly reduced the

calculation amount. The numerical examples given above have illustrated the effectiveness of the proposed algorithm. The optimization method can be also used in wireless sensor networks [13, 14].

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